New Constrained Affine-Projection Adaptive-Filtering Algorithm

Md. Zulfiquar Ali Bhotto and Andreas Antoniou

Department of Electrical and Computer Engineering
University of Victoria, Victoria, BC, Canada
Emails: zbhotto@ece.uvic.ca, aantoniou@ieee.org

ISCAS 19-23 May 2013

Outline

- Objectives

Outline

- Objectives
- Two versions of a new constrained affine-projection (CAP) algorithm, PCAP-I and PCAP-II, are proposed as follows:
  - derivation of PCAP-I algorithm
  - derivation of the PCAP-II algorithm
  - discussion on proposed and conventional CAP algorithms.

Outline

- Objectives
- Two versions of a new constrained affine-projection (CAP) algorithm, PCAP-I and PCAP-II, are proposed as follows:
  - derivation of PCAP-I algorithm
  - derivation of the PCAP-II algorithm
  - discussion on proposed and conventional CAP algorithms.
- Simulation results
  - system identification application
  - interference-suppression application for direct-sequence code-division multiple access (DS-CDMA) communication systems

Outline

- Objectives
- Two versions of a new constrained affine-projection (CAP) algorithm, PCAP-I and PCAP-II, are proposed as follows:
  - derivation of PCAP-I algorithm
  - derivation of the PCAP-II algorithm
  - discussion on proposed and conventional CAP algorithms.
- Simulation results
  - system identification application
  - interference-suppression application for direct-sequence code-division multiple access (DS-CDMA) communication systems
- Conclusions

Objectives

- Develop adaptive-filtering algorithms that can be used for applications where the weight vector must satisfy certain constraints.

Objectives

- Develop adaptive-filtering algorithms that can be used for applications where the weight vector must satisfy certain constraints.
- The developed algorithms should

Objectives

- Develop adaptive-filtering algorithms that can be used for applications where the weight vector must satisfy certain constraints.
- The developed algorithms should
  - yield an unbiased solution

Objectives

- Develop adaptive-filtering algorithms that can be used for applications where the weight vector must satisfy certain constraints.
- The developed algorithms should
  - yield an unbiased solution
  - require a reduced computational effort

Objectives

- Develop adaptive-filtering algorithms that can be used for applications where the weight vector must satisfy certain constraints.
- The developed algorithms should
  - yield an unbiased solution
  - require a reduced computational effort
  - yield a reduced steady-state misalignment

Objectives

- Develop adaptive-filtering algorithms that can be used for applications where the weight vector must satisfy certain constraints.
- The developed algorithms should
  - yield an unbiased solution
  - require a reduced computational effort
  - yield a reduced steady-state misalignment
  - require fewer iterations to converge

Objectives

- Develop adaptive-filtering algorithms that can be used for applications where the weight vector must satisfy certain constraints.
- The developed algorithms should
  - yield an unbiased solution
  - require a reduced computational effort
  - yield a reduced steady-state misalignment
  - require fewer iterations to converge

  compared to the constrained normalized least-mean square (CNLMS), known CAP, and set-membership CAP (CSMAP) algorithms.

New CAP Algorithm

- The two versions of the CAP algorithm are obtained by solving the minimization problem

$$\underset{\mathbf{w}}{\text{minimize}}\ J(\mathbf{w}) = 0.5\|\mathbf{X}_k^T\mathbf{w} - \mathbf{d}_k\|^2$$

New CAP Algorithm

- The two versions of the CAP algorithm are obtained by solving the minimization problem

$$\underset{\mathbf{w}}{\text{minimize}} \;\; J(\mathbf{w}) = 0.5\|\mathbf{X}_k^T\mathbf{w} - \mathbf{d}_k\|^2$$

subject to the constraint

$$\mathbf{C}\mathbf{w} = \mathbf{f}$$

New CAP Algorithm

- The two versions of the CAP algorithm are obtained by solving the minimization problem

$$\underset{\mathbf{w}}{\text{minimize}} \;\; J(\mathbf{w}) = 0.5\|\mathbf{X}_k^T \mathbf{w} - \mathbf{d}_k\|^2$$

subject to the constraint

$$\mathbf{C}\mathbf{w} = \mathbf{f}$$

where $\mathbf{d}_k \in \mathcal{R}^{l\times 1}$ is the desired signal vector, $\mathbf{X}_k \in \mathcal{R}^{m\times l}$ is the input signal matrix, $\mathbf{C} \in \mathcal{R}^{p\times m}$ is a constraint matrix, and $\mathbf{f} \in \mathcal{R}^{p\times 1}$ is a constraint vector.

