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Practical closed-loop dynamic pricing in smart grid for supply and demand balancing☆

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ABSTRACT

Pricing strategy for power systems is an important and challenging problem, due to the difficulties in predicting the demand and the reactions of customers to the price accurately. Any prediction errors may result in higher costs to the supplier. To address this issue, in this paper, we propose a novel, practical closed-loop pricing algorithm (PCPA). Using the closed-loop control to well coordinate the customers and the supplier, the power system can run more efficiently, resulting in both cost saving for customers and higher profit for the supplier. We prove the convergence of PCPA, i.e., a stable price can be achieved. We provide sufficient conditions to guarantee the win-win solution for both the customers and the supplier, and an upper bound of the gain. We also provide a necessary and sufficient condition of that the highest win for both the customers and the supplier can be achieved. Extensive simulations have shown that PCPA can outperform the existing prediction-based pricing algorithms. It shows that the profit gain of the proposed algorithm can up to 100% when the total demand can be fixed to the optimal demand.

1. Introduction

Enabled by new technologies, such as the intelligent and autonomous control, two-way communications between the power supplier and customers, and the advanced software-based data management, traditional power grids can be upgraded to smart grids that can intelligently incorporate distributed energy sources and deliver the power to customers efficiently (Fang, Misra, Xue, & Yang, 2012). Different from the traditional power grid, in smart grids, the supply and demand sides interact with each other by exchanging the price and demand information, aiming to minimize over-provisioning at the supply side (Yu & Hong, 2016). To improve efficiency, reduce peak load and balance the demand and supply, dynamic pricing has been advocated and become a promising technology (Borenstein, Jaske, & Rosenfeld, 2002; Chen, Wei, & Hu, 2013; Liang, Li, Lu, Lin, & Shen, 2013; Liu, Liu, Low, & Wierman, 2014; Samadi, Mohsenian-Rad, Schober, Wong, & Jatskevich, 2010; Sen, Joe-Wong, Ha, & Chiang, 2013; Tarasak, 2011). Based on dynamic pricing, considerable benefits will be gained by encouraging the customers to consume energy in a more efficient way (Deng, Yang, Hou, Chow, & Chen, 2015; Kim, Zhang, Schaar, & Lee, 2014; Wen et al., 2013; Zhang & Papachristodoulou, 2015). A proper dynamic pricing strategy cannot only smooth load demand curves to enhance the robustness and lower the generation cost of the power grid, but also reduce the electricity expenditures of the customers by reasonably scheduling their flexible electricity usage. However, how to design a proper dynamic pricing strategy is still a challenging problem given the difficulty in estimating the load accurately. The estimation errors are unavoidable due to the random demand, and the lack of knowledge in customers' preference and their reactions to price change (Joe-Wong, Sen, Ha, & Chiang, 2012; Qian, Zhang, Huang, & Wu, 2013; Wu et al., 2015). We refer the readers to the survey papers (Annaswamy, Hussainy, Chakrabortty, & Cvetkovic, 2016; Khan, Mahmood, Safdar, Khan, & Khan, 2016) for more details about dynamic pricing, price-based control and the corresponding open issues in smart grids.

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In the past few years, dynamic pricing in smart grids has attracted extensive attention, and many pricing schemes were developed in the literature, including real time pricing (Joe-Wong et al., 2012; Mohsenian-Rad & Leon-Garcia, 2010; Mohsenian-Rad, Wong, Jatskevich, Schober, & Leon-Garcia, 2010; Qian et al., 2013), time of use (Braithwait, Hansen, & Sheasy, 2007), and critical peak pricing (Kii, Sakamoto, Hangai, & Doi, 2014), and many more as discussed in Khan et al. (2016). The existing pricing schemes can be divided into two categories. The first one aims to maximize the profits of customers, and deals with how the customers schedule their flexible electricity usage to achieve their desired level of comfort with a lower electricity bill payment based on the prediction of future price (Mohsenian-Rad & Leon-Garcia, 2010; Mohsenian-Rad et al., 2010). The second takes both the customers' cost and the supplier's profits into consideration, and deals with how to determine the appropriate prices according to the prediction of the customer's energy consumption and their reaction to a given price (Braithwait et al., 2007; Chen, Li, Low, & Wang, 2010; Joe-Wong et al., 2012; Kii et al., 2014; Kim et al., 2014; Li, Chen, & Low, 2011; Paschalidis, Li, & Caramanis, 2012; Qian et al., 2013; Roozbehani, Dahleh, & Mitter, 2010a; Samadi et al., 2010; Tarasak, 2011). There is a common feature for these existing schemes, i.e., the decision was based on the prediction of the future price or the customers' reaction on a given price. In other word, the scheduling at the customer side is based on future price prediction, and the pricing determined by the supplier is based on the demand prediction. Hence, they study an open-loop decision problem from the perspective of control theory given the prediction-based decision, and thus these existing scheduling and pricing strategies are named as open-loop scheduling and pricing in this paper. Since the scheduling at the demand side and the pricing at the supply side are separated, it will cause high cost for both the customers and the supplier when the prediction is not accurate. For instance, a very high cost will be caused to the supplier when the customers' demand determined by their scheduling strategy is greatly deviated from the total amount of the electricity provided by the supplier. In contrast, if the loads are delayed to a high cost time interval, the customers will have much higher utility bills. In order to make wise pricing decisions, the price and demand information should be exchanged between the supply and demand sides, and then we can optimize the strategies for both.

Therefore, Roozbehani et al. (2010a) and Roozbehani, Dahleh, and Mitter (2010b) proposed closed-loop dynamic pricing algorithms to achieve a stable price by constructing a feedback loop between the customers and the supplier. The proposed algorithms can achieve a very good performance when the supplier follows demand precisely. Inspired by these works, in this paper, we further investigate the closed-loop pricing in a more realistic scenario, and the assumption that supply follows demand precisely is removed, so the randomness at the demand side is taken into consideration. We first design a novel practical closed-loop pricing algorithm (PCPA) using a piecewise pricing approach. The proposed algorithm largely improves the system efficiency and results in both cost savings for customers and higher profits for the supplier, and thus achieves a win-win solution. In summary, compared with the existing open-loop pricing, PCPA can largely decrease the probability of high cost and thus potentially save the cost a lot. Compared with the existing closed-loop pricing algorithm, firstly, our algorithm relaxes the assumption. Then, a piecewise pricing approach is adopted in PCPA, where a much higher price is used for the penalty and a lower price is used as incentive to the customers, rather than the single pricing approach used in the most existing literatures. Lastly, PCPA achieves a win-win solution for both the customers and the supplier.

The details of the PCPA have been introduced in our conference paper (He, Zhao, Cai, Cheng, & Shi, 2015). In this paper, we have improved the PCPA, and added an optimal open loop pricing algorithm to obtain the initial price. We also have improved the theoretical results on the win-win solution and added the proof to make it rigorous. In addition, the optimality analysis for the proposed algorithm is provided. We obtain the upper bound of the profit gain (i.e., the win) and its necessary and sufficient condition. The condition to achieve the lowest price using the proposed pricing scheme is obtained. The main contributions of this work are summarized as follows.

