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Accurate clock synchronization in wireless sensor networks with bounded noise☆

Jianping He [a,b], Xiaoming Duan [c], Peng Cheng [c], Ling Shi [d], Lin Cai [e]

[a] The Department of Automation, Shanghai Jiao Tong University, Shanghai, China
[b] The Key Laboratory of System Control and Information Processing, Ministry of Education of China, Shanghai, China
[c] The State Key Lab. of Industrial Control Technology, Zhejiang University, Hangzhou, PR China
[d] The Department of Electric and Computer Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong
[e] The Department of Electrical & Computer Engineering at the University of Victoria, BC, Canada

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ABSTRACT

It is important and challenging to achieve accurate clock synchronization in wireless sensor networks. Various noises, e.g., communication delay, clock fluctuation and measurement errors, are inevitable and difficult to be estimated accurately, which is the main challenge for achieving accurate clock synchronization. In this paper, we focus on how to achieve accurate clock synchronization by considering a practical noise model, bounded noise, which may not satisfy any known distributions. The principle that a bounded monotonic sequence must possess a limit and the concept of maximum consensus are exploited to design a novel clock synchronization algorithm for the network to achieve accurate and fast synchronization. The proposed algorithm is fully distributed, with high synchronization accuracy and fast convergence speed, and is able to compensate both clock skew and offset simultaneously. Meanwhile, we prove that the algorithm converges with probability one, which means that an accurate clock synchronization is achieved. We further prove that the probability of the complete synchronization converges exponentially fast. Experiments and simulations are conducted to verify the noise model and demonstrate the effectiveness of the proposed algorithm.

1. Introduction

Clock synchronization is a fundamental requirement for various applications in wireless sensor networks (WSNs), e.g., data fusion, sensor scheduling and node cooperation (Sundararaman, Buy, & Kshemkalyani, 2005). Many protocols have been proposed for clock synchronization in various scenarios (Ganeriwal, Kumar, & Srivastava, 2003; Maroti, Kusy, Simon, & Ledeczi, 2004; Sichitiu & Veerarittiphan, 2003; Sivrikaya & Yener, 2004). For most of those protocols, clock synchronization can be achieved completely when the noise is ignored. When taking various noises, e.g., communication delay and measurement errors, into consideration, however, a highly accurate synchronization is hard to be guaranteed. For example, the accuracy of the maximum clock synchronization (MTS) protocol proposed in He, Cheng, Shi, Chen, and Sun (2014) decreases with the variance of the random communication delay. Therefore, an accurate clock synchronization in WSNs under noises is still a challenging problem.

Recently, the concept of consensus is developed to design consensus-based clock synchronization protocols to achieve accurate clock synchronization for WSNs in a distributed way (Carli, Elia, & Zampieri, 2011; Carli & Zampieri, 2014; Choi, Liang, Shen, & Zhuang, 2012; He, Cheng, Shi et al., 2014; He, Li, Chen, & Cheng, 2014; Philipp & Roger, 2009; Schenato & Fiorentin, 2011). These consensus-based protocols can be classified into two categories, i.e., the average and the maximum or minimum-consensus based clock synchronization protocols. For example, the protocols in Choi et al. (2012), Philipp and Roger (2009)

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E-mail addresses: jphe@uvic.ca (J. He), xiaomingduan.zju@gmail.com (X. Duan), pcheng@iipc.zju.edu.cn (P. Cheng), eesling@ust.hk (L. Shi), cai@ece.uvic.ca (L. Cai).

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and Schenato and Fiorentin (2011) are average consensus-based, where the basic idea is that each node takes an average of its own clock parameter and its neighboring ones to drive the network to achieve a consensus reference clock. He et al. He, Cheng, Shi et al. (2014) and He, Li et al. (2014) utilized the maximum and minimum consensus to design the clock synchronization protocol, which is able to achieve a much faster convergence speed. These consensus-based protocols are fully distributed, so they are robust against packet losses, node failures, the addition of new nodes etc., and are promising for real applications in the networks.

However, the noises such as communication delay, measurement error and the fluctuation of clock speed, are ignored in the design of above consensus-based protocols. Taking the noises into consideration, accurate clock synchronization may not be achieved by these consensus-based protocols (He, Li et al., 2014). Therefore, it is of great interest to design new distributed clock synchronization protocol to handle the noises, and then improve the robustness and the accuracy of clock synchronization. Some existing works have investigated clock synchronization under different noise models for WSNs (Carli et al., 2011; Carli & Zampieri, 2014; Freris, Borkar, & Kumar, 2009; Garone, Gasparri, & Lamonaca, 2013, 2015; He, Cheng, Shi et al., 2014; Liao & Barooah, 2013a). For example, Freris et al. (2009) proposed a stochastic model-based framework for time synchronization, which achieves highly accurate relative skew estimation with expected value one and bounded variance. The authors in Garone et al. (2013, 2015) and Liao and Barooah (2013a) proposed new average consensus-based protocols to improve the robustness of the typical average clock synchronization (ATS) algorithm proposed in Schenato and Fiorentin (2011), including the drift of clock skew and the fluctuation of clock in the stable state of the algorithm. Specifically, in Carli et al. (2011) and Carli and Zampieri (2014) utilized the second-order consensus to design synchronization algorithm which can outperform ATS in terms of robustness to process and measurement noises and time-varying clock drifts. Garone et al. (2015) considered a practical bounded communication delays noise model, and proposed a novel synchronization protocol, named Robust Average TimeSynch (RoATS), to achieve robust clock synchronization under bounded delay. These algorithms are still average consensus-based algorithms, which have a slow convergence speed similar to ATS. Meanwhile, it is desirable to further improve the synchronization accuracy by designing novel schemes to eliminate the effect caused by the noises.

Motivated by these, we develop a distributed clock synchronization protocol by adopting the concept of maximum consensus which provides higher accuracy (complete synchronization in probability) and faster converging speed than ATS under bounded noise in WSNs He et al. (2014). The main contributions of this work are summarized as follows:

- By exploiting the principle that a bounded monotonic sequence must possess a limit and the idea of maximum consensus, we propose a novel synchronization protocol, including the relative skew estimation, clock skew and offset compensation, to achieve high accuracy and fast synchronization under a practical bounded noise model.
- We prove that the proposed algorithm guarantees that the clock synchronization can be achieved with probability one. We also prove that the probability of the complete synchronization converges to one exponentially fast.
- Experimental results are presented to verify the bounded noise model, and demonstrate the effectiveness of relative skew estimation on which the accurate synchronization depends. Extensive simulations demonstrate a better performance of the proposed algorithm compared to ATS in terms of synchronization accuracy and convergence speed.

The remainder of the paper is organized as follows. In Section 2, the problem of clock synchronization under bounded noise is formulated. Section 3 presents the detailed distributed clock synchronization algorithm. Experiments to verify the modeling and the main idea of algorithm design are described in Section 4. Simulation to evaluate the performance of the proposed algorithm is presented in Section 5. Finally, Section 6 concludes the paper.