New CAP Algorithm

- The two versions of the CAP algorithm are obtained by solving the minimization problem

$$\underset{\mathbf{w}}{\text{minimize}} \;\; J(\mathbf{w}) = 0.5\|\mathbf{X}_k^T\mathbf{w} - \mathbf{d}_k\|^2$$

  subject to the constraint

$$\mathbf{Cw} = \mathbf{f}$$

  where $\mathbf{d}_k \in \mathcal{R}^{l\times 1}$ is the desired signal vector, $\mathbf{X}_k \in \mathcal{R}^{m\times l}$ is the input signal matrix, $\mathbf{C} \in \mathcal{R}^{p\times m}$ is a constraint matrix, and $\mathbf{f} \in \mathcal{R}^{p\times 1}$ is a constraint vector.

- The gradient of $J(\mathbf{w})$ at points $\mathbf{w}_k$ and $\mathbf{w}_{k-1}$ satisfies the inequality

$$\|\nabla J(\mathbf{w}_k) - \nabla J(\mathbf{w}_{k-1})\|_2 \leq \lambda_{k,max}(\mathbf{X}_k\mathbf{X}_k^T)\|\mathbf{w}_k - \mathbf{w}_{k-1}\|_2$$

New CAP Algorithm

- The two versions of the CAP algorithm are obtained by solving the minimization problem

$$\underset{\mathbf{w}}{\text{minimize}} \; J(\mathbf{w}) = 0.5\|\mathbf{X}_k^T\mathbf{w} - \mathbf{d}_k\|^2$$

subject to the constraint

$$\mathbf{C}\mathbf{w} = \mathbf{f}$$

where $\mathbf{d}_k \in \mathcal{R}^{l \times 1}$ is the desired signal vector, $\mathbf{X}_k \in \mathcal{R}^{m \times l}$ is the input signal matrix, $\mathbf{C} \in \mathcal{R}^{p \times m}$ is a constraint matrix, and $\mathbf{f} \in \mathcal{R}^{p \times 1}$ is a constraint vector.

- The gradient of $J(\mathbf{w})$ at points $\mathbf{w}_k$ and $\mathbf{w}_{k-1}$ satisfies the inequality

$$\|\nabla J(\mathbf{w}_k) - \nabla J(\mathbf{w}_{k-1})\|_2 \le \lambda_{k,max}(\mathbf{X}_k\mathbf{X}_k^T)\|\mathbf{w}_k - \mathbf{w}_{k-1}\|_2$$

where $\lambda_{k,max}$ is the maximum eigenvalue of $\mathbf{X}_k\mathbf{X}_k^T$.

- We can, therefore, approximate $J(\mathbf{w}) = 0.5\|\mathbf{X}_k^T\mathbf{w} - \mathbf{d}_k\|^2$ at $\mathbf{w}_k$ as

$$\begin{aligned} J(\mathbf{w}) \approx \hat{J}(\mathbf{w}_k) &= J(\mathbf{w}_{k-1}) + (\mathbf{w}_k - \mathbf{w}_{k-1})^T \nabla J(\mathbf{w}_{k-1}) \\ &\quad + \frac{1}{2\mu_k}\|\mathbf{w}_k - \mathbf{w}_{k-1}\|_2^2 \end{aligned}$$

where $J(\mathbf{w}) < \hat{J}(\mathbf{w}_k)$ and μ_k is called the *Lipschitz constant*.

New CAP Algorithm, Cont'd...

