On the Roots of Digital Signal Processing 1770 to 1970

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Introduction

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- Some landmark advancements in mathematics over the period 1770 to 1970 that pertain to the roots of DSP will be examined.
- It will be shown that the mathematical tools for spectral analysis were introduced by a group of French mathematicians who studied or taught at École Polytechnique in Paris during or soon after the French Revolution over a period of 50 years or so.

- The processing of continuous-time signals by digital means is possible by virtue of *the sampling theorem*.
- It is attributed to Nyquist and/or Shannon.
- To elucidate the origins of the sampling theorem, the contributions of Nyquist and Shannon to this discovery will be examined.

- The construction of machines that can perform numerical calculations has been explored by several engineers and scientists, including Pascal and Leibniz, but the most ambitious attempt was by Babbage who is often regarded to be *the father of computers.*

- The construction of machines that can perform numerical calculations has been explored by several engineers and scientists, including Pascal and Leibniz, but the most ambitious attempt was by Babbage who is often regarded to be *the father of computers.*
- Babbage's work will be examined here from the perspective of the DSP practitioner.

- Time permitting, the talk will also deal with certain landmark achievements during the 1960s which led to the emergence of DSP as a field of study.
- The talk is based on an article to be published in the *IEEE Circuits and Systems Magazine* in November 2007 (see [Antoniou, 2007]).

The French Mathematicians

Five generations of French mathematicians who lived during or after the French Revolution have given us the basic tools for the spectral representation of signals:

- Jean Baptiste Joseph Fourier (1768–1830)
- Siméon-Denis Poisson (1781–1840)
- Augustin Louis Cauchy (1789–1857)
- Johann Peter Gustav Lejeune Dirichlet (1805–1859)
- Pierre Laurent (1813–1854)

Jean Baptiste Joseph Fourier

- Fourier studied at École Normal in Paris where he was taught by Lagrange and Laplace.
- He was appointed at École Polytechnique soon after.
- In 1798, he was selected to accompany Napoleon's army in its invasion of Egypt as a scientific advisor.

Note: The biographical notes on these slides originate from [Indexes of Biographies] and other websites on the Internet.

Fourier *Cont'd*

- He returned to Paris in 1801 along with what remained of the French expeditionary force.
- Soon after, he was appointed by Napoleon as a Prefect in Grenoble.
- In this capacity, he had to supervise the draining of swamps and the construction of a new highway from Grenoble to Turin.

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Discovery of Fourier Series

- During 1804–1807, while in Grenoble, Fourier found time to pursue research work on heat transfer, presumably in his spare time.
- He presented a paper entitled *On the Propagation of Heat in Solid Bodies* at the Institut de France.
- The paper caused controversy from the start: the committee appointed to report on the work, which included Fourier's teachers Lagrange and Laplace, opposed the work
  - on account of *analytic difficulty* of the heat transfer equations involved, and
  - the *extensive use of trigonometric series* in the derivation, now known universally as the Fourier series.

Fourier Series *Cont'd*

- To resolve the issue once and for all, the Institut de France made "Propagation of Heat" the subject of the Grand Prize for 1811.
- There was one more candidate for the prize in addition to Fourier.
- The committee, which included Lagrange and Laplace as members, awarded the prize to Fourier.
- However, the written report of the committee expressed *reservation about the rigor and generality* of the work.

- Formal publication of the work did not take place until 1822 when the Academy of Sciences published a treatise by Fourier entitled *Théory Analytique de la Chaleur*.
- The controversy continued among mathematicians for some years until Dirichlet, one of Fourier's students, published the conditions for the convergence of the Fourier series in 1829.

Siméon-Denis Poisson

- Poisson's father wanted him to become a surgeon and sent him off to serve as an apprentice surgeon under the guidance of an uncle.
- Handicapped by a dreadful lack of dexterity not to mention a lack of motivation for the medical profession, he soon failed.
- In due course, with his father's consent, he began to study his favorite subject, mathematics, at École Polytechnique in 1798.

Poisson *Cont'd*

- Like Fourier, Poisson had Lagrange and Laplace as teachers.
- His big break came about in 1806 when he was appointed to the professorship vacated by Fourier upon the latter's departure for Grenoble.
- He is known for *a probability distribution, an integral, and a summation formula* that carry his name and many other discoveries.

