Stability Analysis of Vehicle Platooning with Limited Communication Range and Random Packet Losses

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***Abstract*—Control performance of vehicle platooning relies on the information flow topology and quality of wireless communications. In this paper, we investigate the constant-time-headway-spacing-policy-based vehicle platooning problem, where multiple predecessors' information is used by the following vehicles and communication impairments, i.e., limited communication range and random packet losses, are considered. In this paper, first, when the leading vehicle moves at a constant speed, we obtain the sufficient and necessary conditions on sampling time, control gains, and internal lag, to ensure the stability of the vehicle platoon based on matrix polynomials' stability for ideal communications. Secondly, for time-independent homogeneous random packet losses, we provide the upper bound for the loss rate to maintain convergence in expectation by matrix eigenvalue perturbation theory when no input is set for lossy information. We also provide sufficient conditions to guarantee mean-square convergence for heterogeneous time-independent random packet loss and show the convergence time for any given accuracy and probability. Third, when historically latest information is used for input, the sufficient and necessary conditions are provided to ensure the internal stability and string stability by Markov jump linear system theory. Furthermore, we discuss the controller design when no feasible solution exists to guarantee the string stability. Extensive numerical results validate our analysis.**

***Index Terms*—Vehicle Platooning, Longitudinal Control, Random Packet Losses, Internal Stability, String Stability.**

I. Introduction

Autonomous driving technologies enable vehicles to drive by themselves without human supervision, which is an important part of intelligent transportation systems [1]. However, if each autonomous car just senses the surroundings and then makes decisions individually, road congestion cannot be alleviated. Thus, it is necessary to apply cooperated automated driving technologies to further enhance the intelligence of transportation systems. Especially, vehicle platooning is a typical application of cooperated automated driving technologies and also one of the most promising internet-of-vehicles (IoV) applications [2], [3]. It is to form a road train by a group of vehicles without physical couplings, where a shorter inter-vehicle distance is maintained by automation and wireless communication technologies [4]. Studies have shown that the aerodynamic drag of a car can be reduced by following at least one predecessor [5]. The smaller air drag of the "follower" vehicles thus effectively brings down their fuel consumption and also leads to the reduction of carbon emissions in the transportation system. More importantly, vehicle platooning helps increase road traffic throughput, which is a promising technology to alleviate traffic congestions.

Platoon control includes longitudinal and lateral control. Longitudinal control is to regulate longitudinal motions and lateral one focuses on tracking the center of the lane accurately. In this paper, we consider longitudinal control, which includes the spacing policy, the distributed controller, the communication mechanism, and performance evaluation. Different spacing policies have been developed such as the constant distance spacing (CDS) policy and the constant time headway spacing (CTHS) policy. Compared to CDS policy, CTHS policy improves the scalability and stability of the vehicle platoon with different information flow topologies (IFTs), e.g., one-look-ahead, leader-following, leader-predecessor following, multiple-look-ahead, multiple predecessors and followers. For performance evaluation, internal stability characterizes the platoon stability without disturbances, and string stability identifies error amplification of the following vehicles from predecessors' external disturbances.

Vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) wireless communication technologies facilitate the application and development of Cooperative Adaptive Cruise Control (CACC) [6]. Compared to Adaptive Cruise Control (ACC) only using the information of on-board sensors (such as radar-based sensors), CACC can achieve a shorter inter-vehicle distance and better robustness against disturbances by V2V communication. However, the quality of wireless communication is affected by many impairments, such as channel fading, shadowing, and interference, and thus packet losses are inevitable in vehicular networks [7]. As a result, it is necessary and meaningful to investigate their effects on vehicle platooning, which provides guidelines on the design of effective platooning strategies.

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On the other hand, much attention has been paid to vehicle platooning with a leader-predecessor IFT, i.e., each following car combines the information from the leading vehicle and one immediate predecessor for local control [8]–[10]. However, this leader-predecessor IFT may be unrealistic especially when the size of the platoon is large given that each vehicle has a limited communication range. Furthermore, if each car knows a higher percentage of global network information, all cars may achieve faster consensus on acceleration and speed. It has been shown that we can decrease the headway time by using multiple predecessors' information in the local controller [11]. It means that the inter-vehicle distance can be reduced by using the information of more predecessors. At the same time, it is meaningful to jointly design the network and the controller for safety applications and cooperative driving. In summary, it is of vital importance to investigate the performance of the vehicle platoon where each car uses the information of multiple predecessors, which motivates us to analyze how non-ideal communications and internal vehicle dynamics interact with each other and how the macroscopic platoon performance is influenced.

Stability conditions for vehicle platoons in imperfect communications have received attention in recent years. In particular, for the leader-predecessor following and one-look-ahead IFTs, how communication delays and random packet losses affect the performance has been studied. However, for vehicle platoons with a more general IFT such as the two-predecessor following, limited communication range, and bidirectional networks, the stability, scalability, and robustness of the platoon system were only investigated based on the assumption that communication networks are perfect. It remains open to analyze how limited communication range-based network and imperfect communications interact with each other and how they affect control performance of the vehicle platoon. To fill this gap, in this paper, we study a vehicle platoon where each vehicle has a limited communication range and adopts a CTHS policy in an imperfect communication environment. The main contributions of this work are summarized as follows.