- We can, therefore, approximate $J(\mathbf{w}) = 0.5\|\mathbf{X}_k^T\mathbf{w} - \mathbf{d}_k\|^2$ at $\mathbf{w}_k$ as

$$
\begin{aligned}
J(\mathbf{w}) \approx \hat{J}(\mathbf{w}_k) &= J(\mathbf{w}_{k-1}) + (\mathbf{w}_k - \mathbf{w}_{k-1})^T \nabla J(\mathbf{w}_{k-1}) \\
&\quad + \frac{1}{2\mu_k}\|\mathbf{w}_k - \mathbf{w}_{k-1}\|_2^2
\end{aligned}
$$

  where $J(\mathbf{w}) < \hat{J}(\mathbf{w}_k)$ and μ_k is called the *Lipschitz constant*.

- The PCAP-I algorithm can now be obtained by solving the minimization problem

$$
\underset{\mathbf{w}_k}{\text{minimize}} \; J(\mathbf{w}_{k-1}) + (\mathbf{w}_k - \mathbf{w}_{k-1})^T \nabla J(\mathbf{w}_{k-1}) + \frac{1}{2\mu_k}\|\mathbf{w}_k - \mathbf{w}_{k-1}\|_2^2
$$

New CAP Algorithm, Cont'd...

- We can, therefore, approximate $J(\mathbf{w}) = 0.5\|\mathbf{X}_k^T\mathbf{w} - \mathbf{d}_k\|^2$ at $\mathbf{w}_k$ as

$$J(\mathbf{w}) \approx \hat{J}(\mathbf{w}_k) = J(\mathbf{w}_{k-1}) + (\mathbf{w}_k - \mathbf{w}_{k-1})^T \nabla J(\mathbf{w}_{k-1}) + \frac{1}{2\mu_k}\|\mathbf{w}_k - \mathbf{w}_{k-1}\|_2^2$$

  where $J(\mathbf{w}) < \hat{J}(\mathbf{w}_k)$ and μ_k is called the *Lipschitz constant*.

- The PCAP-I algorithm can now be obtained by solving the minimization problem

$$\underset{\mathbf{w}_k}{\text{minimize}} \; J(\mathbf{w}_{k-1}) + (\mathbf{w}_k - \mathbf{w}_{k-1})^T \nabla J(\mathbf{w}_{k-1}) + \frac{1}{2\mu_k}\|\mathbf{w}_k - \mathbf{w}_{k-1}\|_2^2$$

  subject to the constraint

$$\mathbf{C}\mathbf{w}_k = \mathbf{f}$$

- The solution of the problem at hand can be obtained by using the *Lagrange* multiplier method as

$$\mathbf{w}_k = \mathbf{Z}\left[\mathbf{w}_{k-1} + \mu_k \mathbf{X}_k \mathbf{e}_k\right] + \mathbf{t}$$

- The solution of the problem at hand can be obtained by using the *Lagrange* multiplier method as

$$\mathbf{w}_k = \mathbf{Z}\left[\mathbf{w}_{k-1} + \mu_k \mathbf{X}_k \mathbf{e}_k\right] + \mathbf{t}$$

where $\mathbf{Z} \in \mathcal{R}^{m \times m}$ is a matrix given by

$$\mathbf{Z} = \mathbf{I} - \mathbf{C}^T(\mathbf{C}\mathbf{C}^T)^{-1}\mathbf{C}$$

and $\mathbf{t} \in \mathcal{R}^{m \times 1}$ is a vector given by

$$\mathbf{t} = \mathbf{C}^T(\mathbf{C}\mathbf{C}^T)^{-1}\mathbf{f}$$

New PCAP-I Algorithm, Cont'd...

- For the PCAP-I algorithm, we solve the minimization problem

$$\underset{\mu_k}{\text{minimize}}\ F(\mu_k) = 0.5\|\mathbf{X}_k^T\mathbf{w}_k - \mathbf{d}_k\|^2$$

to obtain

$$\mu_k = \frac{\mathbf{e}_k^T\mathbf{X}_k^T\mathbf{Z}\mathbf{X}_k(\mathbf{d}_k - \mathbf{X}_k^T[\mathbf{Z}\mathbf{w}_{k-1} + \mathbf{t}])}{\mathbf{e}_k^T\mathbf{X}_k^T\mathbf{Z}\mathbf{X}_k\mathbf{X}_k^T\mathbf{Z}\mathbf{X}_k\mathbf{e}_k}$$

which can be used in the update formula

$$\mathbf{w}_k = \mathbf{Z}\left[\mathbf{w}_{k-1} + \mu_k\mathbf{X}_k\mathbf{e}_k\right] + \mathbf{t}$$

New PCAP-I Algorithm, Cont'd...