Poisson Summation Formula

In the context of signal analysis, the Poisson Summation Formula can be expressed as

$$\sum_{n=-\infty}^{\infty} x(t+nT) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(jn\omega_s) e^{j\omega_s t}$$

where

- $x(t)$ is a signal,
- $X(j\omega)$ is its Fourier transform or frequency spectrum,
- T is a period in s, and
- $\omega_s = 2\pi/T$ is a frequency in rad/s.

Poisson Summation Formula *Cont'd*

By using the Poisson summation formula, one can show that the spectrum of a sampled signal $\hat{x}(t)$ is given by

$$\hat{X}(j\omega) = X_D(e^{j\omega T}) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(j\omega + jn\omega_s)$$

where

- $X_D(e^{j\omega T})$ is the z transform of $x(nT)$ evaluated on the unit circle $z = e^{j\omega T}$ of the z plane,
- T is the sampling period, and
- ω_s is the sampling frequency.

(see [Antoniou, 2005] for derivation.)

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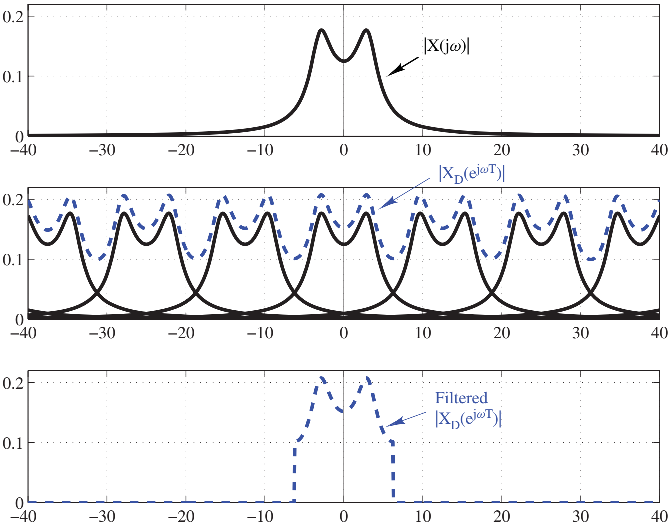
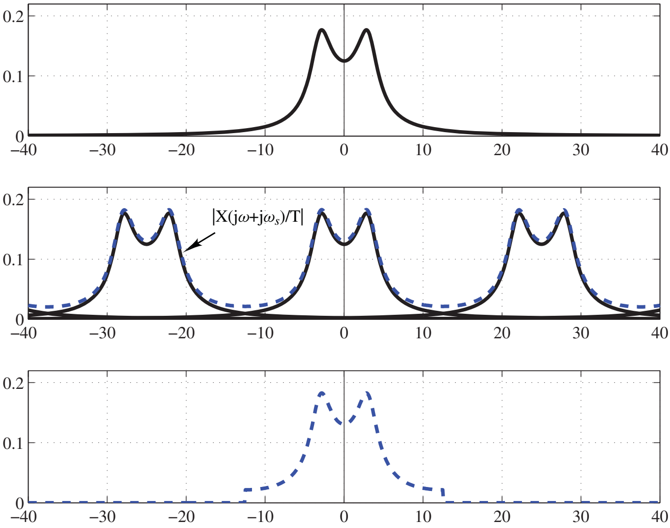
In effect, *given the frequency spectrum $X(j\omega)$ of a continuous-time signal $x(t)$, the spectrum of the corresponding discrete-time signal $x(nT)$ can be readily obtained.*

