Design of Digital Filters Satisfying Prescribed Specifications

Introduction

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State-of-the-art methods for the design of digital filters will be presented placing special emphasis on the design of filters that would satisfy prescribed specifications.

Example

In an application, an equiripple bandstop digital filter is required which should satisfy the following specifications:

- Maximum passband ripple A_p: 0.5 dB
- Minimum stopband attenuation A_a: 50.0 dB
- Lower passband edge ω_{p1}: 0.8 rad/s
- Upper passband edge ω_{p2}: 2.2 rad/s
- Lower stopband edge ω_{a1}: 1.2 rad/s
- Upper stopband edge ω_{a2}: 1.8 rad/s
- Sampling frequency ω_s: 2π rad/s

Design the lowest-order filter that will satisfy the specifications.

The tutorial is organized in four parts as follows:

- *Part 1*: FIR (nonrecursive) filters using the window method.
- *Part 2*: FIR filters using the weighted-Chebyshev method.
- *Part 3*: IIR (recursive) filters based on the bilinear transformation method.
- *Part 4*: IIR filters based on the optimization approach.

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- Prescribed specifications can be achieved by using a *design technique proposed by Kaiser*.

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- The design entails a relatively insignificant amount of computation.

Disadvantages:

- Designs are suboptimal, i.e., the filter order needed to satisfy a given set of prescribed specifications is not the lowest, i.e., other methods are available that yield a lower-order filter (e.g., the weighted-Chebyshev method).

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- A higher-order filter means more computations per sample, which implies that these filters are slower and less efficient in real-time applications.

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- Prescribed specifications can be achieved by using a prediction technique due to Herrmann, Rabiner, and Chan.

Weighted-Chebyshev Method *Cont'd*

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- Designs are optimal, i.e., the required filter order for a set of prescribed specifications is the lowest that can be achieved.
- Minimum filter order implies a more efficient and faster filter for real-time applications.
- The method is very flexible – it can be used to design filters, differentiators, Hilbert transformers, etc.
- The solutions achieved are equiripple.

Disadvantages:

- The design requires a very large amount of computation.

Weighted-Chebyshev Method *Cont'd*

Disadvantages:

- The design requires a very large amount of computation.
- Not suitable for applications where the design has to be carried out in real- or quasi-real time, for example, in programmable or adaptable filters.

IIR Filters

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- The optimization approach.

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- A discrete-time transfer function is obtained by applying the bilinear transformation to the continuous-time transfer function.
- Prescribed specifications can be achieved by using a design method proposed by Antoniou some years ago.

  This method essentially involves deducing the parameters of the transformations involved as well as the minimum filter order from the required specifications.

Advantages

- The bilinear transformation method leads to a complete description of the transfer function *in closed form* either in terms of its zeros and poles or its coefficients.

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- As a consequence, the method requires very little computation and leads to very precise *optimal* designs.
- It can be used to design lowpass (LP), highpass (HP), bandpass (BP), and bandstop (BS) filters of the Butterworth, Chebyshev, inverse-Chebyshev, elliptic types.
- The filters designed are *always stable*.

Disadvantages

- Its main disadvantage is that it is applicable only for the design of *filters with piecewise-constant amplitude responses,* i.e., filters whose passband and stopband gains are constant and zero, respectively, to within prescribed tolerances.

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- A norm of the error function is constructed and is then minimized with respect to the transfer-function coefficients using one of many efficient optimization algorithms.
- As the value of the norm approaches zero, the resulting amplitude and/or phase response approaches the desired amplitude and/or phase response.

Advantages

- The optimization approach is very flexible in that it can be used to design filters with arbitrary amplitude and/or phase responses.

Optimization Approach *Cont'd*

Advantages

- The optimization approach is very flexible in that it can be used to design filters with arbitrary amplitude and/or phase responses.
- A great variety of optimization algorithms can be used such as least-squares, minimax, Newton, quasi-Newton, even genetic algorithms.

Disadvantages

- Like the weight-Chebyshev method for FIR filters, the optimization approach for IIR filters usually involves a large amount of computation.

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- The use of unconstrained optimization often leads to unstable designs.

  However, techniques are available that can be used to convert an unstable into a stable design.
- For filters with piecewise constant amplitude responses, the bilinear transformation method is preferred.

Disadvantages *Cont'd*

- The problem of instability can also be overcome by using constrained optimization.

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  However, the amount of computation required to complete a design is increased considerably, sometimes by a factor of 10 or more.

- For FIR filters, there is a technique for predicting the minimum filter order to achieve any prescribed set of specifications.

  Unfortunately, no such technique is available for IIR filters designed by optimization.

  Typically, the problem is solved through a cut-and-try approach.

This slide concludes the Introduction.
Thank you for your attention.