- We analyze control performance of vehicle platoons where the vehicles have limited communication range and there are random packet losses. It is formulated as the discrete-time vehicle platoon using multiple predecessors' information under random packet losses.
- We obtain sufficient and necessary conditions on the information flow, sampling time, control gains, and internal lag, to guarantee internal stability, by using matrix polynomials for ideal communications.
- We provide an upper bound for time-independent packet loss rate to guarantee convergence in expectation by matrix perturbation theory and obtain a sufficient condition for the mean-square convergence. Moreover, sufficient and necessary conditions are also shown on time-dependent random packet loss rate by Markov jump linear system theory under which the internal stability is ensured.
- We analyze string stability of the considered vehicle platoon model with random packet losses by H_∞ Markov jump linear systems. It is characterized as the feasibility of an optimization problem to avoid the amplification of the spacing error in vehicle platooning.

The remainder of this paper is organized as follows. Related work is summarized in Section II. The preliminaries and problem formulation are provided in Section III. We analyze internal stability in Section IV and string stability in Section V. Section VI verifies the main results through numerical studies. We also discuss the controller design, system modeling, and performance optimization in Section VII. Conclusions and future work are presented in Section VIII.

II. Related Work

Vehicle platooning with packet losses is a hot research topic. Seiler *et al.* investigated how packet losses affect on the discrete-time vehicle following control using linear matrix inequalities, where each vehicle only uses its one predecessor's information for longitudinal control [12]. They also applied their networked control H_∞ theory to the leader-predecessor IFT based vehicle platooning, where a bursty packet loss process is considered [13]. Teo *et al.* explored a discrete-time platoon model with a leader-predecessor-following IFT and packet dropouts, and proposed a mitigation scheme through estimating the leader vehicle's state [14]. A state estimation based method is developed to make CACC gracefully switch to ACC and guarantee the string stability for persistent packet losses [15]. Considering the constant spacing strategy and the leader-predecessor-following-IFT-based platooning with random packet losses and limited communication capacity, Guo *et al.* designed a strategy to guarantee mean square exponential string stability [8]. For a heterogeneous vehicle platoon with inter-vehicle communication losses, an extended dwell-time structure and an adaptive switch controller are designed to guarantee the system stability in [9]. Acciani *et al.* studied a CACC-based vehicle platoon with a one-look-ahead IFT and proposed an H-infinity-based controller to guarantee the stability in expectation [10].

The platoon with a general IFT is also an important topic in recent years. The stability of the vehicle platoon was analyzed by the frequency domain method in [16]. In that model, each vehicle has a limited communication range and broadcasts its spacing error information to neighbors. Zheng *et al.* investigated the scalability and robustness for the platoon with the constant spacing policy and general IFTs [17]. Li *et al.* proposed an eigenvalue-based method to investigate the stability and scalability of the vehicle platoon, where a general IFT and the constant spacing strategy were considered [18]. Stüdli *et al.* investigated the cyclic interconnections for vehicle platoons and provided conditions for the string stability by frequency-domain methods [19]. The stability margin of the vehicle platoon with general undirected IFTs and the constant spacing policy was also studied in [17]. Many efforts have also been made to problems of platoon control with general IFTs [20], [21]. Bian *et al.* studied the vehicle platoon with the constant time headway policy where

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multiple predecessors' information is utilized for the short inter-vehicle distance control [22].

Considering communication delays and communication-based safety enhancement, many have been made to controller synthesis and dynamic analysis of vehicle platoons to avoid vehicle collisions. For example, Considering the constant spacing policy, Liu *et al.* investigated the effects of time-invariant delays on string stability of vehicle platooning and proposed a simple method to avoid the instability caused by communication delays through clock jitter [23]. Peters *et al.* analyzed the effects of uniform delays on the system performance for different communication strategies [24]. Lots of works have been also done regarding the platoon with other simple information structures. For instance, Bernardo *et al.* proposed a consensus-based control strategy for the platoon and investigated the effect of time-varying delays on the internal stability, where the leader vehicle's information is available to all other vehicles [25]–[27]. Jin *et al.* and Zhang *et al.* investigated the effects of communication delays on the stability of heterogeneous ad-hoc platoon and proposed strategies to guarantee string stability [28]–[31]. Meanwhile, to enhance the safety of a leader-predecessor-following IFT based vehicle platooning with unreliable communications, strategy design and analysis have also been investigated recently [32], [33]. For example, considering vehicle platoons with packet losses, Bergenhem *et al.* considered the problem of coordinating emergency brake, where a machine-learning-based parameters estimation method was proposed and quantitative analysis was provided through simulations [34]. Moreover, Wu *et al.* applied Kalman Filter to deal with communication delays and designed an adaptive acceleration for the vehicle platoon [35].