2. System models and problem setup

A WSN is modeled as a strongly connected graph $G = (V, E)$, where V is the set of sensor nodes, with $|V| = n$ $(n \geq 2)$, and E is the set of communication links (edges) between them (i.e., $\langle i, j\rangle \in E$ means that node i can receive the information from node j). The neighbor set of sensor node i is denoted by N_i, where $j \in N_i$ if and only if (iff) the link $\langle i, j\rangle \in E$.

2.1. Clock model

Each sensor has a hardware clock, calculated by counting pulses of its hardware oscillator running at a particular frequency. For a relatively longer period of time (minutes to hours), by referring to Sichitiu and Veerarittiphan (2003) and Sivrikaya and Yener (2004), the hardware clock can be approximated with good accuracy by an oscillator of fixed frequency. Thus, the local hardware clock of sensor node i, denoted by $H_i(t)$, can be approximated as

$$H_i(t) = \alpha_i t + \beta_i, \quad i \in V, \tag{1}$$

where t is the real time, α_i is the hardware clock skew which determines the clock speed and β_i is the hardware clock offset. In practice, $\alpha_i \neq \alpha_j, \forall i \neq j$ as the qualities of sensors' oscillators are usually different which leads to that sensor's oscillators run at slightly different frequencies (Choi et al., 2012). One also has $\beta_i \neq \beta_j, \forall i \neq j$ as the start-up times of sensor nodes are different (He, Li et al., 2014). Hence, different nodes usually have different hardware clocks, due to different clock skew and offset. Since the value of hardware clock cannot be adjusted manually (Philipp & Roger, 2009), a software clock is provided to represent the synchronized time, which is given by

$$S_i(t) = \tilde{\alpha}_i H_i(t) + \tilde{\beta}_i = x_i t + y_i, \quad i \in V,$$

where $\tilde{\alpha}_i$ and $\tilde{\beta}_i$ are adjustable software clock parameters. Then, $x_i = \tilde{\alpha}_i\alpha_i$ and $y_i = \tilde{\alpha}_i\beta_i + \tilde{\beta}_i$ are the software clock skew and offset, respectively.

2.2. Bounded noise model

Let $H_i^+(t)$ be the hardware clock information sent from node i at time t. Each $H_i^+(t)$ is assumed to satisfy

$$H_i^+(t) = H_i(t) + \theta_i(t) = \alpha_i t + \beta_i + \theta_i(t), \quad i \in V, \tag{2}$$

where $\theta_i(t) \in [a, b]$ is defined as the bounded noise due to the communication delay (Garone et al., 2013), measurement error (Garulli & Giannitrapani, 2008), clock fluctuation (Cao, Chen, Zhang, & Sun, 2008; He, Zhou, Cheng, Shi, & Chen, 2016; Yang, Cai, Liu, & Pan, 2012; Yang, Shi, & Chen, 2014), etc. Assume that for each node i the noises $\theta_i(t_1)$ and $\theta_i(t_2)$ are independent of each other for $t_1 \neq t_2$. Since the values of both a and b can be obtained from experiments, it is assumed in this paper that a and b are known to each sensor node. The above model is a general and practical model for the noises, and a typical example is the random bounded communication delay considered in Garone et al. (2013, 2015). Since each noise $\theta_i(t)$ may be any value in $[a, b]$, we state the following assumption.

Assumption 1. Let $f_\theta(x)$ be the probability density function (PDF) of the random noise $\theta_i(t)$. Assume that $f_\theta(x) > 0$ for $x \in [a, b]$ and $f_\theta(x) = 0$ otherwise.

Clearly, from the above assumption, one infers that $\theta_i(t)$ is a bounded noise, i.e., $\theta_i(t) \in [a, b]$, and there exists $0 < \epsilon \leq 1$ such that $\Pr\{\theta_i(t) \in [c-\delta, c+\delta] | \forall c \in [a, b]\} \geq \epsilon$ for $i \in V$, where $b - a > \delta > 0$. Specifically, when $f_\theta(x)$ is available, it is easy to obtain the relationship between δ and ϵ, i.e.,

$$\epsilon = \delta \min_{x\in[a,b]} f_\theta(x). \tag{3}$$

For example, if $\theta_i(t)$ follows a uniform distribution, we have $f_\theta(x) = \frac{1}{b-a}$ for $x \in [a, b]$, and $f_\theta(x) = 0$ for otherwise. Then, we obtain $\epsilon = \frac{\delta}{b-a}$.

Remark 1. In many existing works, it is usually assumed that the noise is a random variable following certain distributions, e.g., Gaussian or exponential distribution (Chaudhari, Serpedin, & Qaraqe, 2008; Mei & Wu, 2011a,b), or with constant mean and variance (He, Cheng, Shi et al., 2014). For these cases, it has been proved in the previous papers that it can achieve a high accurate and even complete clock synchronization in expectation. However, the noise in (2) may have different probability distribution at different time t and may not have a certain constant mean and variance, which may render the clock synchronization unreachable with the previous algorithms in these papers.

Remark 2. There are also some existing works on consensus that have considered the bounded noise (Garone et al., 2015; Garulli & Giannitrapani, 2008; He et al., 2016). The methods given in Garone et al. (2015) and Garulli and Giannitrapani (2008) cannot eliminate the effect of noise on consensus, and thus the consensus algorithm cannot be used to guarantee an accurate clock synchronization. Although the method given in He et al. (2016) can guarantee an accurate and fast consensus, it needs the maximum value of all nodes' initial states as the upper bound for the state updating. Thus, this method cannot be adopted to solve the clock synchronization since the clock reading of each node is changing with time and will not be bounded by a given value.

2.3. Problem setup

The goal of clock synchronization is to find the parameters $(\tilde{\alpha}_i, \tilde{\beta}_i)$ for each node i such that all nodes have the same software clock skews and offsets, i.e., $|x_i - x_j| = 0$ and $|y_i - y_j| = 0, \forall i, j \in V$, which means that $|S_i(t) - S_j(t)| = 0, \forall i, j \in V$. Consensus-based clock synchronization algorithms have been developed to realize this goal (Carli et al., 2011; Philipp & Roger, 2009; Schenato & Fiorentin, 2011), where clock synchronization can be achieved completely using the algorithm when the noises, e.g., communication delay and the fluctuation of hardware clock, are ignored. Unfortunately, as pointed out in Freris, Graham, and Kumar (2011); Freris, Kowshik, and Kumar (2010), the noises are the fundamental limits which affect the synchronization accuracy and even render the synchronization impossible, if the noises cannot be estimated accurately.

Therefore, the objective of this paper is to design an update rule, which forms the distributed synchronization algorithm, to find $(\tilde{\alpha}_i(k), \tilde{\beta}_i(k))$ for each node i, such that

$$\Pr\{\lim_{k\to\infty} |x_i(k) - x_j(k)| = 0\} = 1; \tag{4}$$

$$\Pr\{\lim_{k\to\infty} |y_i(k) - y_j(k)| = 0\} = 1, \tag{5}$$

for $i, j \in V$, where k is the iteration, $x_i(k) = \tilde{\alpha}_i(k)\alpha_i$ and $y_i(k) = \tilde{\alpha}_i(k)\beta_i + \tilde{\beta}_i(k)$. Eqs. (4) and (5) indicate that a highly accurate clock synchronization can be achieved under the bounded noise, and even a complete synchronization is achieved when $t \to \infty$.