- We develop a novel and practical closed-loop pricing framework for supply and demand balancing, where the randomness of the customers' demand and the cost caused by the deviation between the real demand and the desirable load for the supplier, have been modeled.
- We analyze the disadvantages of open-loop-based pricing algorithms, and reveal the potentially higher cost of the algorithms especially when the total demand is larger than the maximum supply. To solve this problem, we propose a novel practical closed-loop pricing algorithm (PCPA) using a piecewise pricing approach, where a much higher price is used for penalty and a lower price is used as incentive to the customers.
- We prove that the proposed algorithm can achieve a stable price and a win-win solution for both the customers and the supplier. Meanwhile, we provide the optimality analysis, where the upper bound of the profit gain and its necessary and sufficient condition are obtained.
- Extensive simulations are conducted to demonstrate the effectiveness of the proposed algorithm. It shows that PCPA can outperform the existing prediction-based pricing algorithm by a profit gain up to 100% (when total demand is fixed to the optimal point).

The remainder of the paper is organized as follows. In Section 2, the problem of the pricing problem is formulated. Section 3 analyzes the disadvantages of the open-loop pricing algorithm. The closed-loop algorithm is introduced in Section 4 and its performance analysis is given in Section 5. Simulation results are presented in Section 6 for performance evaluation. Finally, Section 7 concludes the paper.

2. Modeling and problem setup

2.1. System model

Consider a smart grid consisting of the electricity supplier (supply side), end-users or customers (demand side), and a control center, as shown in Fig. 1. On the supply side, the supplier generates the electricity and sells it to the end-users. On the demand side, each customer purchases the electricity from the supplier to satisfy its electricity demand. The control center is a not-for-profit organization responsible for determining a price in order to balance the supply and the demand. This role of the control center is the same as the Independent System Operator (ISO) proposed in Roozbehani et al. (2010b).

In the above system model, suppose that both the supplier and the customers can communicate with the control center to exchange the price and the demand information. The time of each day is divided into multiple time-slots. The slot duration of each time-slot is given and set by the control center, which is made by a tradeoff between the amount of flexible load (the longer the duration, the less flexible demand) and the system complexity (Tarasak, 2011). In order to determine an appropriate price of a unit electricity, the control center will simultaneously consider the cost and profit functions of both the customers and the supplier at the beginning of each time slot. In this work, the time-correlation

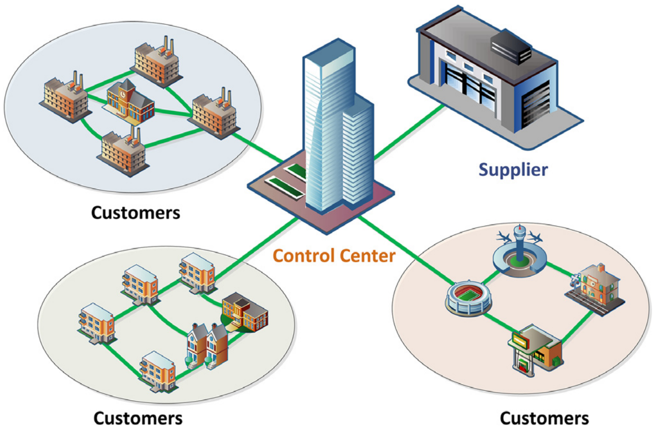


Fig. 1. An example of smart grid system model.

of the demand is not considered, i.e., the situation of the previous slot does not affect the price decision of the future slots, and we omit the time index in the remaining parts of this paper. We aim to design an efficient pricing scheme based on the exchanging information, such that the supply and the demand can be balanced, and both the supplier and customers' profit can be optimized.

2.2. Mathematical modeling and problem setup

On the demand side, assume that there are N customers. Let d_i be the random demand of customer i in the next time slot. Since the demand of each customer will be affected by the price, $d_i : \mathcal{R}^+ \rightarrow \mathcal{R}^+$ is modeled as a function of the price p, which satisfies

$$d_i(p) = x_i + r_i(p), i = 1, 2, \ldots, N, \tag{1}$$

where p is the unit price of electricity, x_i is the fixed electricity demand and $r_i(p)$ is the flexible electricity demand with $r_i : \mathcal{R}^+ \rightarrow \mathcal{R}^+$, respectively. In the above model, the fixed demand x_i is a constant since it denotes the rigid requirement of the customer in the next time-slot.[1] The flexible demand is price-sensitive and can be delayed or canceled according to the electricity price. Let $J_i^c(p)$ be the profit function for customer i, and then $-J_i^c(p)$ could be viewed as the cost function. We use the dollar value of consuming $d_i(p)$ units of electricity (Roozbehani et al., 2010b) to model the cost function, and then the profit function of customer i is given by,

$$J_i^c(p) = -pd_i(p).$$

When $d_i(p)$ is fixed, the profit function is decreased with the price p, which means that the customers obtain higher profits with a lower price. The profit function $J_i^c(p)$ can be viewed as the utility function of the customer i. It should be pointed out that we can use different convex function to model the profit functions of customers, and the pricing scheme proposed in this paper is still applicable.

On the supply side, in each time slot, the supplier plans to generate a certain amount of electricity s. Thus, s is the maximum demand that the supplier can provide. Let $D \leq s$ be the optimal demand that the supplier wishes to serve, and then $D - s$ can be viewed as a safety margin maintained. By referring to the profit model given in Liu et al. (2014), Roozbehani et al. (2010b) and Zhao, He, Cheng, and Chen (2017), the profit function of the supplier, $J^s : \mathcal{R}^+ \rightarrow \mathcal{R}^+$, is modeled as

$$\begin{aligned} J^s(p) &= p\sum_{i=1}^{N} d_i(p) - y(D - \sum_{i=1}^{N} d_i(p)) \\ &= pd(p) - y(D - d(p)), \end{aligned} \tag{2}$$

where $d(p) = \sum_{i=1}^{N} d_i(p)$ is the total demand of customers, and $y : \mathcal{R} \rightarrow \mathcal{R}^+$ is a generic function denoting the cost caused by the deviation between D and $d(p) = \sum_{i=1}^{N} d_i(p)$. It is usually assumed that $y(\cdot)$ is convex, non-negative, and has a global minimum value $y(0) = c$, where c is a positive constant. This modeling and assumption is referred to the penalty function adopted in Liu et al. (2014). It can be seen that the profits of the supplier are increasing with the payment of the customers while decreasing with the cost. We can also add other convex cost functions to $J^s(p)$, e.g., the power generation cost, and it will not invalidate the basic design of our pricing scheme.

The control center will determine the price. It will combine the profit functions of both the customers and the supplier (e.g., the weighted sum of them) as the objective function for appropriate pricing. Let $E(\cdot) = \int_{-\infty}^{\infty} f_{(\cdot)}(\tau)\tau d\tau$ denote the expectation of random variables, where $f_{(\cdot)}$ is the Probability Density Function (PDF) of the random variable $(\cdot)$. An optimization problem for pricing is formulated as follows

$$\begin{aligned} \max_p J(p) &= \mathbf{E}\{\sum_{i=1}^{N} J_i^c(p) + J^s(p)\} \\ &= -\mathbf{E}\{y(D - d(p))\}. \end{aligned} \tag{3}$$

In the above modeling, we set a same weight to the profit functions of the customers and the supplier for simplicity, and we can use a similar approach proposed in the following part to solve the problem when the weights of the profit functions are not the same.

Table 1 summarizes a few important notations in this paper for easy reference.

In the following, we first analyze the disadvantages of an open-loop/prediction-based pricing, where there is no feedback between the customers and the control center in the pricing process, and then reveal the potential high cost of the algorithm. To solve these problems, we establish a communication loop between the customers and the control center, which enables the customers and the control center to communicate with each other during the pricing. Next, we design a closed-loop pricing algorithm to decrease the cost and enlarge the profits of the customers and the supplier. Finally, we provide the performance analysis of the proposed algorithm in both theory and simulation.