Notice that for the case when each vehicle has a limited communication range, how the random packet losses affect the platoon control especially control convergence time and how to design system parameters to enhance vehicle platooning stability speed are still open problems.

III. Preliminaries and Problem Formulation

Notation: Let $\mathbb{R}$ and $\mathbb{N}$ denote the set of rational numbers and the set of integer numbers, respectively. We use $A = [A_{ij}] \in \mathbb{R}^{n\times m}$ to stand for a $n \times m$-dimension matrix, and A_{ij} is the (i, j)-position element. Let $A^\top$ denote the transpose of matrix A. Let $x = [x_1 \ x_2 \ \cdots \ x_n]^\top \in \mathbb{R}^n$ indicate a n-dimension column vector, while its transpose is $x^\top$. We denote the positive definite matrix $A \in \mathbb{R}^{n\times n}$ as $A \succ 0$. The determinant of the square matrix A is denoted by $\mathbf{det}(A)$. We use I_n and $1_n = [1 \ \cdots \ 1]^\top \in \mathbb{R}^n$ to indicate a n-dimension identity matrix and a n-dimension vector, respectively. We use $\otimes$ to denote the Kronecker product symbol, $\mathbf{E}(\cdot)$ takes the expectation, and $\mathbf{Pr}\{\cdot\}$ denotes the probability. If $\mathbb{N}$ is equipped with counting measure, then $\ell_p(\mathbb{N})$ consists of all sequences $\{x(k) \in \mathbb{R} : k \in \mathbb{N}\}$ such that $\|\{x(k)\}\|_{\ell_p} = \sum_{k=0}^{\infty} |x(k)|^p < \infty$, and the norm is denoted by $(\sum_{k=0}^{\infty} |x(k)|^p)^{1/p}$. A continuous function $\alpha : [0, c) \to [0, \infty)$ is said to be of class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$.

A. Network Model

There are $n+1 \geq 2$ vehicles in a platoon, and each vehicle has one unique ID, i.e., $i = 0, 1, \cdots, n$, where $i = 0$ is the ID of the leader vehicle. All vehicles are homogeneous, i.e., they have the same dynamic parameters and hardware devices, meaning that they have the same communication range and receiver sensitivity. The topology of the communication network composed of the following vehicles without the leader is modeled by the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of $\{1, 2, \cdots, n\}$ vehicles and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the edge set. If vehicle i uses vehicle j's information, then we have $(i, j) \in \mathcal{E}$. In this paper, we consider the case where vehicles have a limited communication range and they use information from r predecessors to produce local control input when $i \in [r, n]$. For $1 \leq i < r$, vehicle i communicates with its i predecessors. Let $A = [A_{ij}] \in \mathbb{R}^{n\times n}$ be the adjacency matrix of graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $A_{ij} = 1$ if and only if (iff) $(i, j) \in \mathcal{E}$ and $A_{ij} = 0$ otherwise. Note that self-loop is not considered, which means that $A_{ii} = 0$ for all i in $\mathcal{V}$. The in-degree matrix $D = [D_{ij}] \in \mathbb{R}^{n\times n}$ is a diagonal matrix with $D_{ii} = \sum_{j=1}^{n} A_{ij}$ for all i, j in $\mathcal{V}$. The Laplacian matrix L is defined by $L = D - A$. Let $J \in \mathbb{R}^{n\times n}$ be a diagonal matrix with $J_{ii} = 1$ if vehicle i uses the leader's information and $J_{ii} = 0$ otherwise.

B. Vehicle Dynamic Model

We use m_i and q_i to designate the mass and the position of vehicle i, $i \in \mathcal{V}$, respectively. According to Newton's second law and the vehicle's engine dynamics [36], the dynamic model of vehicle i is

$$m_i\ddot{q}_i = m_i\xi_i - K_i\dot{q}_i^2 - c_{mi}, \dot{\xi}_i = -\frac{\xi_i}{\tau_i(\dot{\xi}_i)} + \frac{\mu_i}{m_i\tau_i(\dot{\xi}_i)}, \quad (1)$$

where $K_i\dot{q}_i^2$ characterizes the air resistance force, $m_i\xi_i$ represents the vehicle's engine force, and c_{mi} is a constant for the mechanical drag. Moreover, $\tau_i(\dot{\xi}_i)$ denotes the engine time constant when vehicle i moves at a speed $\dot{\xi}_i$, and μ_i is the throttle input to the engine. Suppose that the parameters in (1) are priori knowledge. By adopting the control law below

$$\mu_i = m_iu_i + K_i\dot{q}_i^2 + c_{mi} + 2\tau_i(\dot{\xi}_i)K_{di}\dot{q}_i\ddot{q}_i,$$