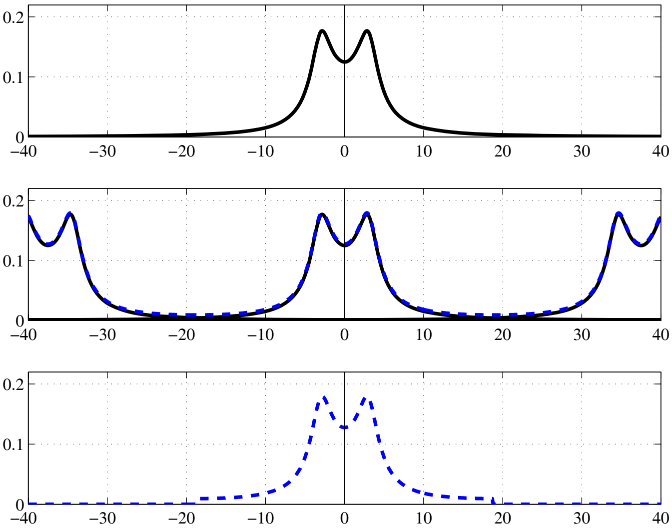
Poisson Summation Formula *Cont'd*



Poisson Summation Formula *Cont'd*



Poisson Summation Formula *Cont'd*



Poisson Summation Formula *Cont'd*

The last slide shows clearly that *a continuous-time signal can be recovered from a sampled version of the signal by using a lowpass filter*, i.e., the validity of the sampling theorem is demonstrated.

Augustine Louis Cauchy

- He studied at École Polytechnique during 1805-1807 and upon graduation he entered École des Ponts and Chaussées (School of Bridges and Roadways) to study engineering.
- From 1815 to 1830 he taught at École Polytechnique.
- He left France in 1830 to get away from an unfavorable political situation to spend some time in Switzerland, Turin, and Prague but returned to Paris in 1838.
- He contributed extensively to the mathematics of physics and in the process he developed new techniques such as transforms, diagonalization of matrices, and the calculus of residues.

Cauchy *Cont'd*

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- By using the residue theorem, the inverse of an arbitrary z transform, $X(z)$, can be deduced as

$$x(nT) = \frac{1}{2\pi j} \oint_{\Gamma} X(z) z^{n-1}\, dz = \sum_{i=1}^{P} \text{Res}_{\ z \to p_i} \left[X(z) z^{n-1} \right]$$

where

$$\text{Res}_{\ z=p_i} \left[X(z) z^{n-1} \right] = \frac{1}{(m_i - 1)!} \lim_{z \to p_i} \frac{d^{m_i - 1}}{dz^{m_i - 1}} \left[(z - p_i)^{m_i} X(z) z^{n-1} \right]$$

(see Chap. 3 of [Antoniou, 2005]).

Johann Peter Gustav Lejeune Dirichlet

- Laplace, Fourier, and Poison were his teachers.
- He married the sister of Felix Mendelssohn.
- As mentioned earlier, he deduced the conditions for the convergence of the Fourier series, which are known as *the Dirichlet conditions*.

Pierre Laurent

- He studied at École Polytechnique.
- He discovered a series which is now know as the Laurent series.
- He submitted a paper on complex analysis that included the Laurent series for the Grant Prize for Mathematics of the Academy of Science.
- Unfortunately, he submitted the paper after the official deadline and, consequently, it was not considered seriously by the Academy.
- Despite several attempts by Cauchy to help Laurent publish his paper, the series he discovered was not published until some years after his death at the early age of 41.

Laurent *Cont'd*

According to Laurent, an analytic function $F(z)$ can be represented by an the infinite series of the form

$$F(z) = \sum_{n=-\infty}^{\infty} a_n (z-a)^{-n}$$

where a is an arbitrary complex constant and

$$a_n = \frac{1}{2\pi j} \oint_{\Gamma} F(z)(z-a)^{n-1}\, dz$$

where Γ is a closed contour in the annulus of convergence that encircles point $z = a$.

The Laurent series is given by

$$F(z) = \sum_{n=-\infty}^{\infty} a_n (z - a)^{-n}$$

The z transform is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x(nT) z^{-n}$$

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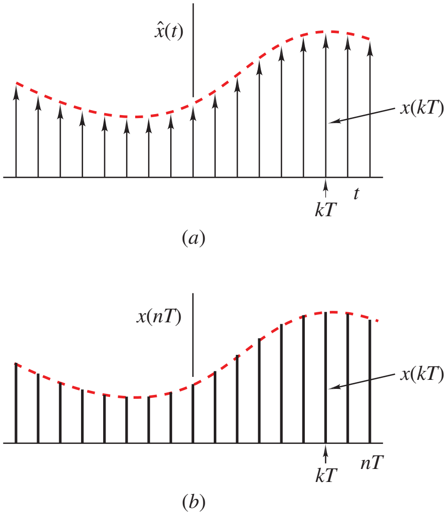
If we now let $a_n = x(nT)$ and $a = 0$ in the Laurent series, we get the z transform and, in effect, *the z transform is one of several possible Laurent series of a rational function*.