3. Distributed clock synchronization algorithm design

In this section, we first give the preliminary which provides a main theoretical support to the algorithm design. Then, we propose a new distributed clock synchronization algorithm to realize the goals in both (4) and (5). In the algorithm, we utilize the principle that a bounded monotonic sequence must possess a limit to design the estimation method to counteract the impact of bounded noise, and use the maximum consensus as the update rule of each iteration, which guarantees a fast convergence speed of the algorithm. This algorithm includes three parts, relative skew estimation, skew compensation and offset compensation.

3.1. Preliminary

Let $\varphi(k) = \mu + \theta(k)$ for $k = 1, 2, \ldots$, be an infinite random sequence, where μ is a constant. Suppose that $\theta(k)$ satisfies Assumption 1, so it is bounded in $[a, b]$. We design an iteration algorithm as follows,

$$\hat{\varphi}(k+1) = \max\{\hat{\varphi}(k), \varphi(k) - b\}, \tag{6}$$

with $\hat{\varphi}(0) = -\infty$. Then, we have the following theorem.

Theorem 1. *Using the algorithm (6), for any small constant $\epsilon > 0$, we have*

$$\Pr\{\hat{\varphi}(k) \in [\mu - \delta, \mu]\} \geq 1 - (1-\epsilon)^k, \tag{7}$$

where δ and ϵ satisfy (3), and

$$\Pr\{\lim_{k\to\infty} \hat{\varphi}(k) \in [\mu - \delta, \mu]\} = 1. \tag{8}$$

Proof. With (6), one infers that $\hat{\varphi}(k)$ is non-decreasing with k and satisfies

$$\hat{\varphi}(k) = \max_{\ell=1,2,\ldots,k-1} \{\varphi(\ell) - b\} \leq \mu.$$

Then, if $\hat{\varphi}(k) \in [\mu - \delta, \mu]$, we have

$$\Pr\{\hat{\varphi}(k+1) \in [\mu - \delta, \mu]\} = 1.$$

If $\hat{\varphi}(k) \notin [\mu - \delta, \mu]$, we have

$$\Pr\{\hat{\varphi}(k+1) \in [\mu - \delta, \mu] | \hat{\varphi}(k) \notin [\mu - \delta, \mu]\} = \int_{b-\delta}^{b} f_\theta(x)dx > 0.$$

It follows that

$$\Pr\{\hat{\varphi}(k+1) \notin [\mu - \delta, \mu] | \hat{\varphi}(k) \notin [\mu - \delta, \mu]\} = 1 - \phi,$$

where $\phi = \int_{b-\delta}^{b} f_\theta(x)dx$. Therefore,

$$\Pr\{\hat{\varphi}(k) \notin [\mu - \delta, \mu] | \hat{\varphi}(0) \notin [\mu - \delta, \mu]\} = (1-\phi)^k,$$

which converges to 0 as $k \to \infty$. Then, we have

$$\begin{aligned} &\Pr\{\hat{\varphi}(k) \in [\mu - \delta, \mu]\} \\ &\quad \geq 1 - \Pr\{\hat{\varphi}(k) \notin [\mu - \delta, \mu] | \hat{\varphi}(0) \notin [\mu - \delta, \mu]\} \\ &\quad = 1 - (1-\phi)^k \\ &\quad \geq 1 - (1-\epsilon)^k, \end{aligned}$$

where we have used the fact that

$$\phi = \int_{b-\delta}^{b} f_\theta(x)dx \geq \delta \min_{x\in[a,b]} f_\theta(x) = \epsilon,$$

which converges to 1 as $k \to \infty$.

In the above theorem, one infers from (7) that algorithm (6) converges exponentially in probability and its convergence rate depends on δ and ϵ, where δ represents the estimation accuracy to be guaranteed and ϵ is a probability characterizing the rate of convergence. Note from (3) that ϵ is a decreasing function of δ, which implies that it needs more iterations for the algorithm (6) to achieve a higher estimation accuracy. Then, one further infers from (8) that under the bounded noise considered in this paper, using algorithm (6) can reach arbitrarily accurate estimation of the constant part in the random variables. This is the main idea for our following clock synchronization algorithm design. It should be pointed out that as long as the noise is bounded and satisfies Assumption 1, one can get an exact estimation of the fixed part of the random variables using (6). Hence, in addition to the clock synchronization, there could be more applications for the above algorithm and theorem. For example, considering a sensor measuring the distance between an object and itself, the measured distance may be equal to the real distance plus a bounded noise. In this scenario, the sensor can do multi-metering, and then an accurate distance estimation can be obtained using the above algorithm.

Consider the case where the noise bound b is not exactly known. Let $\hat{b}$ be the estimation of b. Consider the algorithm

$$\begin{aligned}\hat{\varphi}(k+1) &= \max\{\hat{\varphi}(k), \varphi(k) - \hat{b}\} \\ &= \max\{\hat{\varphi}(k), \varphi(k) - b + b - \hat{b}\}, \end{aligned} \tag{9}$$

with $\hat{\varphi}(0) = -\infty$. Then, with the similar analysis as Theorem 1, we obtain the following corollary.

Corollary 1. *Using the algorithm (9), for any small constant $\epsilon > 0$, we have*

$$\Pr\{\hat{\varphi}(k) \in [\mu - \delta + b - \hat{b}, \mu + b - \hat{b}]\} \geq 1 - (1-\epsilon)^k, \tag{10}$$

where δ and ϵ satisfy (3), and

$$\Pr\{\lim_{k\to\infty} \hat{\varphi}(k) \in [\mu - \delta + b - \hat{b}, \mu + b - \hat{b}]\} = 1. \tag{11}$$

The above corollary shows that under (9), the estimation accuracy of μ will depend on $b - \hat{b}$, i.e., the estimation accuracy of noise bound b. It follows from (11) that $\hat{\varphi}(\infty) - \mu \in [b - \hat{b} - \delta, \mu + b - \hat{b}]$, and thus a smaller value of $b - \hat{b}$ provides a more accurate estimation of μ. Hence, in this case, the key issue is how to estimate the noise bound accurately. In experiment, how to obtain the noise bound is presented in Section 4.2.

3.2. Relative skew estimation

Since the ideal time t is unavailable to each node, α_i and β_i cannot be computed (Freris et al., 2011; Schenato & Fiorentin, 2011). However, note that the local hardware clock readings are available, then we can obtain a relative clock between any two nodes i and j, which is defined as

$$H_i(t) = \frac{\alpha_i}{\alpha_j} H_j(t) + \left(\beta_i - \frac{\alpha_i}{\alpha_j}\beta_j\right) = \alpha_{ji} H_j(t) + \beta_{ji}, \tag{12}$$

where $\alpha_{ji} = \frac{\alpha_i}{\alpha_j}$ is the **relative skew** and $\beta_{ji} = \beta_i - \alpha_{ji}\beta_j$ is the **relative offset** (He, Cheng, Chen, & Cao, 2014; He, Cheng, Shi, & Chen, 2013; He, Cheng, Shi et al., 2014; Schenato & Fiorentin, 2011). After obtaining the relative clock, each node i thus can synchronize its clock with the neighbor j's clock. Then the problem becomes how to obtain an accurate estimation of the relative skew under the noise.