we have $\dddot{q}_i = -\frac{1}{\tau_i}\ddot{q}_i + \frac{1}{\tau_i}u_i$, where $\tau_i = \tau_i(\dot{\xi}_i)$ is constant when $\tau_i(\dot{\xi}_i)$ is small enough [37]. Let $v_i(t)$ and $a_i(t)$ denote the velocity and the acceleration of vehicle i at time t, respectively. The dynamics of vehicle i can be written as

$$\begin{aligned} \dot{q}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= a_i(t), \\ \dot{a}_i(t) &= -\frac{1}{\tau}a_i(t) + \frac{1}{\tau}u_i(t), \end{aligned}$$

where $u_i(t)$ is the control input and $\tau = \tau_i = \tau_j$, $\forall i, j \in V$. Let $s_i(t) = [q_i(t) \ v_i(t) \ a_i(t)]^\top$, and one obtains

$$\dot{s}_i(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & \frac{-1}{\tau} \end{bmatrix} s_i(t) + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau} \end{bmatrix} u_i(t), \forall i \in \mathcal{V}. \quad (2)$$

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We then discretize the vehicle dynamics in (2). Let η be the sampling time and therefore, there holds

$$s_i(k+1) = \begin{bmatrix} 1 & \eta & \frac{\eta^2}{2} \\ 0 & 1 & \eta \\ 0 & 0 & 1-\frac{\eta}{\tau} \end{bmatrix} s_i(k) + \begin{bmatrix} 0 \\ 0 \\ \frac{\eta}{\tau}. \end{bmatrix} u_i(k). \quad (3)$$

where $s_i(k)$ and $u_i(k)$ are state and input at time slot k, respectively.

C. Distributed Controller

CTHS policy is used to maintain a small relative distance between neighboring vehicles. The desirable distance $d_{i,i-1}(k)$ between vehicle i and its predecessor $i-1$ is

$$d_{i,i-1}(k) = d + hv_i(k), \forall i \in \mathcal{V}, \quad (4)$$

where $h > 0$ is the constant headway time and d is a constant distance. The actual distance between vehicle i and its predecessor $i-1$ is $d_i(k) = q_{i-1}(k) - q_i(k) - l$, where l is the length of vehicle i. Note that for simplicity, we assume that all vehicles have the same length, and our approach can be easily extended to consider heterogeneous length. Then, the spacing error between vehicle $i \geq 1$ and its predecessor $i-1$ is calculated by

$$\begin{aligned} e_{qi}(k) &= d_i(k) - d_{i,i-1}(k), \\ &= q_{i-1}(k) - q_i(k) - l - (d + hv_i(k)). \end{aligned} \quad (5)$$

Since l and d are the same and can be omitted when we consider the relative position, the relative velocity, and the relative acceleration to characterize the system dynamic. For any pair of vehicles i and j in the communication network, the desired spacing distance is denoted by

$$d_{i,j}(k) = \sum_{\eta=j+1}^{i} hv_\eta(k), j < i. \quad (6)$$

Note that for system (3), if we aim to make the relative distance between neighboring vehicles be monotonically increasing with the following vehicle's velocity or be some constant distance, the equilibrium point will not be zero. Inspired by [17] and [22], we study the case where each vehicle uses spacing errors, velocity errors, and acceleration errors with respect to its communication neighbors to produce the control input. For each vehicle $i \geq 1$, the control input $u_i(k)$ can be denoted as

$$\begin{aligned} u_i(k) = \sum_{j=1}^{n} & A_{ij}(\kappa_q(q_j(k) - q_i(k) - d_{i,j}(k)) \\ &+ \kappa_v(v_j(k) - v_i(k)) + \kappa_a(a_j(k) - a_i(k))) \\ &+ J_{ii}(\kappa_q(q_0(k) - q_i(k) - d_{i,0}(k)) \\ &+ \kappa_v(v_0(k) - v_i(t)) + \kappa_a(a_0(k) - a_i(k))), \end{aligned} \quad (7)$$

where κ_q, κ_v, and κ_d are the corresponding control gain parameters.

D. Communication Mechanism with Random Packet Losses

Let $p_{ij}(k)$ be the probability that the packet transmission on communication link (i,j) at the k-th time interval is lost. We use $\vartheta_{ij}(k)$ to characterize the availability of the communication link (i,j). As a result, $\vartheta_{ij}(k)$ is a binary random variable with the distribution given by

$$\mathbf{Pr}\{\vartheta_{ij}(k) = 1\} = 1 - p_{ij}(k), \mathbf{Pr}\{\vartheta_{ij}(k) = 0\} = p_{ij}(k). \quad (8)$$