- Cauchy tried unsuccessfully to help Laurent publish his series.
- Interestingly, coefficients a_n for $-\infty < n < \infty$ are the residues of function $F(z)$, which can be evaluated using the residue theorem and, as mentioned, the residue theorem is based on Cachy's integral.

Laurent *Cont'd*

- Fourier and Laurent are related through their association with École Polytechnique.
- Their contributions to the roots of DSP are also related: *The Fourier transform of an impulse modulated signal is numerically equal to the z transform of a corresponding discrete-time signal evaluate on the unit circle of the z plane.*

Laurent *Cont'd*



The Sampling Theorem

- The next major discovery in spectral analysis, after the Fourier series, is the formulation of the sampling theorem during the early part of the 20th century.
- Notable contributions to the understanding of this important theorem were made by
  - Harry Nyquist (1889–1976), and
  - Claude Shannon (1916–2001)

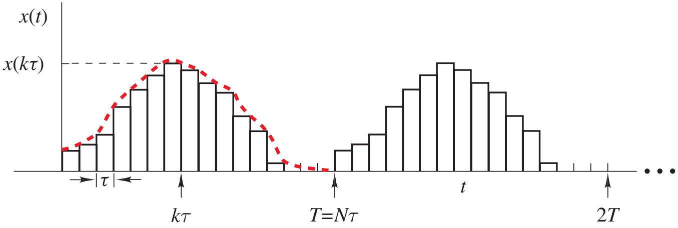
Harry Nyquist

- Nyquist was born in Nilsy, Sweden.
- He emigrated to the USA in 1907.
- Received the bachelor's and master's degrees from the University of North Dakota and the PhD degree from Yale University in 1914, 1915, and 1917, respectively.
- He spent his professional life until his retirement in 1954 at Bell Telephone Laboratories.

Nyquist *Cont'd*

- In addition to his association with the sampling theorem, Nyquist is known for his work on the stability of amplifiers.
- He carried out important work on thermal noise which is often referred to as Johnson-Nyquist noise.

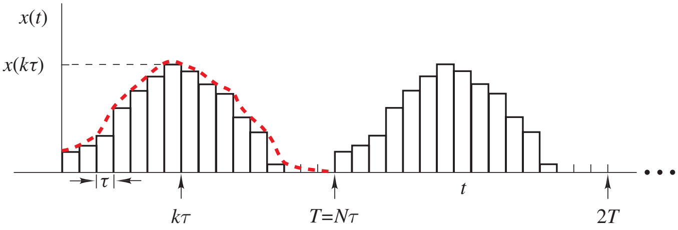
What Nyquist did in connection with the sampling theorem was to show that a periodic pulse signal constructed from a sequence of N equally spaced rectangular pulses of arbitrary amplitudes *can be uniquely determined from the amplitudes and phase angles of the first $N/2$ sinusoidal components of the Fourier series of the periodic signal by solving a set of $N/2$ simultaneous equations* [Nyquist, 1929].



The fundamental of such a signal in Hz is given by

$$f_0 = \frac{1}{T} = \frac{1}{N\tau}$$

where T is the period of the pulse signal and τ is the duration of each rectangular pulse.



...

$$f_0 = \frac{1}{T} = \frac{1}{N\tau}$$

If B is the bandwidth from 0 up to and including harmonic $N/2$, then we have

$$B = \frac{N}{2} f_0 = \frac{N}{2} \cdot \frac{1}{N\tau} = \frac{1}{2\tau}$$

and if we let $1/\tau = f_s$, we get

$$B = \frac{f_s}{2} \text{ in Hz} \quad \text{or} \quad \frac{\omega_s}{2} \text{ in rad/s}$$

where $\omega_s = 2\pi f_s$.