Table 1
Important notations.

Symbol	Definition
d_i	The random demand of customer i
$\mathcal{R}^+$	The set of positive real number
D	The optimal demand that the supplier wishes to see
s	The maximum demand that the supplier can support
J_i^c	The profit function of customer i
J^s	The profit function of the supplier
y	The cost function
$\mathbf{E}$	The expectation of random variables
f_p^o	The PDF of flexible demands under open-loop pricing
f_p^c	The PDF of flexible demands under closed-loop pricing
p_o	The price under open-loop pricing
p_l	The lower price under closed-loop pricing
p_m	The intermediate price under closed-loop pricing
p_h	The higher price under closed-loop pricing
ρ	The flexible ratio
z_i	The lowest guaranteed demand by customer i

[1] Unless otherwise specified, all the parameters in this paper have non-negative real values.

3. High costs of open-loop pricing algorithm

In this section, we investigate the open-loop pricing strategy, and reveal that such open-loop strategies introduce high costs unavoidably.

Note that the total demand $d(p)$ in problem (3) is a random variable since $r_i(p)$ is random in (1). The control center needs to predict $d(p)$. Let $f_p^o(\tau)$ be the PDF of $r(p)$ $(=\sum_{i=1}^N r_i(p))$ under a given price p, which is regressed from the historical consumption data (Kim et al., 2014; Liu et al., 2014). Then, the objective function (3) is rewritten as

$$J(p) = -\int_0^\infty f_p^o(\tau) y(D - x - \tau) \mathrm{d}\tau, \tag{4}$$

where $x = \sum_{i=1}^N x_i$ is the total fixed demand. To obtain the optimal price, the main challenge is to solve the regression of $f_p^o(\tau)$. If $f_p^o(\tau)$ is obtained and the feasible value of p is finite, then the optimal price is obtained by comparing $J(p)$ under different setting of p. In particular, if the closed-form of $f_p^o(\tau)$ is obtained, the optimal price is obtained by solving the stationary point(s) of $J(p)$. In the following, we denote that the optimal price obtained from the open-loop pricing approach is p_o^*, and the corresponding PDF is $f_{p_0^*}^o(\tau)$.

For this pricing approach, there are three problems: (i) the price highly depends on the accuracy of the prediction, and thus has low robustness against the fluctuation of customers' demands; (ii) unified pricing is used for different demand (fixed and flexible demand), and thus the randomness of the flexible demand cannot be well constrained by the price; and (iii) the profit gain may be very small even to the optimal pricing and be sensitive with the total demand due to high cost under the open-loop pricing.

We give an example to illustrate the third problem. When $d(p) > s$, i.e., the real demand is larger than the amount of electricity generated by the supplier, which leads to a large cost to the supplier. For example, the spinning reserve will be used by the supplier when $d(p) > s$ to satisfy the demands of the customers, which results in a much higher cost to the supplier. Furthermore, if the supplier has insufficient spinning reserve to meet the demand, the supplier has to purchase the electricity from other suppliers temporarily with a high price. Suppose the $y(D - d(p)) \geq M$ if $d(p) > s$, where M is a large constant. Then, we have

$$\int_s^\infty f_p^o(\tau) y(D - \tau) d\tau \geq M \Pr\{d(p) > s\}. \tag{5}$$

Since the randomness of the flexible demand cannot be well constrained by the price, $\Pr\{d(p) > s\}$ would not be a very small value. Hence, $M \Pr\{d(p) > s\}$ could still be a large constant, which means that the supplier still faces a large cost. See Fig. 2 as an example for illustration, although the probability that the total demands exceed D is low, the corresponding cost is very high, and thus the mean of the cost (average cost) could still be high. In addition, when the estimation error of $f_p^o(\tau)$ in interval $[s, \infty)$ cannot be ignored, the value of the cost may change significantly following a similar analysis, and then the obtained optimal price may cause a much higher cost than that from the initial estimation. To overcome these problems and decrease the average cost, the approach used in this paper is to decrease the probability of the excessive demands and the corresponding cost simultaneously by using the idea of closed-loop control.

4. Practical closed-loop pricing: design, strategy and algorithm

In this section, we introduce a novel practical closed-loop pricing algorithm (PCPA) inspired by the stabilizing pricing algorithm proposed in Roozbehani et al. (2010b). Different from Roozbehani et al. (2010b), the assumption that supply follows demand precisely is relaxed in PCPA. Meanwhile, a piecewise pricing approach is adopted in PCPA, rather than the single pricing approach used in most existing literature.

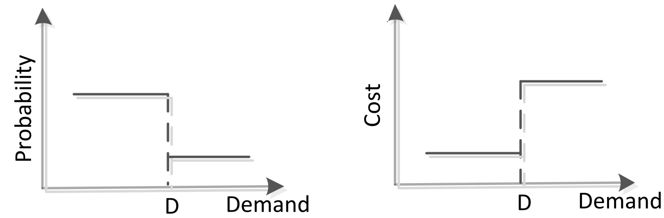


Fig. 2. Low probability corresponding to high cost.

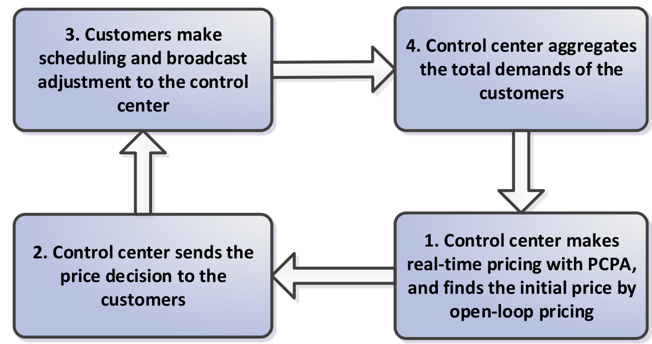


Fig. 3. The architecture of the closed-loop pricing algorithm.

The procedure of our algorithm is shown in Fig. 3. First, the control center will find an initial price through open-loop pricing (Algorithm 1 in Section 4.3). Second, the obtained price will be broadcast to the customers. Third, the customers then schedule (or re-schedule) their electricity usage and send their adjusted demand the control center. Fourth, the control center makes the demand aggregation, and it may change the price based on PCPA (Algorithm 2 in Section 4.3), and thus start a new round. Such a process will continue in the same time slot until a stable price is achieved. In this paper, we focus on the pricing, and the other parts in the closed-loop either have been studied in existing works, e.g., data aggregation (Rottondi, Verticale, & Krauss, 2013), or left as our future works, e.g., the optimal scheduling by the customers in the loop.

4.1. Pricing design

The basic idea of the PCPA is that piecewise pricing can give the customers incentive to fix part of the flexible demand (i.e., making the decision on whether to schedule flexible demand in this slot or not) in order to decrease the randomness of demand, and make the aggregated load close to the desired load D, which is further transformed into the cost saving and profits of both the customers and the supplier. Meanwhile, we set a much higher price for the excessive demands, which exceeds the maximum supply amounts, to largely decrease the probability of supply shortage. Also, a much higher price results in the customers undertake part of the high cost for the supplier. Thus, the highly cost part for the supplier can be decreased largely.