During the k-th time interval for all $k \in \mathbb{N}$, from (8), the information received by vehicle i from vehicle j is

$$\begin{aligned} \bar{q}_{ij}((k+1)) &= \vartheta_{ij}(k)\bar{q}_{ij}((k-1)) + (1 - \vartheta_{ij}(k))q_j(k), \\ \bar{v}_{ij}((k+1)) &= \vartheta_{ij}(k)\bar{v}_{ij}((k-1)) + (1 - \vartheta_{ij}(k))v_j(k), \\ \bar{a}_{ij}((k+1)) &= \vartheta_{ij}(k)\bar{a}_{ij}((k-1)) + (1 - \vartheta_{ij}(k))a_j(k), \end{aligned} \quad (9)$$

where $\bar{q}_{ij}((k+1))$, $\bar{v}_{ij}((k+1))$, and $\bar{a}_{ij}((k+1))$ are available position, available velocity, and available acceleration of vehicle j through communication link $(i,j) \in \mathcal{E}$, respectively. In this paper, we consider both time-independent and the time-dependent packet loss rate model on all communication links. In the time-independent model, we suppose that the constant homogeneous packet loss rate p and heterogeneous packet loss rate p_{ij}. For the time-dependent model, the Markov chain packet loss rate model is formulated.

Remark 3.1: There are many factors that cause a packet transmission failure in vehicular networks, e.g., collisions, channel errors, and packet dropping due to the end of the common control channel [38]. As a result, the packet loss probability is time-varying. But we can still use a random process to govern the packet delivery characteristics of the network, which has been commonly used in the literature [13], [39]. For example, the packet losses occur in bursts in wireless networks, which is analyzed by the Gilbert-Elliott model of fading channels [39]. This random packet loss/delivery model has also been widely used in the design and analysis of networked control systems [13]. This paper applies the simple model of randomly distributed packet losses. As a result, the resulting theoretical conclusion of the effect of packet losses on the control performance can be of the explicit form, which can further facilitate future research on more general cases.

E. Problem of Interests

To ensure the performance of a large-scale vehicle platoon, it is crucial to solve the following problems.

- When the communication network is ideal, what is the sufficient and necessary condition on the sampling time, control gains, and IFTs to guarantee the internal stability of the platoon system?
- When there are random packet losses, how can we characterize the communication impairments on the internal stability?
- How will the random packet losses affect the string stability of the platoon system? How can we mitigate its side effects if the string stability cannot be achieved by the system?

IV. Internal Stability Analysis

To address the above problems, in this section, we first investigate the internal stability of the considered vehicle platoon with ideal communication networks. Then, we investigate the effect of time-independent packet loss rate on the internal stability by matrix perturbation and random matrix eigenvalue analysis. We also model the time-dependent random packet losses as a Markov chain process and analyze the internal stability through Markov jump linear system theory.

A. Cases without Packet Losses

To make the analysis more formal and tractable, we first make the following variable transformation. Let $\hat{q}_i(k)$, $\hat{v}_i(k)$, and $\hat{a}_i(k)$ be

$$\begin{aligned}\hat{q}_i(k) &= q_i(k) - q_0(k) - d_{i,0}(k),\\ \hat{v}_i(k) &= v_i(k) - v_0(k),\\ \hat{a}_i(k) &= a_i(k) - a_0(k),\\ \hat{u}_i(k) &= u_i(k) - u_0(k),\end{aligned}$$

where $d_{i,0}(k)$ is the desirable distance between vehicle i and the leader vehicle. By (3), one obtains

$$\begin{aligned}\hat{q}_i(k+1) &= \hat{q}_i(k) + \eta\hat{v}_i(k) + \frac{\eta^2}{2}\hat{a}_i(k) - \sum_{j=1}^{i} h\eta\hat{a}_j(k)\\ &\quad - \sum_{j=1}^{i} h\eta a_0(k), \qquad (10)\\ \hat{v}_i(k+1) &= \hat{v}_i(k) + \eta\hat{a}_i(k),\\ \hat{a}_i(k+1) &= (1-\frac{\eta}{\tau})\hat{a}_i(k) + \frac{\eta}{\tau}\hat{u}_i(k).\end{aligned}$$

Let $\hat{q}(k) = [\hat{q}_1^\top(k) \ \cdots \ \hat{q}_n^\top(k)]^\top$, $\hat{v}^\top(k) = [\hat{v}_1^\top(k) \ \cdots \ \hat{v}_n^\top(k)]^\top$, and $\hat{a}(k) = [\hat{a}_1^\top(k) \ \cdots \ \hat{a}_n^\top(k)]^\top$. Considering the vehicle platoon with vehicle dynamics in (3), control input (7), $v_0(k) = c$, and $a_0(k) = 0$ in an ideal communication network, from (10)

$$\begin{bmatrix}\hat{q}(k+1)\\ \hat{v}(k+1)\\ \hat{a}(k+1)\end{bmatrix} = \begin{bmatrix} I_n & \eta I_n & \frac{\eta^2}{2}I_n - \eta H\\ 0 & I_n & \eta I_n\\ 0 & 0 & (1-\frac{\eta}{\tau})I_n\end{bmatrix}\begin{bmatrix}\hat{q}(k)\\ \hat{v}(k)\\ \hat{a}(k)\end{bmatrix} + \begin{bmatrix}0_n\\ 0_n\\ \frac{\eta}{\tau}I_n\end{bmatrix}u(k), \qquad (11)$$