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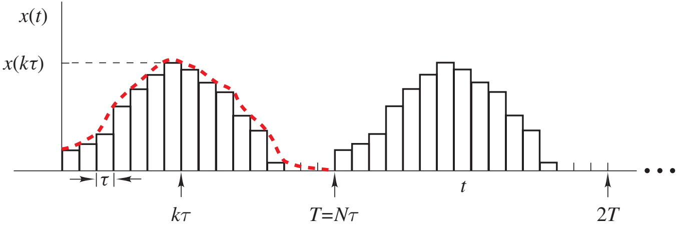
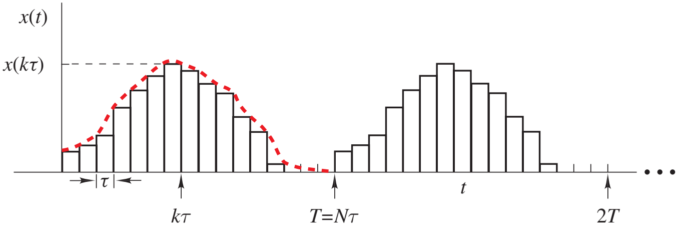
In other words, *the pulse signal can be uniquely determined from the spectrum of the signal over the frequency range 0 to $f_s/2$ where $f_s/2$ is commonly known as the Nyquist frequency*.

Nyquist *Cont'd*

- Nyquist derived his result in the context of telegraphy – no sampled signals were involved.
- However, if τ becomes infinitesimally small, then the dashed curve may be deemed to be a sampled signal.



- In order to extend the validity of his result to the case of nonperiodic signals, Nyquist suggested that period T could be made very large, a day or a year, in his words, by adding pulses of zero amplitude.



- Unfortunately, Nyquist's analysis breaks down because the Fourier-series coefficients which are given by

$$X_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}\,dt$$

would become zero when T becomes infinite.

Claude Elwood Shannon

- Shannon studied at the University of Michigan graduating with two Bachelor of Science degrees, one in electrical engineering and the other in mathematics in 1936.
- He pursued graduate studies at the Massachusetts Institute of Technology earning a master's degree in electrical engineering and a PhD in mathematics in 1940.
- He joined the mathematics department at Bell Labs in 1941 and remained affiliated with Bell Labs until 1972.
- He was appointed as Donner Professor of Science at MIT in 1958 and continued from 1978 on as professor emeritus.

Shannon *Cont'd*

- His contributions are both numerous and diverse.
- He proposed the application of Boolean algebra for the description of switching circuits, which became in due course the standard design methodology for digital circuits and computers.
- From 1940 on he began to be involved with the emerging field of communication theory and over the years he laid the foundation of what is now known as *information theory*.
- What is of interest here is his contribution to the understanding of the sampling theorem.

- Essentially, what Shannon did was to provide a more general proof that a signal which satisfies the Nyquist condition can be recostructed from its values $x(nT)$ for $-\infty < n < \infty$ (see [Shannon, 1949]).

If a signal $x(t)$ that satisfies the Nyquist condition is passed through an ideal channel with a frequency response

$$H(j\omega) = \begin{cases} 1 & \text{for } -\omega_s/2 < \omega < \omega_s/2 \\ 0 & \text{otherwise} \end{cases}$$

then a signal of the form

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT) \frac{\sin \omega_s (t - nT)/2}{\omega_s (t - nT)/2} \qquad \text{(A)}$$

would be obtained at the receiving end.

...

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT) \frac{\sin \omega_s (t - nT)/2}{\omega_s (t - nT)/2} \qquad \text{(A)}$$

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- The formula in Eq. (A) is essentially an interpolation formula that reconstructs the original signal from its values $x(nT)$ for $-\infty < n < \infty$.

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- Shannon used the Fourier transform in his proof and, in effect, he has shown that the Nyquist condition applies to periodic as well as nonperiodic signals that have a Fourier transform.