For the piecewise pricing, a lower price, denoted by p_l, is set for the demand that the customers can be guaranteed for usage (i.e., updated fixed demand) and has been broadcast to the control center; an intermediate price, denoted by p_m, is set for the flexible electricity usage which is in a given flexible interval, where the flexible ratio of the interval is defined as ρ ($\rho \geq 0$); and there is a much higher price, denoted by p_h, for the electricity usage exceeding the flexible interval, which can be viewed as a penalty price. Thus, the price p satisfies

$$p = \begin{cases} p_l, & \text{guaranteed demand}, \\ p_m, & \text{flexible demand}, \\ p_h, & \text{exceed demand}. \end{cases} \quad (6)$$

Customers' Decision: After receiving a price from the control center, the customer will fix some of his flexible demand for the next following time-slot, and update the fixed demand as the lowest demand from himself. Then, the lowest demand as a feedback is sent to the control center. Let $z_i(p, \rho)$ be the lowest guaranteed demand for the decision of customer i. $z_i(p, \rho)$ is modeled as a function of the price p and the flexible ratio ρ due to the following reasons.[2] Intuitively, a lower price can give incentive to the customers to fix more demand, and thus $z_i(p, \rho)$ is assumed to be a decreasing function with p. Meanwhile, it is an increasing function of ρ, because a lower flexible ratio means that the customers need to fix more flexible demand and thus have less flexibility to handle the uncertain demand. Then, we re-write the demand function of the customers (1) to

$$d_i(p) = z_i(p, \rho) + e_i(p), i = 1, 2, \ldots, N, \quad (7)$$

where $z_i(\cdot) \geq x_i$ is the updated fixed demand, and $e_i(p) \geq 0$ is the remainder flexible demand. Hence, under the piecewise pricing strategy, the payment of each customer satisfies

$$\begin{cases} p_l z_i + p_m(d_i - z_i), & d_i \leq (1+\rho)z_i, \\ p_l z_i + p_m z_i \rho + p_h[d_i - (1+\rho)z_i], & d_i > (1+\rho)z_i, \end{cases} \quad (8)$$

where $(z_i, (1+\rho)z_i]$ is the flexible interval for customer i, and ρ is the flexible ratio which is discussed as follows.

Flexible Ratio Setting: We consider how the control center sets the flexible ratio. Let $z = \sum_{i=1}^N z_i$ be the total lowest usage power aggregated from the customers' demand feedback. For simplicity, suppose that $s \geq z$, i.e., the power generation plan is larger than the lowest requirement of customers.[3] Noting that $s - z$ could be the flexible electricity for the customers use, we thus set ρ with

$$\rho = \frac{s - z}{z}. \quad (9)$$

From (9), we have $z(1+\rho) = s$. When $d_i > z_i(1+\rho)$, the total fixed requirement satisfies $z > s$, which will cause a very high cost. Hence, we set ρ according to (9), which ensures that the customers pay for the flexible usage exceeding with the high price p_h.[4]

4.2. Pricing strategy

We consider how to set the piecewise prices, i.e., p_l, p_m and p_h. We use the optimal price p_o^*, which can be computed from the equation shown in step 3 of Algorithm 1, as the benchmark price. Let $f_p^c(\tau)$ be the PDF of $e(p)$, where $e(p) = \sum_{i=1}^N e_i(p)$. It is reasonable to assume that $f_p^c(\tau)$ depends on p_m when $\tau \in [z, (1+\rho)z]$, and depends on p_h when $\tau \in [(1+\rho)z, \infty]$, but does not depend on p_l. $f_p^c(\tau)$ can also be regressed from the historical consumption data, or using $f_p^o(\tau)$ and the decreased randomness for the estimation.

[2] The customers can select $z_i(p, \rho)$ based on their preferences, and they can also adopt a utility function (e.g., the function (2) in Roozbehani et al. (2010b)) to determine the value of $z_i(p, \rho)$.

[3] When $z > s$, the control center will raise the price p_0^* to encourage customers to defer or cancel some flexible demand.

[4] We can also use the other setting of ρ, e.g., set $\rho = \frac{s-z}{z}$, which ensures that the customers pay for the usage exceeding the optimal demand with the high price p_h.

Let $J(p) - J(p_o^*)$ be the profit gain under the closed-loop pricing compared to the open-loop pricing. Then, we have

$$J(p) - J(p_o^*) = -\int_0^\infty f_p^c(\tau) y(D - z - \tau)\mathrm{d}\tau + \int_0^\infty f_{p_o^*}^o(\tau) y(D - x - \tau)\mathrm{d}\tau, \quad (10)$$

where $x = \sum_{i=1}^N x_i$ is the total fixed demand under open-loop pricing. Considering the full range of τ, we define

$$F_{p_o^*}^o(\tau) = \begin{cases} 0, & \tau \in [0, x), \\ f_{p_o^*}^o(\tau - x), & \tau \in [x, \infty), \end{cases} \quad (11)$$

and

$$F_p^c(\tau) = \begin{cases} 0, & \tau \in [0, z), \\ f_p^c(\tau - z), & \tau \in [z, \infty), \end{cases} \quad (12)$$

respectively. (10) thus can be simplified as

$$\begin{aligned} J(p) - J(p_o^*) &= \int_0^\infty [F_{p_o^*}^o(\tau) - F_p^c(\tau)] y(D - \tau)\mathrm{d}\tau \\ &= \int_0^\infty F_{p_o^*}^o(\tau) y(D-\tau)\mathrm{d}\tau - \int_0^\infty F_p^c(\tau) y(D - \tau)\mathrm{d}\tau \\ &= \mathbf{E}^o\{p_0^*\} - \mathbf{E}^c\{p_c\}, \end{aligned} \quad (13)$$

where $\mathbf{E}^o\{p_0^*\}$ and $\mathbf{E}^c\{p_c\}$ denote the expectation of the cost under open-loop pricing and closed-loop pricing (p_c denotes the corresponding price), respectively. It follows from (13) that when the randomness especially in the high cost part decreases or shifts from the high cost part to the low cost part, the profit gain will increase. This is the key factor guiding our pricing algorithm design.

Considering the fairness, the control center will equally allocate the profit gain to both the customers and the supplier (also can set different weights to them here) to make the decision. Note the expectation of the profit gain for the customers and the supplier can be calculated by $\mathbf{E}\{\sum_{i=1}^N (J_i^c(p) - J_i^c(p_o^*))\}$ and $\mathbf{E}\{J^s(p)\} - \mathbf{E}\{J^s(p_o^*)\}$, respectively, and the total profit gain is given by (13). Thus, splitting the profit gain equally gives the results that

$$\frac{1}{2}(J(p) - J(p_o^*)) = \mathbf{E}\{\sum_{i=1}^N (J_i^c(p) - J_i^c(p_o^*))\} \quad (14)$$

and

$$\frac{1}{2}(J(p) - J(p_o^*)) = \mathbf{E}\{J^s(p)\} - \mathbf{E}\{J^s(p_o^*)\}. \quad (15)$$