The estimation of the relative skew is crucial in clock synchronization, since both the clock skew and offset compensation depend on it, and the accuracy of its estimation will directly affect the synchronization accuracy (Liao & Barooah, 2013b). Define $e_{ij}(k)$ as a one-step estimation of the relative skew, given by

$$\begin{aligned} e_{ij}(k) &= \frac{H_j^+(t_k) - H_j^+(t_{k-1}) - (b-a)}{H_i(t_k) - H_i(t_{k-1})} \\ &= \frac{H_j(t_k) - H_j(t_{k-1}) + (\theta_j(t_k) - \theta_j(t_{k-1})) - (b-a)}{H_i(t_k) - H_i(t_{k-1})}, \end{aligned} \tag{13}$$

for $i, j \in V$, where $H_j^+(t_k)$ and $H_j^+(t_{k-1})$ are the hardware clock information received from node j at iterations k and $k-1$, respectively, and $H_i(t_k)$ and $H_i(t_{k-1})$ are the two times of the time-sampling of node i. Let $\hat{\alpha}_{ij}(1) = e_{ij}(1)$ be the initial estimation of relative skew. Then, based on (6), we design the following equation to iteratively estimate the relative skew α_{ij},

$$\hat{\alpha}_{ij}(k+1) = \max\{e_{ij}(k+1), \hat{\alpha}_{ij}(k)\}, \quad i \in V, j \in N_i. \tag{14}$$

In (13), note that $\theta_j(t_k)$ and $\theta_j(t_{k-1}) \in [a, b]$ are bounded noise satisfying Assumption 1, which means that $\theta_j(t_k) - \theta_j(t_{k-1}) \in [a-b, b-a]$ is also a bounded noise and satisfies Assumption 1. However, from (14), $\hat{\alpha}_{ij}(k)$ is an increasing (non-decreasing) function of iteration k with an upper bound $\frac{\alpha_j}{\alpha_i}$. Hence, according to the principle that a bounded monotonic sequence must possess a limit, $\hat{\alpha}_{ij}(k)$ will converge to a constant. Then, by referring to Theorem 1, we give a lemma as follows.

Lemma 1. *Given any small constant $\sigma > 0$, using (14) to estimate the relative skew, there exists a small constant ϵ such that*

$$\Pr\{\alpha_{ij} - \hat{\alpha}_{ij}(k) \leq \sigma \,|\, \langle i,j\rangle \in E\} \geq 1 - (1-\epsilon^2)^k, \tag{15}$$

and we have

$$\Pr\{\lim_{k\to\infty} \alpha_{ij} - \hat{\alpha}_{ij}(k) \leq \sigma \,|\, \langle i,j\rangle \in E\} = 1. \tag{16}$$

Proof. Since the time interval between two iterations is positive, there exists $\tau > 0$ such that $H_i(t_k) - H_i(t_{k-1}) \geq \tau$.

From (1), (2), it follows that

$$\begin{aligned} e_{ij}(k) &= \frac{H_j(t_k) - H_j(t_{k-1})}{H_i(t_k) - H_i(t_{k-1})} + \frac{\theta_j(t_k) - \theta_j(t_{k-1}) - (b-a)}{H_i(t_k) - H_i(t_{k-1})} \\ &= \frac{\alpha_j}{\alpha_i} + f_{ij}(t_k, t_{k-1}), \quad i, j \in V, \end{aligned} \tag{17}$$

where

$$f_{ij}(t_k, t_{k-1}) = \frac{\theta_j(t_k) - \theta_j(t_{k-1}) - (b-a)}{H_i(t_k) - H_i(t_{k-1})}.$$

According to Assumption 1, there exist δ and ϵ such that $\Pr\{|\theta_j(t_k) - b| \leq \delta\} \geq \epsilon$ and $\Pr\{|\theta_j(t_{k-1}) - a| \leq \delta\} \geq \epsilon$. Then, we have

$$\begin{aligned} &\Pr\left\{f_{ij}(t_k, t_{k-1}) \geq -\frac{2\delta}{\tau}\right\} \\ &\quad \geq \Pr\{|\theta_j(t_k) - b| \leq \delta \cap |\theta_j(t_{k-1}) - a| \leq \delta\} \geq \epsilon^2, \end{aligned}$$

which means that for all k,

$$\Pr\left\{\alpha_{ij} - e_{ij}(k) \leq \frac{2\delta}{\tau}\right\} \geq \epsilon^2, \quad i, j \in V. \tag{18}$$

From (14) and (18), we have

$$\begin{aligned}
&\Pr\left\{\alpha_{ij} - \hat{\alpha}_{ij}(k) \le \frac{2\delta}{\tau}\right\} \\
&= \Pr\left\{\alpha_{ij} - \max\{e_{ij}(1), e_{ij}(2), \ldots, e_{ij}(k)\} \le \frac{2\delta}{\tau}\right\} \\
&= 1 - \Pr\left\{\alpha_{ij} - e_{ij}(l) > \frac{2\delta}{\tau} | l = 1, 2, \ldots, k\right\} \\
&\ge 1 - (1 - \epsilon^2)^k, \qquad (19)
\end{aligned}$$

for each iteration k. Therefore, by setting $\sigma = \frac{2\delta}{\tau}$ and $k \to \infty$, Eqs. (15) and (16) hold.

From Assumption 1, we can find a positive ϵ, $\forall \delta > 0$, which guarantees that (16) holds. Since $\sigma = \frac{2\delta}{\tau}$, we have that for any estimation accuracy $\sigma > 0$, it can be achieved with probability one guaranteed by Lemma 1. Hence, (14) can guarantee any accuracy requirement with probability one, i.e., the estimation of the relative skew converges to the real relative skew almost surely. Meanwhile, from (19), it follows that the probability of an accurate estimation of the relative skew converges to one exponentially fast, which means that the estimation algorithm (14) has an exponential speed in probability. Moreover, from Lemma 1, note that for any small positive δ, we have $\Pr\{|\theta_j(t_k) - b| \le \delta\} \ge \epsilon > 0$ and $\Pr\{|\theta_j(t_{k-1}) - a| \le \delta\} \ge \epsilon > 0$ which can guarantee the estimation accuracy as $\sigma = \frac{2\delta}{\tau}$, especially when $\delta \to 0$, a completely accurate estimation is achieved, i.e.,

$$\Pr\{\lim_{k\to\infty} \alpha_{ij} - \hat{\alpha}_{ij}(k) = 0 | \langle i, j\rangle \in E\} = 1.$$

Thus, in the remainder of this paper, we assume that $\Pr\{\theta_i(t) = c\} \ge \epsilon$ for all $i \in V$, where $c = a$ or $c = b$, and $0 < \epsilon < 1$ is a small positive constant.