Shannon *Cont'd*

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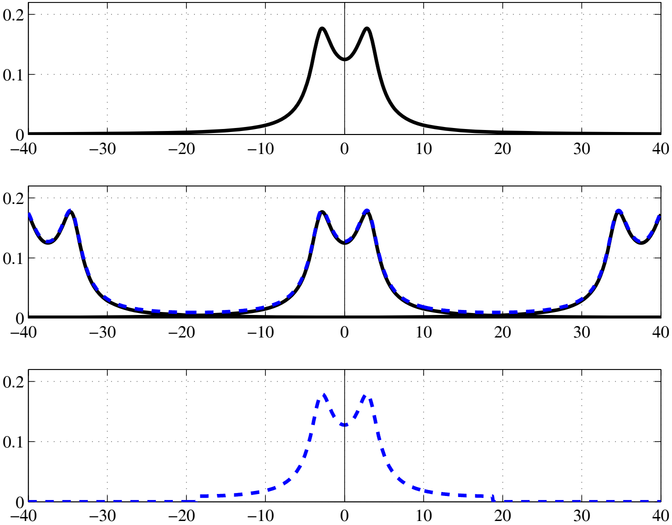
- However, given a continuous-time signal $x(t)$ with a spectrum $X(j\omega)$ that satisfies the Nyquist condition, the spectrum of the sampled signal would simply be

$$\frac{1}{T}X(j\omega) \quad \text{for} \quad -\omega_s/2 < \omega < \omega_s/2$$

by virtue of the Poisson summation formula.

Shannon *Cont'd*



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Shannon *Cont'd*

- In effect, Shannon's proof applies equally well to the situation where a sampled signal is passed through an ideal lowpass filter.
- Since a lowpass filter will also reject the sidebands introduced by the sampling process, Shannon's proof also incorporates a practical technique that can be used to recover continuous-time signals from their sampled versions.

Shannon *Cont'd*

- It should be mentioned that Shannon pointed out in his paper that the sampling theorem was common knowledge in the art of communications and that it had been given previously in other forms by mathematicians; in fact, he cites a mathematical treatise by Whittaker published in 1913.

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- In recent years it has been found out that the sampling theorem was 'discovered' independently by several others, e.g., *Kotelnikov in 1933*, *Raabe in 1939*, and *Someya in 1949*, according to a recent article by Lüke [Lüke, 1999].

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- In fact, the underlying principle is closely related to the *barycentric interpolation formula* of Lagrange.

Emergence of Computers

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- Therefore, a survey of the events that led to DSP would be incomplete without a word or two on the emergence of computers.

Emergence of Computers *Cont'd*

- The rapid advancements in mathematics and most other sciences during the Renaissance led to commensurate advancements in engineering, manufacturing, transportation, navigation, trade, banking, etc.
- Consequently, a great need for numerical calculations emerged be it to estimate the position of a ship using astronomical measurements, to establish the trajectory of a heavenly body, or to design a bridge or steam engine.

Emergence of Computers *Cont'd*

- To expedite such calculations, published numerical tables, such as logarithm and trigonometric tables, had been in use since the 1600s.
- The calculations necessary to construct numerical tables were carried out by people who spent endless monotonous hours performing manual calculations.
- The end result was that published tables contained numerous typographical errors.

Emergence of Computers *Cont'd*

- From the 17th century on, a number of notable scientists and engineers, including Pascal and Leibniz, attempted to construct calculating machines to alleviate the burden of numerical calculations.
- The most ambitious of these individuals was Charles Babbage (1791–1871) who attempted to construct machines he called *difference engines* that would perform the necessary computations as well as print the numerical tables without human intervention.

  In this way, he hoped *to produce numerical tables that were free of numerical errors*.

Charges Babbage

- Babbage studied at Cambridge University, during 1810-1814 earning a BA degree.
- He was elected member of the Royal Society of London in 1816 at the early age of 24.
- In 1819 he began work on his first difference engine having obtained a grant from the British government.
- He was to spend the rest of his professional life trying to accomplish this task.

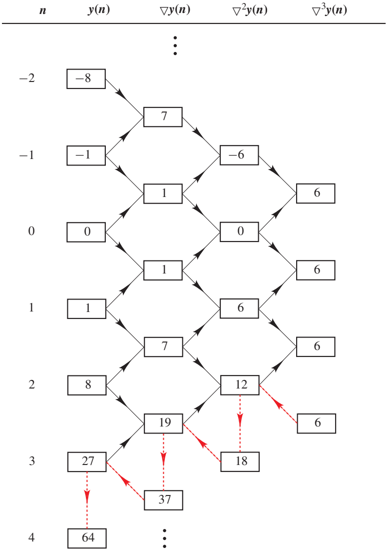
Babbage *Cont'd*

- The purpose of the difference engine was to evaluate polynomials of arbitrary orders with high precision.
- It was to be constructed using the technology of the 1800s, namely, in terms of cams, gears, and levers.
- Its theoretical basis was a simple numerical extrapolation technique.
- The underlying principle is illustrated in the two or three slides for the case where the function

$$y(n) = n^3$$

is to be evaluated.