where $H \in \mathbb{R}^{n\times n}$ is a lower triangular matrix with all entries in/below the main diagonal equal to h. From (7) and (10)

$$\begin{aligned}u_i(k) = &\sum_{j=1}^{n} A_{ij}(\kappa_q(\hat{q}_j(k) - \hat{q}_i(k)) + \kappa_v(\hat{v}_j(k) - \hat{v}_i(k))\\ &+ \kappa_a(\hat{a}_j(k) - \hat{a}_i(k))\\ &+ J_{ii}(-\kappa_q\hat{q}_i(k) - \kappa_v\hat{v}_i(k) - \kappa_a\hat{a}_i(k)).\end{aligned}$$

Let $x^o(k) = \begin{bmatrix}\hat{q}(k)^\top & \hat{v}(k)^\top & \hat{a}(k)^\top\end{bmatrix}^\top \in \mathbb{R}^{3n}$. Hence, the compact form of the input can be written as

$$\begin{aligned}u(k) &= -\kappa_q(L+J)\hat{q}(k) - \kappa_v(L+J)\hat{v}(k) - \kappa_a(L+J)\hat{a}(k)\\ &= -\kappa_q L_J\hat{q}(k) - \kappa_v L_J\hat{v}(k) - \kappa_a L_J\hat{a}(k)\\ &= \begin{bmatrix}-\kappa_q L_J & -\kappa_v L_J & -\kappa_a L_J\end{bmatrix} x^o(k),\end{aligned} \qquad (12)$$

where $L_J = L + J$. Then, combining (11) and (12), we obtain the following closed-loop system

$$\begin{aligned}x^o(k+1) &= \begin{bmatrix} I_n & \eta I_n & \frac{\eta^2}{2}I_n - \eta H\\ 0 & I_n & \tau I_n\\ 0 & 0 & (1-\frac{\eta}{\tau})I_n\end{bmatrix} x^o(k)\\ &\quad + \begin{bmatrix}0_n\\ 0_n\\ \frac{\eta}{\tau}I_n\end{bmatrix}\begin{bmatrix}-\kappa_q L_J & -\kappa_v L_J & -\kappa_a L_J\end{bmatrix} x^o(k)\\ &= W x^o(k),\end{aligned} \qquad (13)$$

where

$$W = \begin{bmatrix} I_n & \eta I_n & \frac{\eta^2}{2}I_n - \eta H\\ 0 & I_n & \eta I_n\\ -\frac{\eta}{\tau}\kappa_q L_J & -\frac{\eta}{\tau}\kappa_v L_J & (1-\frac{\eta}{\tau})I_n - \frac{\eta}{\tau}\kappa_a L_J\end{bmatrix}.$$

Then, we provide the following lemma to guarantee the stability of the platoon system (13).

Lemma 4.1: The platoon system (13) can achieve asymptotic stability iff

$$\max(|W|) < 1. \qquad (14)$$

From Theorem 1.4 in [40], we have the following lemma, which plays a crucial role to analyze the internal stability of the platoon system.

Lemma 4.2: Given real numbers ι_0, ι_1, and ι_2, iff

$$|\iota_2 + \iota_0| < 1 + \iota_1, |\iota_2 - 3\iota_0| < 3 - \iota_1, \iota_0^2 + \iota_1 - \iota_0\iota_2 < 1,$$

all roots of the third-degree polynomial equation in (15) lie in the open disk $|\lambda| < 1$,

$$\lambda^3 + \iota_2\lambda^2 + \iota_1\lambda + \iota_0 = 0. \qquad (15)$$

By analyzing the structure of the matrix W and combining with Lemma 4.1 and Lemma 4.2, we establish the sufficient and necessary conditions on the system parameters to ensure the internal stability of the platoon system.

Theorem 4.3: Suppose that the leading vehicle moves at a constant speed. For any initial states, each vehicle i, $i > 0$ in the platoon system (13) can achieve stable states, i.e., $a_i = a_0$, $v_i = v_0$, $q_i - q_{i-1} = d_{i,i-1}(k)$, iff