Then, the piecewise prices will be made by the control center using (14) or (15). Substituting the piecewise prices into (14), it follows that

$$\begin{aligned} \frac{1}{2}(J(p) - J(p_o^*)) &= \mathbf{E}\{\sum_{i=1}^N (J_i^c(p) - J_i^c(p_o^*))\} \\ &= p_o^* \int_0^\infty F_{p_o^*}^o(\tau)\tau\mathrm{d}\tau - \int_0^s F_p^c(\tau)[p_l z + p_m(\tau - z)]\mathrm{d}\tau \\ &- \int_s^\infty F_p^c(\tau)[p_l z + p_m(s - z) + p_h(\tau - s)]\mathrm{d}\tau, \end{aligned}$$

where we have used the fact that $z\rho = s - z$. Then, from (13), the above equation can be simplified as

$$\begin{aligned} \frac{\mathbf{E}^o\{p_0^*\} - \mathbf{E}^c\{p_c\}}{2} &= -p_l z + p_m z \\ &- \left[\int_0^s F_p^c(\tau) p_m \tau \mathrm{d}\tau + \int_s^\infty F_p^c(\tau)[p_m s + p_h(\tau - s)]\mathrm{d}\tau\right] \\ &+ p_o^* \int_0^\infty F_{p_o^*}^o(\tau)\tau\mathrm{d}\tau \\ &= -p_l z + p_m z - A_0 + A_1 \end{aligned} \quad (16)$$

where A_0 satisfies

$$\begin{aligned}A_0 &= \int_0^s F_p^c(\tau)p_m\tau\mathrm{d}\tau + \int_s^\infty F_p^c(\tau)[p_m s + p_h(\tau - s)]\mathrm{d}\tau \\ &= \int_0^s F_p^c(\tau)p_m\tau\mathrm{d}\tau + \int_s^\infty F_p^c(\tau)p_m s\mathrm{d}\tau - \int_s^\infty F_p^c(\tau)p_h s\mathrm{d}\tau \\ &\quad + \int_s^\infty F_p^c(\tau)p_h\tau\mathrm{d}\tau \\ &= p_m\int_0^s F_p^c(\tau)\tau\mathrm{d}\tau + (p_m - p_h)s\int_s^\infty F_p^c(\tau)\mathrm{d}\tau \\ &\quad + p_h\int_s^\infty F_p^c(\tau)\tau\mathrm{d}\tau \end{aligned} \tag{17}$$

when $x \le z < s$ and

$$A_0 = p_m s + p_h\int_s^\infty F_p^c(\tau)(\tau - s)\mathrm{d}\tau \tag{18}$$

when $z = s$, which is the expectation of the maximum payment of the customers under the setting $p_l = p_m$, and A_1 satisfies

$$A_1 = p_o^*\int_0^\infty F_{p_o^*}^o(\tau)\tau\mathrm{d}\tau, \tag{19}$$

which is the expectation of the payment of the customers under the open-loop pricing. From (16), we have that the expected payment under closed-loop pricing is $A_0 - (p_m - p_l)z$. Hence, by solving (16), it follows that

$$p_l = p_m - \frac{B_0 + A_0 - A_1}{z}, \tag{20}$$

where

$$B_o = \frac{\mathbf{E}^o\{p_0^*\} - \mathbf{E}^c\{p_c\}}{2}. \tag{21}$$

In the above equation, $\mathbf{E}^o\{p_0^*\} - \mathbf{E}^c\{p_c\}$ is the profit gain from the closed-loop pricing which can be obtained from (13), and $A_0 - A_1$ is the expected saving for the customers.

Note that p_h is the highest price which can be viewed as the penalty price to the excessive usage of customers. When the customers' demand exceeds the supply, the supplier needs a fast power generation to meet the requirement. Hence, the value of p_h can be equal to and usually higher than the unit cost of fast power generation, and is assumed to be a known constant in our modeling. Meanwhile, by increasing the value of p_h, the probability of higher than exceeded demand and the associated cost can be further decreased while it may increase the payment of the customers, which is a tradeoff. We set $p_h = 2p_o^*$ in our algorithm for a simple illustration. Since p_m denotes the price of flexible part for the customers, it is reasonable to set $p_m \le p_o^*$, and we set $p_m = p_o^*$ in the PCPA. When both p_h and p_m are fixed, it is not difficult to obtain p_l by solving (20). Therefore, when z is fixed, the closed-loop pricing is given by

$$\begin{cases} p_l = p_o^* - \dfrac{B_0 + A_0 - A_1}{z}; \\ p_m = p_o^*; \\ p_h = 2p_o^*. \end{cases} \tag{22}$$

From (20), one infers that the price p_l is a decreasing function of profit improvement while it is an increasing function of the total demands z. Hence, in the following subsection, we design PCPA to achieve a stable price of p_l, such that the customers and supplier achieve a win-win situation.

4.3. Practical closed-loop pricing algorithm

In this subsection, we provide the details of PCPA. First, we use an open-loop pricing scheme to obtain the optimal price p_o^* and using this price as the initial price in the closed-loop pricing. We thus design the optimal open-loop pricing under our modeling as follows.

Algorithm 1 : *Optimal Open-loop Pricing Algorithm*

1: **Input:** s, D, and $y(\cdot)$.
2: Regresses from the historical data to obtain $f_p^o(\tau)$, which is a function of the price p.
3: The control center calculates the optimal price p_o^* by

$$\begin{aligned} p_o^* &= \arg\left\{\frac{\mathrm{d}\,(\mathbf{E}\{y(D - d(p))\})}{\mathrm{d}p} = 0\right\} \\ &= \arg\left\{\frac{\mathrm{d}\int_0^\infty f_p^o(\tau)y(D - x - \tau)\mathrm{d}\tau}{\mathrm{d}p} = 0\right\}, \end{aligned}$$

where $\arg\{\cdot\}$ is a value of the variable such that the equation $\{\cdot\}$ holds true, i.e., p_o^* is the point where the derivative of $\mathbf{E}\{y(D - d(p))\}$ equals to 0.
4: **Output:** p_o^*.

For the above algorithm, the pricing decision is made by the control center. When the control center executes Algorithm 1, it needs to know the cost function of supplier y and the PDF $f_p^o(\tau)$ which is regressed from the historical data. Then, the control center obtains p_o^* from step 3.

In the closed-loop pricing, since there is no constraint for the customer in the open-loop price, we set the flexible ratio $\rho = \infty$ initially. Then, the customer will set $z_i(0)$ with p_o^* and $\rho = \infty$. Next, the control center will do the pricing with the strategy proposed in the above subsection, and broadcast them to the customers. The customers will reset their z_i based on the updated prices and flexible ratio. Such a loop will be updated iteratively until p_l cannot be decreased or the gain cannot be enhanced. We describe the details of this in Algorithm 2 .

Algorithm 2 : *Practical Closed-loop Pricing Algorithm*

1: **Input:** s, D, $y(\cdot)$, p_o^* (obtained in Algorithm 1), and $z_i(0)$ which is calculated by

$z_i(0) = z_i(p_o^*, \infty)$.

2: **Loop:** At each iteration k, the control center calculates the total lowest demand $z(k-1)$ from the aggregation. Set $z(k) = z(k-1)$ and regress the f_p^c under the setting of $z = z(k)$.
3: Compute the flexible ratio $\rho(k)$ using (9) under the setting of $z = z(k)$. Then, calculate the value of $B_0(k)$ with (13) and (21), and the value of $A_0(k)$ and $A_1(k)$ with (17) and (19), respectively, under the setting of $z = z(k)$.
4: Set the price $p_l(k)$ as

$$p_l(k) = p_m - \frac{B_0(k) + A_0(k) - A_1(k)}{z(k)}. \tag{23}$$

5: If $B_0(k) > B_0(k-1)$ and $p_l(k) < p_l(k-1)$, where $B_0(0) = 0$ and $p_l(0) = p_m$, then broadcast $p_l(k)$ and $\rho(k)$ to the customers. Otherwise, broadcast $p_l(k-1)$ and $\rho(k-1)$ (where we set $\rho(0) = \frac{z(0)-s}{s}$) to the customers and stop the iteration.
6: If a customer receives the feedback signal $p_l(k)$ and $\rho(k)$, he calculates

$z_i(k) = z_i(p_l(k), \rho(k))$

and broadcasts $z_i(k)$ to the aggregation center. Otherwise output $z_i = z_i(p_l(k-1), \rho(k-1))$.
7: **End Loop**
8: **Output:** p_l, p_m, and p_h.