Babbage *Cont'd*



- The first and second columns of the difference table give the values of independent variable n and the corresponding values of the function.

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- The third, fourth, and fifth columns give the first, second, and third backward differences which are defined as

$$\begin{aligned} \nabla y(n) &= y(n) - y(n-1) \\ \nabla^2 y(n) &= \nabla\nabla y(n) \\ \nabla^3 y(n) &= \nabla\nabla^2 y(n) \end{aligned}$$

Babbage *Cont'd*

- We note that the entries in the fifth column, namely, the third backward differences, are all equal to 6.
- The reason behind this phenomenon is connected to the fact that the third derivative of n^3 is a constant.
- On the basis of this fact, we can generate a new set of differences for the table by starting with the next entry in the fifth column, which is known to be 6, and progressing towards the left, ending with the next value of the function in the second column.

  Thus an arbitrarily long table of the cubes of n can be generated.

Babbage *Cont'd*

- Extending the same principle, one can show that the *n*th backward differences of an *n*th-order polynomial are also all equal to a constant.
- Therefore, the extrapolation technique described can be used to evaluate arbitrary polynomials just as well.

- If we now extend the difference table for the evaluation of $y(n) = n^3$ to entries $n-3$, $n-2$, $n-1$, n, we obtain

$$\begin{aligned} \nabla^2 y(n) &= 6 + \nabla^2 y(n-1) \\ \nabla y(n) &= \nabla^2 y(n) + \nabla y(n-1) \\ y(n) &= \nabla y(n) + y(n-1) \end{aligned}$$

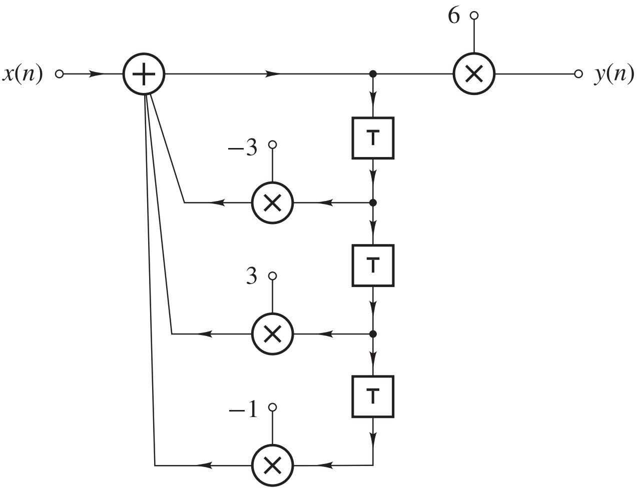
- By solving for $y(n)$, we get the difference equation

$$\begin{aligned} y(n) &= 6x(n) + 3y(n-1) - 3y(n-2) \\ &\quad + y(n-3) \end{aligned} \qquad \text{(B)}$$

where $x(n) = u(nT)$ and $u(nT)$ is the unit-step function.

- Therefore, from Eq. (B) we conclude that what Babbage was trying to construct was a *recursive discrete system* in today's language.

Network representation of Babbage's difference engine:



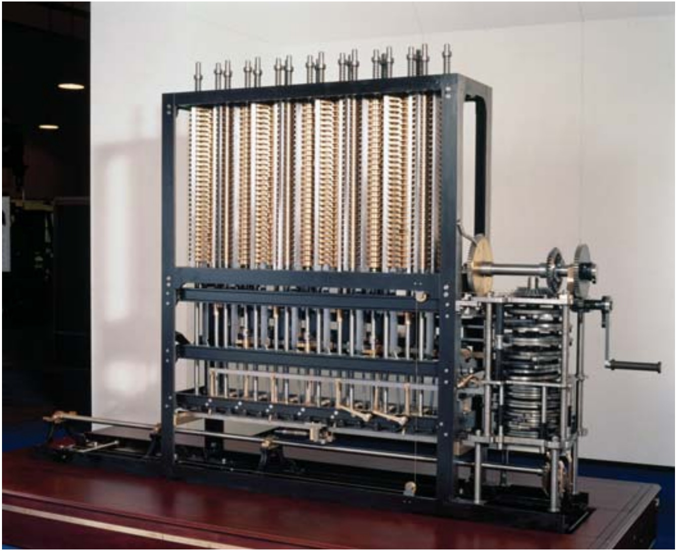
Babbage *Cont'd*

- For various reasons Babbage failed to build a working difference engine (see [Swade, 2000]).
- All that was left to posterity is an almost complete set of drawings of Difference Engine No. 2 and certain parts that escaped recycling.
- However, he was fully vindicated when a team led by Donald D. Swade, sponsored by the Science Museum in London, actually built a working model of Difference Engine No. 2, minus the printing mechanism, based on Babbage's drawings.

Babbage's Difference Engine No. 2

- Was designed during the period 1847 to 1849.
- Was built at the Science Museum, London, U.K., in 1991 (see [Swade, 1991]).
- Measures 2.1 $\times$ 3.4 $\times$ 0.5 m.
- Weighs 3 tons.
- Can evaluate 7th-order polynomials.
- Was designed to calculate to 30 significant figures.

Babbage's Difference Engine No. 2



Emergence of Modern Era of DSP

- The pressures of World War II during the 1940s rekindled strong interest in constructing machines that would perform calculations accurately and efficiently, and several machines were built during that period based on the new emerging electronics technology.
- The most notable of these machines was the Electronic Numerical Integrator and Computer, or ENIAC, which was conceived and designed by John Mauchly and J. Presper Eckert of the University of Pennsylvania.

- ENIAC bears no ancestral relationship to Babbage's difference and analytical engines.
- However, it is of interest to note that just like the difference engines of Babbage, ENIAC was designed *to construct numerical tables*, actually artillery firing tables for the U.S. Army's Ballistics Research Laboratory.

- By the late fifties, a cohesive collection of techniques referred to as '*data smoothing and prediction*' began to emerge through the efforts of pioneers such as Blackman, Bode, Shannon, and others.

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Modern Era of DSP *Cont'd*

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Note: See [Antoniou, 2007] for references.

Modern Era of DSP *Cont'd*

- Digital filters in hardware form began to appear during the late sixties and an early design was reported by Jackson, Kaiser, and McDonald.

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- During the 1960s, the discrete Fourier transform was formalized and efficient algorithms for its computation, usually referred to as *Fast Fourier Transforms,* were proposed by Cooley, Tukey, and others.
- From that time on, the analysis and processing of signals in the form of numerical data began to be referred to as *digital signal processing*, and algorithms, computer programs, or systems that could be used for the processing of these signals became firmly established as *digital filters*.

Modern Era of DSP *Cont'd*

More recently, DSP has mushroomed into a multifaceted collection of related areas with applications in

> *long-distance and cellular telephone systems; radar systems; high-definition TVs; audio systems , CD players, and iPods; speech synthesis; image processing and enhancement; the Internet; instrumentation; photography; processing of biological signals such as ECGs; processing of seismic signals; artificial cochleas; remote sensing; astronomy; economics; genetic and proteomic signal processing; movie making*

to name just a few.

Conclusions

- It has been shown that a small number of mathematicians who taught or studied at École Polytechique in Paris laid the mathematical foundations of modern spectral analysis.

  However, none of them knew anything about signals.

Conclusions *Cont'd*

- The contributions of Nyquist and Shannon to the sampling theorem have been examined from a modern perspective.
- Nyquist's proof was based on the Fourier series and as such it was limited to periodic signals.
- Shannon extended the proof to include nonperiodic signals as well by using the Fourier transform.
- However, the underlying principles of the sampling theorem are related to an interpolation method due to the great Lagrange himself who, as mentioned, was the teacher of Fourier.

Conclusions *Cont'd*

- The work of Babbage has then been examined in the context of DSP.
- Although Babbage is often referred to as the father of computers what he invented was actually a discrete system that would implement a difference equation just like a modern digital filter.

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This slide concludes the presentation.
Thank you for your attention.