$$\begin{aligned}&2\eta^2\kappa_q h r_i - 2\eta^2\kappa_v r_i + 4\eta(\kappa_a r_i + 1) - 5\tau < 0\\ &\eta^3\kappa_q r_i + 2\tau > 0\\ &\eta^3\kappa_q r_i - 2\eta^2\kappa_q h r_i + 2\eta^2\kappa_v r_i - 4\eta\kappa_a r_i - 4\eta + 8\tau > 0\\ &2\eta^2\kappa_q h r_i - 2\eta^2\kappa_v r_i + 4\eta(\kappa_a r_i + 1) - 5\tau < 0\\ &\eta^3\kappa_q r_i + 2\tau > 0\\ &\eta^3\kappa_q r_i - 2\eta^2\kappa_q h r_i + 2\eta^2\kappa_v r_i - 4\eta\kappa_a r_i - 4\eta + 8\tau > 0\\ &(2\tau + 2\eta + 2\eta\kappa_a r_i - 2\eta^2\kappa_v r_i + \eta^3\kappa_q r_i + 2\eta^2\kappa_q h r_i)\\ &\times(8\tau - 2\eta^2\kappa_v r_i + \eta^3\kappa_q r_i + 2\eta^2\kappa_q h r_i)\\ &+ 2\tau(\eta^3\kappa_q r_i - 2\eta^2\kappa_q h r_i + 2\eta^2\kappa_v r_i - 4\eta\kappa_a r_i - 4\eta + 6\tau)\\ &- 4\tau^2 < 0.\end{aligned} \qquad (16)$$

Proof: We postpone the proof to Appendix A. ■

Remark 4.4: Note that the above sufficient and necessary conditions for a discrete-time platoon system with multiple predecessors are different from that one for a continuous-time system as the sampling time η is shown. Here, we provide the relationship between the sampling time η, internal lag τ, the headway time constant h, control gains, and IFTs to guarantee the inter-vehicle distance can be unified through platooning control in (7). Moreover, how to guarantee the feasibility of the condition in (16) can be complex and difficult to obtain theoretically since multiple variables and nonlinear inequalities there. However, this problem can be solved numerically. Furthermore, the conditions provide a convenient tool to quickly determine whether the given system parameter settings are acceptable or not. If not, the conditions provide important insights on the tuning of these parameters to ensure internal stability.

B. Cases with Time-Independent Packet Losses

In this part, we first consider homogeneous time-independent packet losses rates , i.e., $p_{ij}(k) = p$ for all $(i, j) \in \mathcal{E}$ and k. Then, we provide a sufficient conditions on p to guarantee convergence in expectation. For the more general time-independent packet loss rate, we investigate the effect of random packet losses through the mean-square convergence (MSC). Meanwhile, we here assume that once the broadcast packet is lost, the car in the platoon system will have no input part for that lossy link.

Let $x(k)$ be the system state vector at time k and we provide one basic convergence definition as follows.

Definition 4.5: The system achieves convergence in expectation if for any initial state $x(0)$, $\lim_{k\to\infty} \mathbf{E}(x(k)) = 0$. The system achieves MSC if for any initial state $x(0)$, $\lim_{k\to\infty} \mathbf{E}(\|x(k)\|^2) = 0$.

Then, let

$$\Delta = \begin{bmatrix} 0_n & 0_n & 0_n \\ 0_n & 0_n & 0_n \\ \frac{\eta}{\tau}\kappa_q L_J & \frac{\eta}{\tau}\kappa_v L_J & \frac{\eta}{\tau}\kappa_a L_J \end{bmatrix} \tag{17}$$

and $\tilde{W} = W + p\Delta$. We use $\sigma(W) = \{\lambda_1, \cdots, \lambda_{3n}\}$ and $\sigma(\tilde{W}) = \{\tilde{\lambda}_1, \cdots, \tilde{\lambda}_{3n}\}$ to denote the set of eigenvalues of matrix W and that of $\tilde{W}$, respectively, where $|\lambda_1| \le |\lambda_2| \le |\lambda_{3n}|$ and $|\tilde{\lambda}_1| \le |\tilde{\lambda}_2| \le |\tilde{\lambda}_{3n}|$. Then, from [41], one obtains the definition of the optimal matching distance

$$\mathrm{md}(\sigma(W), \sigma(\tilde{W})) = \min_{\pi} \max_{i}(|\tilde{\lambda}_{\pi(i)} - \lambda_i|),$$

where π is taken over all permutations of $\{1, \cdots, 3n\}$. Geometrically, the optimal matching distance is the radius of the smallest circle among circles that center at $\lambda_1, \cdots, \lambda_n$ respectively and cover $\sigma(\tilde{W})$. From [42], we have the following lemma.

Lemma 4.6: For any square matrices W and $\tilde{W}$,

$$\begin{aligned} &\mathrm{md}(\sigma(W), \sigma(\tilde{W})) \\ &\le 4(\|W\|_\infty + \|\tilde{W}\|_\infty)^{1-1/3n} \|W - \tilde{W}\|_\infty^{1/3n}. \end{aligned}$$

As a result, we can conclude the following result to ensure that all vehicles can track their predecessors with the same inter-vehicle distance.