Remark 3. It should be noticed that in the previous literature (e.g., He, Cheng, Shi et al., 2014; Schenato & Fiorentin, 2011), $e_{ij}(k)$ was estimated by

$$e_{ij}(k) = \frac{H_j^+(t_k) - H_j^+(t_{k-1})}{H_i(t_k) - H_i(t_{k-1})}.$$

Then, they used the weighted average of multi-step estimations ($e_{ij}(\ell)$, $\ell = 1, 2, \ldots, k$) to estimate the relative skew. For such relative skew estimation approach, if the random noise in (2) have different expected value at different time t, then

$$\mathbf{E}\{e_{ij}(k)\} = \frac{\alpha_j}{\alpha_i} + \frac{\mathbf{E}\{\theta_j(t_k)\} - \mathbf{E}\{\theta_j(t_{k-1})\}}{H_i(t_k) - H_i(t_{k-1})} \ne \frac{\alpha_j}{\alpha_i}.$$

Hence, a weighted average process cannot obtain an accurate estimation of relative skew, so an accurate clock synchronization cannot be achieved.

Remark 4. If the noise bounds, a and b, are not exactly known to each node i, then we can use the estimation of $b - a$, denoted by $\hat{d}$, to substitute $b - a$ in (13), where the method of obtaining $\hat{d}$ is given in Section 4.2. Then, (13) is changed to

$$e_{ij}(k) = \frac{H_j^+(t_k) - H_j^+(t_{k-1}) - (b-a)}{H_i(t_k) - H_i(t_{k-1})} + \frac{(b-a) - \hat{d}}{H_i(t_k) - H_i(t_{k-1})}.$$

Thus, the accuracy of the relative skew estimation will depend on $\frac{b-a-\hat{d}}{H_i(t_k) - H_i(t_{k-1})}$. In this case, we can set a large communication time interval $(H_i(t_k) - H_i(t_{k-1}))$ to guarantee highly accurate estimation of the relative skew.

3.3. Skew compensation

By referring to our earlier work (He, Cheng, Shi et al., 2014), based on the maximum consensus approach, we design the following skew compensation iteration algorithm. When node i receives information $H_j^+(t_k)$ and $\tilde{\alpha}_j(k)$ from neighbor node j, it updates its clock skew as

$$\tilde{\alpha}_i(t_k^+) = \max\{\tilde{\alpha}_i(t_k), \hat{\alpha}_{ij}(t_k)\tilde{\alpha}_j(t_k)\},$$

where t_k^+ is the time just after updating at time t_k (at iteration k), with the initial condition $\tilde{\alpha}_i(t_0) = 1$. By multiplying with α_i on both sides of the above equation, we have

$$\begin{aligned}
x_i(t_k^+) &= \max\{x_i(t_k), x_j(t_k) - \alpha_i[\alpha_{ij} - \hat{\alpha}_{ij}(t_k)]\} \\
&= \max\{x_i(t_k), x_j(t_k) - g_{ij}(t_k)\}, \qquad (20)
\end{aligned}$$

where $g_{ij}(t_k) = \alpha_i[\alpha_{ij} - \hat{\alpha}_{ij}(t_k)]$.

Theorem 2. *Consider the skew update equation given by (20) with the initial condition $\tilde{\alpha}_i(t_0) = 1$. Then, there exists a constant c_x such that*

$$\Pr\{\lim_{k\to\infty} x_i(t_k) = c_x\} = 1, \quad \forall i \in V. \qquad (21)$$

Proof. Note that $\hat{\alpha}_{ij}(t_k)$ is an increasing function of iteration k with an upper bound α_{ij}, which implies that $g_{ij}(t_k)$ is a positive decreasing function of iteration k with a lower bound 0, i.e., $0 < g_{ij}(t_k) \le g_{ij}(t_{k-1})$. Then, by Lemma 1, we have

$$\Pr\{\lim_{k\to\infty} g_{ij}(t_k) = 0 | i, j \in V\} = 1. \qquad (22)$$

From (20), we have

$$\max_{l=1,2,\ldots,n} x_l(t_0) \ge \max_{l=1,2,\ldots,n} x_l(t_1) \cdots \ge \max_{l=1,2,\ldots,n} x_l(t_k),$$

and

$$x_i(t_k) \ge x_i(t_{k-1}).$$

Hence, $x_i(t_k)$ is an increasing function of iteration k with an upper bound $\max_{l=1,2,\ldots,n} x_l(t_0)$, i.e., $\max_{l=1,2,\ldots,n} x_l(t_0) \ge x_i(t_k) \ge x_i(t_{k-1})$, $i \in V$. It implies that the limit of each $x_i(t_k)$ exists and is given by

$$\lim_{k\to\infty} x_i(t_k) = c_i, \quad i \in V, \qquad (23)$$

where $c_i \le \max_{l=1,2,\ldots,n} x_l(t_0)$ is a constant.

Let i_0 be the node with $x_{i_0}(t_k) = \max_{l=1,2,\ldots,n} x_l(t_k)$, and let i_m be one of the m-hop neighbor node of i_0. Since the network is strongly connected, we have $m \le N - 1$. Then, from (20), we also have

$$\begin{aligned}
x_{i_m}(t_{k+m}) &\ge x_{i_{m-1}}(t_{k+m-1}) - g_{i_m i_{m-1}}(t_{k+m-1}) \\
&\ge \cdots \\
&\ge x_{i_0}(t_k) - \sum_{l=1}^{m} g_{i_{m-l+1} i_{m-l}}(t_{k+m-l}) \\
&\ge \max_{l=1,2,\ldots,n} x_l(t_k) - m \max_{l=1,2,\ldots,m} g_{i_{m-l+1} i_{m-l}}(t_k),
\end{aligned}$$

i.e., for each node i, we have

$$x_i(t_{k+m}) \ge \max_{l=1,2,\ldots,n} x_l(t_k) - m \max_{l=1,2,\ldots,m} g_{i_{m-l+1} i_{m-l}}(t_k). \qquad (24)$$

Taking limitation on both sides of (24) leads to

$$c_i \ge \max_{l=1,2,\ldots,n} c_l - m \max_{l=1,2,\ldots,m} \lim_{k\to\infty} g_{i_{m-l+1} i_{m-l}}(t_k), \quad i \in V,$$

and then it follows from (22) that

$$\Pr\{c_i = \max_{l=1,2,\ldots,n} c_l | i \in V\} = 1.$$

Hence, from (23), we have

$$\Pr\{\lim_{k\to\infty} |x_i(t_k) - x_j(t_k)| = 0\} = 1, \quad i, j \in V,$$

which means that (29) holds.