- For the PCAP-I algorithm, we solve the minimization problem

$$\underset{\mu_k}{\text{minimize}}\ F(\mu_k) = 0.5\|\mathbf{X}_k^T \mathbf{w}_k - \mathbf{d}_k\|^2$$

to obtain

$$\mu_k = \frac{\mathbf{e}_k^T \mathbf{X}_k^T \mathbf{Z} \mathbf{X}_k (\mathbf{d}_k - \mathbf{X}_k^T [\mathbf{Z}\mathbf{w}_{k-1} + \mathbf{t}])}{\mathbf{e}_k^T \mathbf{X}_k^T \mathbf{Z} \mathbf{X}_k \mathbf{X}_k^T \mathbf{Z} \mathbf{X}_k \mathbf{e}_k}$$

which can be used in the update formula

$$\mathbf{w}_k = \mathbf{Z}\left[\mathbf{w}_{k-1} + \mu_k \mathbf{X}_k \mathbf{e}_k\right] + \mathbf{t}$$

- With the initialization $\mathbf{w}_0 = \mathbf{t}$ the update formula of the PCAP-I algorithm simplifies to

$$\mathbf{w}_k = \mathbf{w}_{k-1} + \mu_k \mathbf{Z} \mathbf{X}_k \mathbf{e}_k$$

New PCAP-I Algorithm, Cont'd...

- For the PCAP-I algorithm, we solve the minimization problem

$$\underset{\mu_k}{\text{minimize}} \; F(\mu_k) = 0.5\|\mathbf{X}_k^T\mathbf{w}_k - \mathbf{d}_k\|^2$$

to obtain

$$\mu_k = \frac{\mathbf{e}_k^T\mathbf{X}_k^T\mathbf{Z}\mathbf{X}_k(\mathbf{d}_k - \mathbf{X}_k^T[\mathbf{Z}\mathbf{w}_{k-1} + \mathbf{t}])}{\mathbf{e}_k^T\mathbf{X}_k^T\mathbf{Z}\mathbf{X}_k\mathbf{X}_k^T\mathbf{Z}\mathbf{X}_k\mathbf{e}_k}$$

which can be used in the update formula

$$\mathbf{w}_k = \mathbf{Z}\left[\mathbf{w}_{k-1} + \mu_k\mathbf{X}_k\mathbf{e}_k\right] + \mathbf{t}$$

- With the initialization $\mathbf{w}_0 = \mathbf{t}$ the update formula of the PCAP-I algorithm simplifies to

$$\mathbf{w}_k = \mathbf{w}_{k-1} + \mu_k\mathbf{Z}\mathbf{X}_k\mathbf{e}_k$$

where

$$\mu_k = \frac{\mathbf{e}_k^T\mathbf{X}_k^T\mathbf{Z}\mathbf{X}_k\mathbf{e}_k}{\mathbf{e}_k^T\mathbf{X}_k^T\mathbf{Z}\mathbf{X}_k\mathbf{X}_k^T\mathbf{Z}\mathbf{X}_k\mathbf{e}_k}$$

- The solution of the system of equations $\mathbf{Cw} = \mathbf{f}$ can be obtained as

$$\mathbf{w} = \mathbf{V}_r \boldsymbol{\omega} + \mathbf{C}^{+}\mathbf{f}$$

where $\mathbf{C}^{+}$ denotes the *Moore-Penrose pseudo-inverse* of $\mathbf{C}$, $\mathbf{V}_r \in \mathcal{R}^{m \times r}$ is a matrix consisting of the last $m = r - p$ columns of $\mathbf{V}$ which is obtained by using using the singular-value decomposition of $\mathbf{C}$, and $\boldsymbol{\omega} \in \mathcal{R}^{r \times 1}$ is a vector.