In Algorithm 2, each customer has fully freedom to fix its lowest guaranteed demand. However, since the pricing depends on the total lowest guaranteed demands z but not on the lowest guaranteed demand of single customer z_i, the pricing will not be affected by individual customers even if they are selfish. Meanwhile, step 5 of PCPA can guarantee that p_l is decreased but the profit gain is increased with the iteration, which are desirable for both customers and supplier. More performance analysis of PCPA will be given in the following section.

5. Performance analysis of PCPA

In this section, we analyze the performance of PCPA. First, we will prove that PCPA can converge, i.e., a stable price p_l can be achieved. Then, we prove that with PCPA, a win-win situation can be achieved, where the win is the profit gain. Finally, we analyze the optimality of the win.

5.1. Convergence

First, we prove the convergence of PCPA by using the principle that bounded monotonic sequence must possess a limit.

Theorem 1. *Under PCPA, a stable price can be achieved, i.e.,*

$$\lim_{k\to\infty} p_l(k) = p_l^*,$$

where $p_l^* \le p_m$ is a constant.

Proof. From (13) and (21), one infers that

$$\begin{aligned} B_o(k) =& \frac{\mathbf{E}^o\{y(k)\} - \mathbf{E}^c\{y(k)\}}{2} \\ \le& \frac{\int_0^\infty F^o_{p_o^*}(\tau)y(D-\tau)\mathrm{d}\tau}{2}. \end{aligned} \tag{24}$$

It can be seen that $B_o(k)$ has an upper bound. In step 5 of the PCPA, it guarantees that $B_o(k)$ is an increasing function of iteration k. According to the fact that a bounded increasing sequence should have a limitation, one infers that $B_o(k)$ will converge to a constant. After $B_o(k)$ becomes a constant, the PCPA will stop based on step 5, and then $p_l(k)$ will not change again and converge to a constant. Meanwhile, since $p_l(k) \le p_l(k-1) \le \cdots \le p_l(0) = p_m$, we have $p_l(k) \le p_m$. The proof is completed.

From the proof of Theorem 1, it is observed that PCPA is a greedy-based algorithm since each customer will set $z_i(k)$ to maximize the utility of itself at each iteration. The convergence speed of PCPA depends on the step size (i.e., the value of $z(k)-z(k-1)$) and it usually has a fast convergence speed due to the greedy, while it cannot obtain a global optimal solution which will be further discussed in Section 5.3.

5.2. Win-win solution

Then, we provide the conditions under which a win-win solution is achieved, i.e., both the customers and the supplier can obtain a higher profit under PCPA than that under open-loop pricing. We also analyze how much the profit can be improved by PCPA.

Theorem 2. *If $B_0(1) > 0$ and $B_0(1)+A_0(1)-A_1(1) > 0$, then under PCPA, we have*

$$\begin{aligned} \frac{\mathbf{E}^o\{p_0^*\} - c}{2} \ge& \mathbf{E}\{\sum_{i=1}^N (J_i^c(p_l(k)) - J_i^c(p_o^*))\} \\ =& \mathbf{E}\{J^s(p_l(k)) - J^s(p_o^*)\} \\ \ge& B_0(1) > 0, \end{aligned} \tag{25}$$

hold for $k \ge 0$, i.e., a win-win solution is achieved.

Proof. Under PCPA, from step 6, one infers that

$$p_l(1) - p_m \le -\frac{B_0(1)+A_0(1)-A_1(1)}{z(1)} < 0$$

i.e., $p_l(1) < p_m$, where we use the conditions that $z(1) > 0$ and $B_0(1)+A_0(1)-A_1(1) > 0$. And, we have $B_0(1) > 0$. Hence, the conditions in step 5 can be satisfied at iteration $k = 1$, which means that the PCPA converges when $k \ge 1$ and the pricing process (23) has been applied. Hence, the profit of both the customers and the supplier should satisfy (14) or (15).

First, we prove the win for the customers. From (14) and step 5, one infers that

$$\begin{aligned} B_o(k) =& \mathbf{E}\{\sum_{i=1}^N (J_i^c(p_l(k)) - J_i^c(p_o^*))\} \\ >& \mathbf{E}\{\sum_{i=1}^N (J_i^c(p_l(k-1)) - J_i^c(p_o^*))\} \\ >& \cdots \\ >& \mathbf{E}\{\sum_{i=1}^N (J_i^c(p_l(1)) - J_i^c(p_o^*))\} \\ =& B_0(1) > 0. \end{aligned} \tag{26}$$

Based on (26), we have

$$\mathbf{E}\{\sum_{i=1}^N (J_i^c(p_l^*) - J_i^c(p_o^*))\} \ge B_0(1) > 0,$$

i.e., the closed-loop pricing helps the customers save more than $B_0(1)$ payment compared with the open-loop pricing.

Second, we prove the win for the supplier. Similarly, it follows from (15) and step 5 that

$$\begin{aligned} B_o(k) =& \mathbf{E}\{J^s(p_l(k)) - J^s(p_o^*)\} \\ >& \mathbf{E}\{J^s(p_l(k-1)) - J^s(p_o^*)\} \\ >& \cdots \\ >& \mathbf{E}\{J^s(p_l(1)) - J^s(p_o^*)\} \\ =& B_0(1) > 0. \end{aligned} \tag{27}$$

Therefore, we have

$$\mathbf{E}\{\sum_{i=1}^N (J^s(p_l^*) - J^s(p_o^*))\} \ge B_0(1) > 0,$$

which means that the supplier also obtains a higher profit.

Next, we derive the upper bound of the gain of the customers or the supplier. Since at each iteration k, $k \ge 1$, the customers' gain satisfies $B_o(k) = \mathbf{E}\{J^s(p_l(k)) - J^s(p_o^*)\}$ and the supplier's gain satisfies $B_o(k) = \mathbf{E}\{\sum_{i=1}^N (J_i^c(p_l(k)) - J_i^c(p_o^*))\}$, we have

$$\mathbf{E}\{J^s(p_l(k)) - J^s(p_o^*)\} = \mathbf{E}\{\sum_{i=1}^N (J_i^c(p_l(k)) - J_i^c(p_o^*))\}.$$

For each $B_0(k)$, it follows from (13) and (21) that

$$\begin{aligned} B_o(k) =& \frac{1}{2}\int_0^\infty [F^o_{p_0^*}(\tau) - F^c_p(\tau)]y(D-\tau)\mathrm{d}\tau \\ =& \frac{1}{2}[\mathbf{E}^o\{p_0^*\} - \int_0^\infty F^c_p(\tau)y(D-\tau)\mathrm{d}\tau] \\ \le& \frac{1}{2}[\mathbf{E}^o\{p_0^*\} - \int_0^\infty F^c_p(\tau)c\mathrm{d}\tau] \\ \le& \frac{\mathbf{E}^o\{p_0^*\} - c}{2}, \end{aligned} \tag{28}$$

for $\forall k \ge 1$, where we have used the fact that $F^c_p(\tau) \ge 0$ and $y(D-\tau) \ge c$. Accordingly, (25) holds.