Theorem 2 guarantees that skew compensation converges with probability one, which means that the clock skews of all nodes are able to be synchronized completely. Moreover, from (19) and the definition of $g_{ij}(t_k)$, we have

$$\Pr\{g_{ij}(t_k) = 0\} = \Pr\{\alpha_{ij} - \hat{\alpha}_{ij}(t_k) = 0\} \geq 1 - (1-\epsilon^2)^k.$$

Hence, it follows that

$$\begin{aligned} &\Pr\{g_{ij}(t_k) = 0 | \langle i, j\rangle \in E\} \\ &\geq [1-(1-\epsilon^2)^k]^{n^2} \geq 1 - n^2(1-\epsilon^2)^k. \end{aligned} \tag{25}$$

Meanwhile, if each $g_{ij}(t_k) = 0$, Eq. (20) is maximum consensus, and then it has $x_i(t_{k+n}) = x_j(t_{k+n}) = \max_{l=1,2,\ldots,n} x_l(t_k)$ holds for $i, j \in V$, i.e., clock skew compensation is accomplished. Hence, the clock skew compensation under (20) converges exponentially in probability.

3.4. Offset compensation

Similar to the skew compensation, the maximum consensus is also adapted to design the clock offset compensation algorithm. Specifically, when node i receives information $H_j^+(t_k)$ and $\tilde{\beta}_j(k)$ from neighbor node j, it updates its clock offset as

$$\begin{aligned} \tilde{\beta}_i(t_k^+) = &\max\{\tilde{\beta}_i(t_k), \tilde{\alpha}_j(t_k)(H_j^+(t_k) - b) \\ &+ \tilde{\beta}_j(k) - \tilde{\alpha}_i(t_k)H_i(t_k)\}, \end{aligned} \tag{26}$$

with initial condition $\tilde{\beta}_i(t_1) = 0$. The above equation satisfies

$$\begin{aligned} \tilde{\beta}_i(t_k^+) + \tilde{\alpha}_i(t_k)\beta_i = &\max\{y_i(t_k), \tilde{\alpha}_j(t_k)H_j(t_k) + \tilde{\beta}_j(k) \\ &+ \tilde{\alpha}_j(t_k)(\theta_j(t_k) - b) - x_i(t_k)t_k\} \\ = &\max\{y_i(t_k), (x_j(t_k) - x_i(t_k))t_k + y_j(t_k) \\ &+ \tilde{\alpha}_j(t_k)(\theta_j(t_k) - b)\}. \end{aligned} \tag{27}$$

Thus, we have

$$\begin{aligned} &y_i(t_k^+) - (\tilde{\alpha}_i(t_k^+) - \tilde{\alpha}_i(t_k))\beta_i \\ &= \max\{y_i(t_k), (x_j(t_k) - x_i(t_k))t_k + y_j(t_k) \\ &\quad + \tilde{\alpha}_j(t_k)(\theta_j(t_k) - b)\}. \end{aligned} \tag{28}$$

Theorem 3. *Consider the offset update equation given by (28) with the initial condition $\tilde{\beta}_i(t_1) = 0$. Then, there exists a constant c_y such that*

$$\Pr\{\lim_{k\to\infty} y_i(t_k) = c_y\} = 1, \quad \forall i \in V. \tag{29}$$

Proof. From Theorem 2, we have $\lim_{k\to\infty} \tilde{\alpha}_i(t_k) = \frac{c_i}{a_i}$, which means that

$$\begin{aligned} &\lim_{k\to\infty} |\tilde{\alpha}_i(t_k^+) - \tilde{\alpha}_i(t_k)| \\ &\leq \lim_{k\to\infty} |\tilde{\alpha}_i(t_k^+) - \frac{c_i}{a_i}| + \lim_{k\to\infty} |\tilde{\alpha}_i(t_k) - \frac{c_i}{a_i}| \leq 0. \end{aligned} \tag{30}$$

Since the clock skew compensation under (20) converges exponentially in probability, by Theorem 2, one can infer

$$\Pr\{\lim_{k\to\infty} (x_j(t_k) - x_i(t_k))t_k = 0 | i, j \in V\} = 1. \tag{31}$$

When (30) and (31) hold, we can simplify (28) as

$$y_i(t_k^+) = \max\{y_i(t_k), y_j(t_k) + \tilde{\alpha}_j(t_k)(\theta_j(t_k) - b)\}.$$

Since the limitation of each $\tilde{\alpha}_j(t_k)$ is $\frac{c_j}{\alpha_j}$ and $\Pr\{\theta_j(t_k) = b\} \geq \epsilon$, we have

$$\Pr\{\lim_{k\to\infty} |y_i(t_k) - y_j(t_k)| = 0\} = 1,$$

for $\forall i, j \in V$.

Theorem 3 guarantees that offset compensation converges with probability one, which means that the clock offsets of all nodes are able to be synchronized completely.

Since both of the skew and offset compensation converge with probability one, i.e., the goals (4) and (5) have been achieved using the above clock synchronization algorithm, which guarantees that a highly accurate clock synchronization can be achieved under the bounded noise, and a complete synchronization is realized when $t \to \infty$. Meanwhile, following from (19) and (25), a larger ϵ will make the probability converge to one faster. Moreover, the convergence of both the skew and the offset compensation are not affected by the value of the bounded noise.

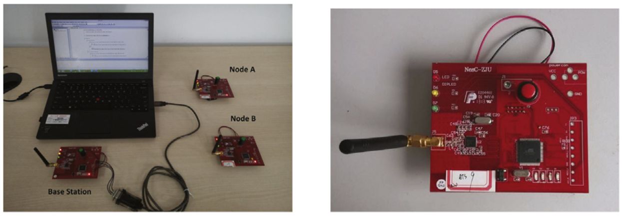
Remark 5. It follows from (20) that the convergence of skew compensation depends on the convergence of maximum consensus and the relative skew estimation. From (28), the convergence of offset compensation depends on the convergence of maximum consensus, the skew compensation, and delay compensation (caused by $(\theta_j(t_k) - b)$). Theorem 1 guarantees the relative skew estimation and delay compensation can converge exponentially, and they are independent of the network topology. Hence, the proposed algorithm can converge within less than $N - 1$ (the diameter of the network topology) for any strongly connected network topology, provided that the relative skew is estimated accurately and the delay compensation is finished.

4. Experimental verification

In this section, we present experimental results to validate the basic assumption (Assumption 1) and the effectiveness of the proposed approach of the relative skew estimation, on which the synchronization accuracy depends.

4.1. Testbed

The experimental testbed, shown in Fig. 1(a), is composed of 3 Micaz sensor nodes and a PC. Among the 3 nodes, two are normal nodes, and another is the base station. The communication processes are completed by the two normal nodes. The base station connected with the PC collects communication packets (from normal nodes) and reports them to the PC through serial port communication. The PC processes the collected data, and then verifies the assumption and the effectiveness of our approach. Micaz is a wireless sensor mote developed by Crossbow Technology. It is a combination of an ATmega 128L micro-processor and a CC2420 wireless communication chip. The ATmega 128L micro-processor equips with an internal 8 MHz crystal oscillator and an external 32 kHz crystal oscillator, and the latter one serves as the hardware clock to be synchronized with a synchronization algorithm. The clock period is 30.5 μs. The CC2420 chip, which is based on IEEE 802.15.4/ZigBee communication protocol, operates within the 2.4 to 2.48 GHZ band and allows a data rate of 250 kbps. A picture of real micaz mote (assembled by our research group) used in the experiment is illustrated in Fig. 1(b).



(a) Testbed. (b) Micaz node.

Fig. 1. The hardware platform.