New PCAP-I Algorithm, Cont'd...

- The solution of the system of equations $\mathbf{Cw} = \mathbf{f}$ can be obtained as

$$\mathbf{w} = \mathbf{V}_r \omega + \mathbf{C}^+ \mathbf{f}$$

where $\mathbf{C}^+$ denotes the *Moore-Penrose pseudo-inverse* of $\mathbf{C}$, $\mathbf{V}_r \in \mathcal{R}^{m \times r}$ is a matrix consisting of the last $m = r - p$ columns of $\mathbf{V}$ which is obtained by using using the singular-value decomposition of $\mathbf{C}$, and $\boldsymbol{\omega} \in \mathcal{R}^{r \times 1}$ is a vector.

- With this solution, the optimization problem

$$\underset{\mathbf{w}}{\text{minimize}} \;\; J(\mathbf{w}) = 0.5\|\mathbf{X}_k^T \mathbf{w} - \mathbf{d}_k\|^2$$

subject to the constraint

$$\mathbf{Cw} = \mathbf{f}$$

New PCAP-I Algorithm, Cont'd...

- The solution of the system of equations $\mathbf{C}\mathbf{w} = \mathbf{f}$ can be obtained as

$$\mathbf{w} = \mathbf{V}_r\omega + \mathbf{C}^+\mathbf{f}$$

where $\mathbf{C}^+$ denotes the *Moore-Penrose pseudo-inverse* of $\mathbf{C}$, $\mathbf{V}_r \in \mathcal{R}^{m\times r}$ is a matrix consisting of the last $m = r - p$ columns of $\mathbf{V}$ which is obtained by using using the singular-value decomposition of $\mathbf{C}$, and $\boldsymbol{\omega} \in \mathcal{R}^{r\times 1}$ is a vector.

- With this solution, the optimization problem

$$\underset{\mathbf{w}}{\text{minimize}} \;\; J(\mathbf{w}) = 0.5\|\mathbf{X}_k^T\mathbf{w} - \mathbf{d}_k\|^2$$

subject to the constraint

$$\mathbf{C}\mathbf{w} = \mathbf{f}$$

can be expressed as

$$\underset{\boldsymbol{\omega}}{\text{minimize}} \;\; J(\omega) = 0.5\|\mathbf{X}_k^T\mathbf{V}_r\omega + \mathbf{X}_k^T\mathbf{C}^+\mathbf{f} - \mathbf{d}_k\|^2$$

New PCAP-II Algorithm

- The PCAP-II algorithm is obtained by solving the minimization problem

$$\underset{\boldsymbol{\omega}}{\text{minimize}} \;\; J(\boldsymbol{\omega}) = 0.5\|\mathbf{X}_k^T\mathbf{V}_r\boldsymbol{\omega} + \mathbf{X}_k^T\mathbf{C}^+\mathbf{f} - \mathbf{d}_k\|^2$$

by using the same steps as for the PCAP-I algorithm.

New PCAP-II Algorithm

- The PCAP-II algorithm is obtained by solving the minimization problem

$$\underset{\boldsymbol{\omega}}{\text{minimize}} \;\; J(\omega) = 0.5\|\mathbf{X}_k^T\mathbf{V}_r\omega + \mathbf{X}_k^T\mathbf{C}^+\mathbf{f} - \mathbf{d}_k\|^2$$

  by using the same steps as for the PCAP-I algorithm.

- The update formula of the PCAP-II algorithm becomes

$$\boldsymbol{\omega}_k = \boldsymbol{\omega}_{k-1} + \mu_k\mathbf{V}_r^T\mathbf{X}_k\mathbf{e}_k$$

  where

$$\mu_k = \frac{\mathbf{e}_k^T\mathbf{X}_k^T\mathbf{V}_r\mathbf{V}_r^T\mathbf{X}_k\mathbf{e}_k}{\mathbf{e}_k^T\mathbf{X}_k^T\mathbf{V}_r\mathbf{V}_r^T\mathbf{X}_k\mathbf{X}_k^T\mathbf{V}_r\mathbf{V}_r^T\mathbf{X}_k\mathbf{e}_k}$$