The above theorem provides a sufficient condition for PCPA such that a win-win solution can be achieved. It is noted that the profit gain is larger than $B_0(k)$ which will increase with iteration, and is bounded by $\frac{\mathbf{E}^o\{p_0^*\}-c}{2}$. Next, we analyze this sufficient condition. The first condition $B_0(1) > 0$ implies that $\mathbf{E}^o\{y(1)\} > \mathbf{E}^c\{y(1)\}$, which means that the customers' first feedback can decrease the value of the cost function in expectation. Thus, the feedback should decrease the cost. Since under the closed-loop pricing, the high cost part corresponds to a much higher price for the customer, the probability that the demand of customers exceeding s would be much smaller than that under open-loop pricing and near 0. Hence, the condition $B_0(1) > 0$ is not difficult to be satisfied. Note that $A_0(1) - A_1(1)$ is the maximum expected saving for customers, and the physical meaning of the second condition $B_0(1) + A_0(1) - A_1(1) > 0$ is that the decreased cost is larger than the maximum saving payment. The control center can always obtain a positive profit from the closed-loop pricing when the second condition holds. Then, the positive profit will be transferred to the profits of both the customers and the supplier on average. As a result, this second condition is actually a necessary condition for a win-win solution, and we thus state a corollary as follows.

Corollary 1. *If a win-win solution is achieved under PCPA, then we have*

$B_0(1) + A_0(1) - A_1(1) > 0.$

When $B_0(1)+A_0(1)-A_1(1) \le 0$, the control center can increase the value of p_m and p_h to increase the value of $A_0(1)$ such that $B_0(1) + A_0(1) - A_1(1) > 0$ holds. That is, this second condition is also not difficult to be satisfied. Therefore, according to the above discussion, it is not difficult to achieve a win-win solution under PCPA.

5.3. Optimality

In this subsection, we will investigate when the customers and the supplier can get the highest profit gain (the global optimal win and cannot be archived by PCPA).

From Theorem 2, one sees that there is an upper bound, $\frac{\mathbf{E}^o\{p_0^*\}-c}{2}$, of the win-win solution for both the customers and supplier. The following theorem provides a necessary and sufficient condition that the upper bound is achieved, i.e., the optimal win is obtained.

Theorem 3. *The optimal win for both the customers and the supplier is obtained, i.e.,*

$$B_o^* = \frac{\mathbf{E}^o\{p_0^*\} - c}{2},$$

if and only if the customers fix their total demands to D, i.e., $\Pr\{d(p) = D\} = 1$.

Proof. From the proof of Theorem 2, we know that the win for both customer and supplier is B_0 and satisfies

$$B_0 = \frac{\mathbf{E}^o\{p_0^*\}}{2} - \frac{1}{2}\int_0^\infty F_p^c(\tau)y(D-\tau)\mathrm{d}\tau. \tag{29}$$

Note that $\mathbf{E}^o\{p_0^*\}$ is fixed and

$$\int_0^\infty F_p^c(\tau)y(D-\tau)\mathrm{d}\tau \ge \int_0^\infty F_p^c(\tau)c\mathrm{d}\tau = c.$$

Hence, the following equation holds,

$$B_0 = \frac{\mathbf{E}^o\{p_0^*\} - c}{2} \Leftrightarrow \int_0^\infty F_p^c(\tau)y(D-\tau)\mathrm{d}\tau = c \\ \Leftrightarrow \Pr\{d(p) = D\} = 1, \tag{30}$$

where the fact that $y(D-\tau) > c$ when $\tau \ne D$ is used. From Theorem 2, we know that $\frac{\mathbf{E}^o\{p_0^*\}-c}{2}$ is the upper bound of the win, which means that $\frac{\mathbf{E}^o\{p_0^*\}-c}{2}$ is the optimal win. We thus have completed the proof.

Suppose that there are random demands of customers, i.e., there exists an interval such that $F_p^c(\tau) > 0$, then

$$B_0 = \frac{\mathbf{E}^o\{p_0^*\}}{2} - \frac{1}{2}\int_0^\infty F_p^c(\tau)y(D-\tau)\mathrm{d}\tau \\ < \frac{\mathbf{E}^o\{p_0^*\}}{2} - \frac{1}{2}\int_0^\infty F_p^c(\tau)c\mathrm{d}\tau = B_o^*, \tag{31}$$

where we have used the fact that $y(D-\tau) > c$ when $\tau \ne D$. It means that the randomness of demands will decrease the win, which is correspondent to the intuition. In Theorem 3, one sees that $\mathbf{E}^o\{p_0^*\}$ is the expectation of the cost under open-loop pricing, and c is the lowest cost under cost function y and is achieved only when $d(p) = D$. Hence, only when the customers fix all their demands and the total demands should equal to D, the optimal win can be achieved. We then obtain a corollary as follows.

Corollary 2. *Suppose the customers can fix their total demands to D, then the optimal win for both the customers and the supplier is obtained, and the customers will obtain the lowest price as*

$$p = p_o^* - \frac{\mathbf{E}^o\{p_0^*\} - c}{2D}.$$

This corollary can be obtained from the above discussion directly, we thus omit its proof.

However, for PCPA, each customer has an independent decision function $z_i(p, \rho)$ and there is no cooperation among customers, so the optimal win cannot be guaranteed by PCPA. How to design the cooperation scheme among customers to obtain a higher profit gain than PCPA, and how to optimize the tradeoff between the profit gain and the flexibility of customers' demands, are interesting and challenging problems, which will be left as our future works.

6. Performance evaluation

In this section, we conduct extensive simulations to evaluate the performance of the proposed algorithm PCPA, and compare it with the open-loop pricing algorithm.

6.1. Parameter setting

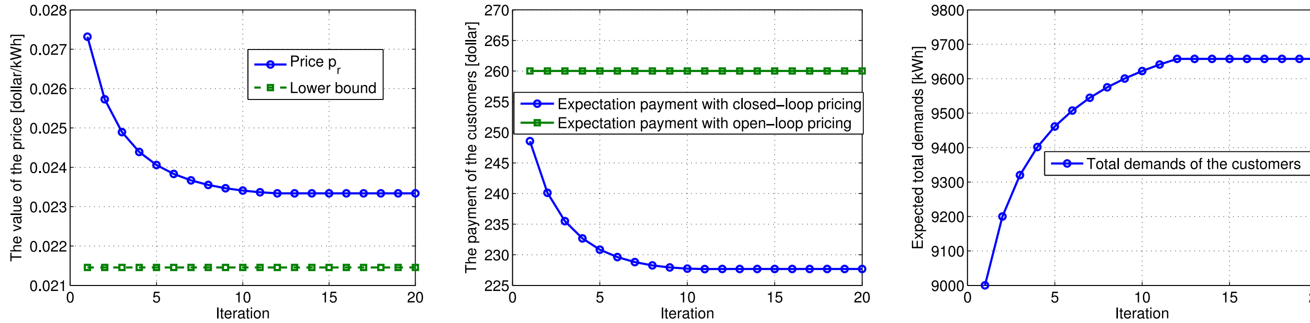
Consider a system with $s = 10^4$. Let the ideal usage of the customers be $D = s \times 95\%$, and the fixed demand of the customers be $x = s \times 80\%$. For the cost function, by referring to the penalty function in Liu et al. (2014), we set $y(p) = 0.1|D - \sum_{i=1}^N d_i(p)|$, where 0.1 is the weight and $d_i(p)$ is defined by (1). The optimal open-loop price is the initial price in PCPA and is used as a reference price. We have discussed how to obtain p_o^* in Section 3. Herein, assume that $p_o^* = 0.03$ dollar/kWh and the PDF is