4.2. Verification

During the experiment, two normal nodes A and B transmit packets containing the timestamp of the transmission time every 10 s based on their own hardware clock. Upon receiving the packet, the node records the reception time and assembles a clock pair, i.e., the transmission time and reception time of the same packet. Then the node includes the clock pair in the packet to be transmitted in the next period. Based on these clock pairs, we can estimate the relative skew and the communication delays between nodes.

Let $(H_A^b(t_k), H_B^r(t_k))$ be a pair of the clock reading, where $H_A^b(t_k)$ is the clock reading of node A at kth period and $H_B^r(t_k)$ is the clock reading of node B when it receives $H_A^b(t_k)$ from node A. Note that

$$\frac{H_B^r(t_k)-H_B^r(t_{k-1})}{H_A^b(t_k)-H_A^b(t_{k-1})} = \frac{\alpha_B(t_k-t_{k-1})+\alpha_B(d_k-d_{k-1})}{\alpha_A(t_k-t_{k-1})} = \frac{\alpha_B}{\alpha_A}+\frac{\alpha_B(d_k-d_{k-1})}{10}, \tag{32}$$

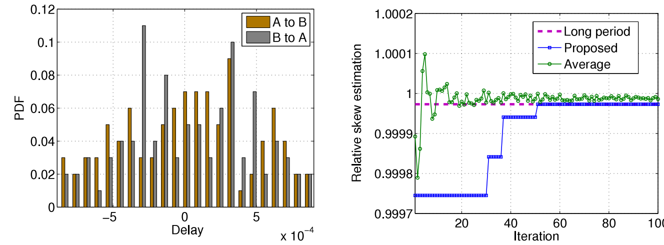
where d_k is the communication delay. In (32), the value of the left hand is obtained directly using the value of the clock reading pairs while for the right hand, $\frac{\alpha_B}{\alpha_A}$ is approximated by the following equation

$$\frac{\alpha_B}{\alpha_A} \approx \frac{H_B^r(t_K)-H_B^r(t_0)}{H_A^b(t_K)-H_A^b(t_0)}, \tag{33}$$

where K is a large integer which guarantees a highly accurate approximation (we set $K = 100$ in our experiments), and α_B is approximated by 1 since the clock speed of each node is in 1 ± 10^{-5} and 10^{-5} is much smaller than the delay (He, Li et al., 2014). Hence, we can obtain a series of the difference of two times of delay $d_k - d_{k-1}$ for $k = 1, 2, \ldots$, and the distribution of them (statistics from 100 times experiments) are shown in Fig. 2(a). It is observed that the difference of two times of delay is bounded by a symmetric interval and could be any values in the interval, and also have same distribution for the delays from A to B and B to A. Hence, it verifies the soundness of Assumption 1, which can also be the verification of the model in Garone et al. (2015).

Next, we use the upper bound of $d_k - d_{k-1}$ obtained from the experiments as the value of $b - a$ in (13), and use (14) for relative skew estimation. Then, we compare the different relative skew estimation methods in Fig. 2(b), where our method is marked as Proposed, the approximation using (33) (which can be viewed as the benchmark) is marked as Long period, and the average of multi-step estimation for the relative skew is marked as Average. It is clear to see that our method can converge to a much higher accurate relative skew estimation than other ones. Hence, it verifies the effectiveness of the relative skew estimation method.

From the above experiments, one sees that the bound of the delay and the accurate relative skew estimation can be obtained with our approach, on which a highly accurate clock synchronization depends. Therefore, our modeling and algorithm can guarantee highly accurate clock synchronization, and it will be further demonstrated with simulation in the following section.



(a) Difference of two times delay. (b) Relative skew estimation.

Fig. 2. Verification and comparison.

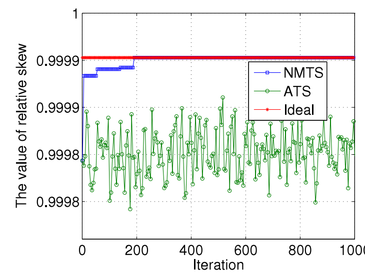


Fig. 3. Comparison of relative skew estimation. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

5. Performance evaluation

In this part, we compare our algorithm with a typical ATS algorithm (Schenato & Fiorentin, 2011). Since the maximum consensus is used in our algorithm and an accurate synchronization can be achieved under bounded noise, we name it as Noise-resilient Maximum-consensus-based clock synchronization (NMTS). The parameter settings in the simulation have referred to the experimental results obtained in Section 4.

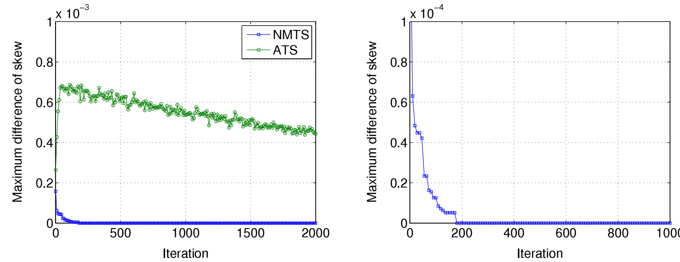
Consider the network with 50 nodes which are randomly deployed in a 100 m × 100 m area, and the maximum communication range of each node is 20 m. For the simulation examples, we set the initial condition $\hat{\alpha}(0) = 1$ and $\hat{\beta}(0) = 0$ for both NMTS and ATS. As in ATS, we set the common broadcast period to be one second. Note that the typical error for a quartz crystal oscillator is between 10 ppm and 100 ppm, (Choi et al., 2012), which corresponds to a 10 to 100 microsecond (μs) during the broadcast interval. Thus, each skew α_i in simulation is randomly selected from the set [0.9999, 1.0001], and the offset b_i is randomly selected from the set [0, 0.0002]. The parameters used in ATS is set as $\rho_o = \rho_v = 0.5$ and $\rho_\eta = 0.2$, which are the same as those in Schenato and Fiorentin (2011). By referring to the experimental results obtained in Section 4, we set $a = 0$ and $b = 0.0005$, and $\Pr\{\theta_i(t_k) = a\} = \Pr\{\theta_i(t_k) = b\} = 0.04$ for each iteration k. But it should be pointed out that the value of the bounded noise does not affect the convergence of our algorithms as proved in our theoretical results. We also define two functions as follows:

$$d_s(t) = \max_{i,j\in\mathcal{V}} \left(x_i(t) - x_j(t)\right),$$

$$d_o(t) = \max_{i,j\in\mathcal{V}} \left(y_i(t) - y_j(t)\right),$$

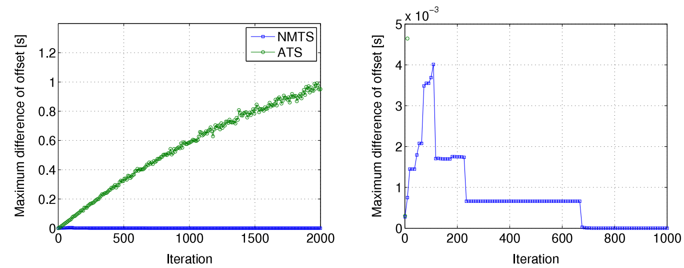
where $d_s(t)$ and $d_o(t)$ denote the maximum differences of the software skew and of the software offset between any two nodes, respectively. Clearly, the clock synchronization is reached completely iff $d_s(t) = 0$ and $d_o(t) = 0$.