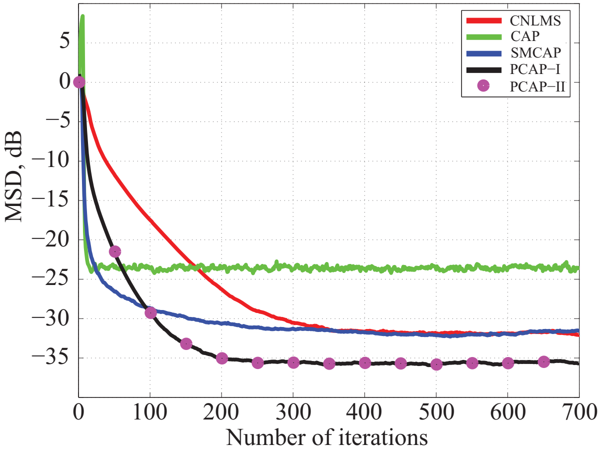
Discussion

- The PCAP-I and PCAP-II algorithms do not require the inverse of $\mathbf{X}_k^T \mathbf{Z} \mathbf{X}_k$ and hence they require less computation than the conventional CAP algorithm.

Discussion

- The PCAP-I and PCAP-II algorithms do not require the inverse of $\mathbf{X}_k^T \mathbf{Z} \mathbf{X}_k$ and hence they require less computation than the conventional CAP algorithm.
- The PCAP-II algorithm requires reduced computation as compared to the PCAP-I algorithm due to the reduced dimensions of $\boldsymbol{\omega}_k$ and $\mathbf{V}_r^T$.

- MSD learning curves for system identification application:



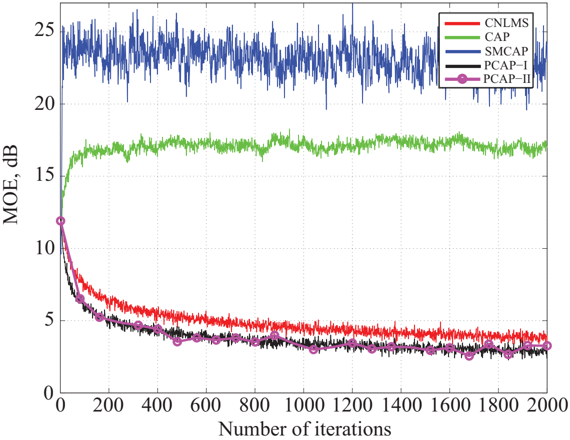
Simulation Results, Cont'd...

Table: Average CPU Time, in Microseconds

CNLMS	CAP	SMCAP	PCAP-I	PCAP-II
27	61	8	38	36

Simulation Results, Cont'd...

- Learning mean output-error (MOE) curves for DS-CDMA interference suppression application:



Conclusions

Two new closely related CAP adaptive-filtering algorithms, the PCAP-I and PCAP-II, that use a new step size have been proposed. The new algorithms

- produce an unbiased output in applications where the desired signal is unavailable or not required

Conclusions

Two new closely related CAP adaptive-filtering algorithms, the PCAP-I and PCAP-II, that use a new step size have been proposed. The new algorithms

- produce an unbiased output in applications where the desired signal is unavailable or not required
- require reduced computational effort than the conventional CAP algorithm

Conclusions

Two new closely related CAP adaptive-filtering algorithms, the PCAP-I and PCAP-II, that use a new step size have been proposed. The new algorithms

- produce an unbiased output in applications where the desired signal is unavailable or not required
- require reduced computational effort than the conventional CAP algorithm
- yield reduced steady-state misalignment relative to the CNLMS algorithm as well as some recent CAP algorithms

Conclusions

Two new closely related CAP adaptive-filtering algorithms, the PCAP-I and PCAP-II, that use a new step size have been proposed. The new algorithms

- produce an unbiased output in applications where the desired signal is unavailable or not required
- require reduced computational effort than the conventional CAP algorithm
- yield reduced steady-state misalignment relative to the CNLMS algorithm as well as some recent CAP algorithms
- offer faster convergence than the CNLMS algorithm.