$$F_{p_o^*}^o(\tau) = \begin{cases} -2\dfrac{1-\varrho}{(s-x)^2}(\tau - s), & \tau \in [x, s), \\ -2\dfrac{\varrho}{s^2}(\tau - 2s), & \tau \in [s, 2s], \end{cases} \tag{32}$$

where $\varrho = 10^{-4}$ is the high cost probability that the demand of the customers is larger than s. Similarly, we set

$$F_{p_I(k)}^c(\tau) = \begin{cases} \dfrac{1-\omega\varrho}{(s-z(k))}, & \tau \in [z(k), s), \\ -2\dfrac{\omega\varrho}{s^2}(\tau - 2s), & \tau \in [s, 2s], \end{cases} \tag{33}$$

where $\omega\varrho$ (with $\omega = 0.2$) is the high cost probability in PCPA. The weight $\omega < 1$ can ensure that the high cost decreases with the high price constraint in PCPA. Then, in the iteration of PCPA, we set $z(k+1) = \min\{z(k) + \frac{s\times 0.06}{k+2}, s\}$ with the setting of $z(0) = x$, where k is the number of iterations. It should be pointed out that except the cost function, most of the above settings can be changed and will not affect the following results.



(a) The lower price p_l. (b) The payment of customer. (c) The expected total demands.

Fig. 4. The comparison between closed-loop and open-loop pricing.

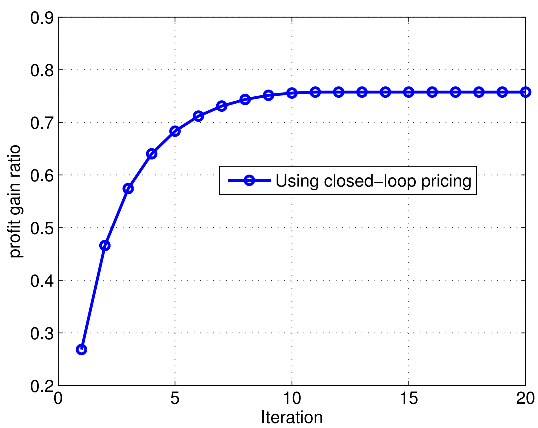


Fig. 5. The profit gain ratio with closed-loop pricing.

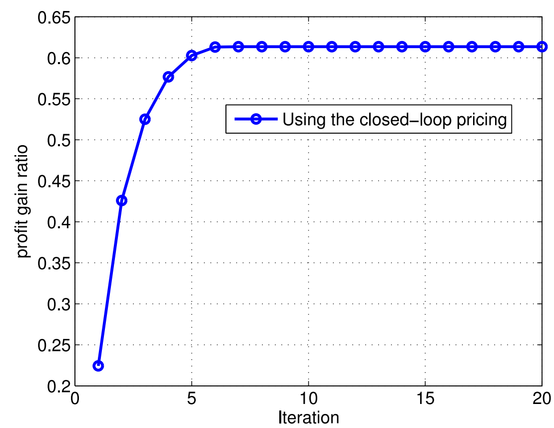


Fig. 6. The profit gain ratio with closed-loop pricing.

6.2. Simulation results

Lower price, lower payment but get more electricity: The price p_l and the customers' payment under PCPA are shown in Fig. 4. The lower bound in Fig. 4(a) is the price under the assumption that $z = s$ and there is no random demand for the customers. It is observed that p_l is decreasing with the iteration while larger than the lower bound, and the payment for the customers is much smaller than that under open-loop pricing in expectation. Meanwhile, it can be noted that not only the payment is decreased, but also the total amount of the demand is increased in expectation which is shown in Fig. 4(c). Therefore, under PCPA, the customers can use more power electricity with a lower price and lower payment compared with the open-loop pricing.

Win-win solution: Then, we consider the profit gain and cost savings for the supplier and the customers. We define $\frac{J(p_o^*)-J(p_l)}{J(p_o^*)}$ as the profit gain ratio. Clearly, a larger ratio means a better improvement for PCPA compared with the open-loop pricing algorithm, and then more profits can be obtained by both the customers and the supplier. The result is shown in Fig. 5. It is observed that the ratio increases with iterations. After convergence, the ratio can reach 0.76, which is a significant gain. Note that under our pricing strategy, the supplier and the customers will have the same profit gain, thus the curve in Fig. 5 shows the profit gain ratio of both the supplier and customers. Therefore, a win-win solution can be achieved by PCPA. It should be pointed out that if the optimal win is achieved, we have $\frac{J(p_o^*)-J(p_l)}{J(p_o^*)} = \frac{2B_0^*}{J(p_o^*)} = 1$ (where we have used the fact that $y(0) = 0$), i.e., the profit gain ratio will be equal to 1 (> 0.76). It means that PCPA cannot obtain the optimal win due to the randomness and the total demands are not D, which illustrates the theoretical results given in Section 5.3.

However, when the PDF $F_{p_l}^c$ under a given z changes to

$$F_{p_l}^c(\tau) = \begin{cases} \frac{1}{(D-z)}, & \tau \in [z, D), \\ 0, & \text{otherwise}, \end{cases} \tag{34}$$

which means that only when $z < D$, the customers have the flexible ratio, otherwise the total demands are fixed without any randomness. Then, we can obtain the results as shown in Table 2. It is observed that when the customers fix their demand to D, they can get the lowest price and the correspondent profit gain ratio will equal to 1. This result is consistent with Corollary 2. In addition, the proposed algorithm can still achieve a high profit gain ratio for both customers and the supplier when the values of parameters, s, D and x, change. For example, as shown in Fig. 6, if we change the ideal usage of the customers to $D = s \times 90\%$, and the fixed demand of the customers to $x = s \times 70\%$, a profit gain ratio above 0.61 can still be achieved by the proposed closed-loop pricing algorithm.

7. Conclusions

In this paper, we have developed a novel closed-loop pricing framework considering the randomness of the demand side and the cost in the supplier side for smart grids. By exploiting the information feedback between the customers and the control center, we proposed a practical closed-loop pricing algorithm using a piecewise pricing mechanism. Based on the proposed pricing algorithm, the cost caused by the uncertainties is decreased and

Table 2
Demand, price, and profit gain.

Guarantee demands	D−200	D−150	D−100	D−50	D	D+50	D+100
Price p_l	0.0259	0.0258	0.0257	0.0256	0.0255	0.0258	0.0261
Profit gain ratio	0.8836	0.9128	0.9421	0.9713	1.0000	0.9415	0.8830

transformed into the profit iteratively. Thus, the proposed algorithm can achieve a win-win solution for both the customers and the supplier. We proved that the stability of the pricing algorithm and the bounds of the maximum profit for both customers and supplier have been derived. We also provided a necessary and sufficient condition, under which the maximum profit gain can be achieved. Simulations show that the total profits can be improved by about 76% (and even 100% when the total demands of customers can be fixed to D) using the proposed algorithm compared with the open-loop pricing algorithm.

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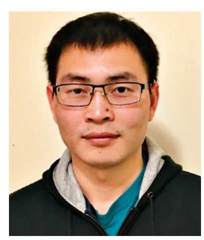

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