First, taking the bounded noise into consideration, we compare our relative skew estimation method used in NMTS and that in ATS, where in NMTS the method is given by (14) while in ATS it uses the weighted average of the current one-step estimation and the



(a) NMTS and ATS. (b) NMTS.

Fig. 4. Comparison of skew compensation.



(a) NMTS and ATS. (b) NMTS.

Fig. 5. Comparison of offset compensation.

last-time estimation as the current relative skew estimation. Fig. 3 shows the estimated results of relative skew α_{12}. It is observed that using (14), α_{12} can be estimated accurately as $\hat{\alpha}_{12}(k)$ (the blue line) will converge to the ideal value α_{12} (the red line), while using the method in ATS, the average estimate error is about 0.0001.

We then compare the performance of skew compensation for the two algorithms. As shown in Fig. 4, NMTS has a faster convergence speed and a higher synchronization accuracy than those of ATS. A more clear result about NMTS is shown in Fig. 4(b), in which we can see that the skew compensation converges after iteration 190 completely, which means that by NMTS all nodes' clock skews can be synchronized completely even with the noise. The results of NMTS validate the theoretical results in Theorem 2.

Finally, we compare the performance of offset compensation for NMTS and ATS. The result is given in Fig. 5. It is clear that NMTS has a much better accuracy in offset compensation. For ATS, the maximum difference of the software clock offset becomes worse as the iteration increases, which means that the synchronization is not fully achieved. The main reason is that ATS cannot synchronize the node's software clock skews completely under bounded noise. The difference between nodes' software clocks increases with time, and the offset cannot be compensated fast enough. Thus, Garone et al. (2013) have proposed a novel robust ATS algorithm to prevent the error from becoming larger under random bounded communication delay. For NMTS, it is clear to see from Fig. 5(b) that although the maximum difference between nodes' software clocks may also increase with time as the software clock skew has not yet been compensated sufficiently in the early time, the difference can be reduced and converged to 0, which corresponds to Theorem 3.

6. Conclusions

This paper has investigated distributed clock synchronization for WSNs with bounded noise. By taking the advantage of the unique features of bounded monotonic sequence and the concept of maximum consensus, we proposed a novel distributed clock synchronization algorithm, including the relative skew estimation and software clock skew and offset compensations, to achieve accurate clock synchronization. It has been proved that our proposed algorithm can achieve complete clock synchronization with probability one under bounded noise. Experimental results are presented to validate the modeling and the basic idea used for algorithm design. Extensive simulations demonstrated that the proposed algorithm has a faster convergence speed and a higher synchronization accuracy than that of typical average consensus-based clock synchronization algorithms.

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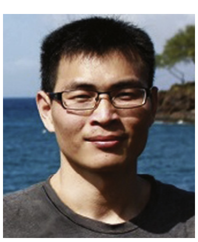
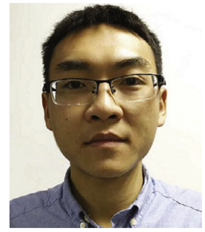


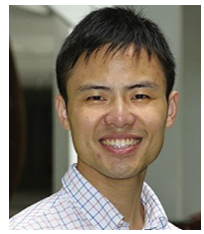
Jianping He is currently an Associate Research Fellow in the Department of Automation at Shanghai Jiao Tong University.

He received the Ph.D. degree in Control Science and Engineering in 2013, at Zhejiang University, Hangzhou, China.

His research interests include the control and optimization of sensor networks and cyber-physical systems, the scheduling and optimization in VANETs and social networks, and the investment decision in financial market and electricity market. He served as an Associate Editor for a special issue on Consensus-based Applications in Networked Systems in the International Journal of Robust and Nonlinear Control in 2016.

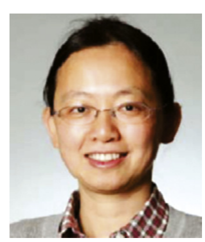


Xiaoming Duan obtained his B.E. degree in Automation from the Beijing Institute of Technology, Beijing, China, in 2013, and his Master's Degree in Control Science and Engineering from Zhejiang University, Hangzhou, China, in 2016. He is currently working toward the Ph.D. degree in the Department of Mechanical Engineering, UC Santa Barbara. His research interests include control and synchronization in networked systems and distributed computation and optimization in multi-agent networks.



Peng Cheng received the B.E. degree in Automation, and the Ph.D. degree in Control Science and Engineering in 2004 and 2009 respectively, from Zhejiang University, Hangzhou, China. Currently he is Professor with College of Control Science and Engineering, Zhejiang University. He serves as Associate Editor of IEEE Transactions on Control of Network Systems, Wireless Networks, and International Journal of Communication Systems. He has served as the Guest Editor of IEEE Transactions on Control of Network Systems, and IEEE Transactions on Signal and Information Processing over Networks. He served as the tpc co-chair of IEEE IOV 2016, local arrangement co-chair for ACM MobiHoc 2015, and the publicity co-chair for IEEE MASS 2013. His research interests include networked sensing and control, cyber-physical systems, control system security.

Ling Shi received the B.S. degree in Electrical and Electronic engineering from Hong Kong University of Science and Technology, Kowloon, Hong Kong, in 2002 and the Ph.D. degree in Control and Dynamical Systems from California Institute of Technology, Pasadena, CA, USA, in 2008. He is currently an Associate Professor at the Department of Electronic and Computer Engineering, Hong Kong University of Science and Technology. His research interests include cyber-physical systems security, networked control systems, sensor scheduling, and event-based state estimation. He has been serving as a Subject Editor for International Journal of Robust and Nonlinear Control from March 2015 and as Asociate Editor for IEEE Transactions on Control of Network Systems from July 2016. He also served as Asociate Editor for a special issue on Secure Control of Cyber Physical Systems in the IEEE Transactions on Control of Network Systems in 2015–2016.



Lin Cai received her M.A.Sc. and Ph.D. degrees in Eelectrical and Computer Engineering from the University of Waterloo, Waterloo, Canada, in 2002 and 2005, respectively. Since 2005, she has been with the Department of Electrical & Computer Engineering at the University of Victoria, where she is currently a Professor. Her research interests span several areas in communications and networking, with a focus on network protocol and architecture design supporting emerging multimedia traffic over wireless, mobile, ad hoc, and sensor networks. She has been a recipient of the NSERC Discovery Accelerator Supplement Grants in 2010 and 2015, and the best paper awards of IEEE ICC 2008 and IEEE WCNC 2011. She has served as a TPC symposium co-chair for IEEE Globecom'10 and Globecom'13, and as Associate Editor for IEEE Transactions on Wireless Communications, IEEE Transactions on Vehicular Technology, EURASIP Journal on Wireless Communications and Networking, International Journal of Sensor Networks, and Journal of Communications and Networks (JCN).