Theorem 4.7: Suppose that the platoon system (13) is stable when there is no packet loss over all communication links, i.e., (16) holds. When each link has a packet loss rate p and $1 + \eta + \eta(n-1)h + |\eta^2/2 - \eta h| \ge 2r\eta(\kappa_q + \kappa_v + \kappa_a)/\tau + |1 - \eta/\tau|$, the platoon system can achieve the internal stability in expectation if

$$\begin{aligned} p < &\frac{1}{4}(1 - |\lambda_{3n}|)(2r\eta(\kappa_q + \kappa_v + \kappa_a)/\tau)^{-1/3n} \\ &\times (2(1 + \eta + \eta(n-1)h + |\eta^2/2 - \eta h|))^{1/3n-1}. \end{aligned} \tag{18}$$

Proof: We postpone the proof to Appendix B. ■

Remark 4.8: Given homogeneous packet loss rate p on all communication links, Theorem 4.7 provides a conservative method to design the lower bound on p to ensure the internal stability in expectation of the platoon. Since the eigenvalue perturbation is a complex problem to solve, we only provide sufficient conditions on convergence in expectation and MSC. Internal stability in expectation means that for any initial positions, velocities, and accelerations of the following cars, the road train can be formed in expectation with the inter-vehicle distance satisfying (4). The mean-square convergence is another metric of the internal stability on average, which can provide a lower bound on the probability of the final state converging to the stable state with any given accuracy. However, these are only guarantees on average, which means that the final state may deviate from the expected one once the covariance does not go to zero with time. Markov jump linear system theory can be used to ensure the convergence of covariance. It also should be pointed out that the assumption on homogeneous packet loss rate p on all communication links is rather simplified, but the obtained results of the sufficient condition to guarantee the platoon stability are explicit. Moreover, when the worst-case packet loss rates are the same on different communication links, our conclusion can provide a simple and effective way to determine the stability of the platoon system. Furthermore, we provide thorough analysis for more realistic scenarios where the packet loss rates on different links are different.

In the following part, we consider the mean-square convergence of the platoon system with more general random packet losses, where packet loss rates on different links can be spatially correlated but temporally independent.

Lemma 4.9: Supposing the communication errors are independently and identically distributed (iid) over time, the platoon system (13) achieves MSC if

$$\mathbf{E}(\rho(\tilde{W})) < 1, \tag{19}$$

where $\rho(\cdot)$ takes the spectral radius of a matrix.

Proof: We postpone the proof to Appendix C. ■

Based on the above lemma, we provide a conclusion on the effect of random packet losses on the convergence time of the platoon control process below.

Theorem 4.10: When the communication errors is iid over time, if $\|y(0)\| \ne 0$ and $\mathbf{Pr}\{\|y(K)\| < \epsilon\} = \delta$, then the convergence time K of the platoon system (13) can be characterized by

$$K > \log(\frac{\epsilon(1-\delta)}{\|y(0)\|}) / \log(\mathbf{E}(\rho(\tilde{W}))). \tag{20}$$

Proof: We postpone the proof to Appendix D. ■

Remark 4.11: Lemma 4.9 provides a general sufficient condition to ensure strong stability, i.e., mean-square convergence, for the iid packet losses. It can provide guidance for communication design. More importantly, Theorem 4.10 means that given any convergence accuracy $\epsilon > 0$ and probability $\delta \ge 0$, we can obtain the lower bound on the convergence time for the control process under random packet losses. We provide an effective way to characterize the impact of packet losses on the convergence time of the platoon control process. Note that to ensure convergence in expectation and MSC, we only need to analyze the system matrix variance and the expectation of its spectral radius. Furthermore, the obtained results of Lemmas 4.9 and Theorem 4.10 are applicable to iid communication errors,

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which means that our solution is feasible to heterogeneous packet drop rates on different communication links. However, it is assumed that the controller has no related input if the packet is lost. In practice, it is desirable to use the latest received information for control. How to extend Lemma 4.9 using historical information in control will be addressed in the following subsection.

C. Cases with Time-Dependent Packet Losses

In this part, we consider that once the packet is lost, the latest received information will be used for control. Since the system model is time-dependent and W cannot be found, the previous analytical tool is not applicable. To solve this problem, we first characterize the random packet losses in the communication network as a Markov Decision Process and then obtain the Markov jump linear system expression for the platoon system. We then derive the sufficient and necessary condition on the packet loss rate to guarantee the internal stability of the vehicle platoon system.

1) Markov Decision Process: For the platoon system with a multiple predecessors IFT, the communication network has $r(r-1)/2+r(n-r-1)$ randomly connected links. Suppose that the leading vehicle's information can be transmitted reliably as it is more important in the platoon process. Note that the following approach can be extended to remove this assumption. Let $m=r(r-1)/2+r(n-r-2)$, all links are ordered as $\{1,2,\cdots,m\}$. Then, the state of the connectivity of the communication network at time k is characterized by a diagonal matrix $\theta(k)\in\mathbb{R}^{m\times m}$. As a result, there are 2^m possible states $\{S_1,S_2,\cdots,S_{2^m}\}$ that $\theta(k)$ could be, which is a $m\times m$ dimension diagonal matrix with each element $\{S_j\}_{ii}$ describing the connectivity of the i-th link. The possible state set is denoted by $\mathbb{K}$. Let $P\in\mathbb{R}^{2^m\times 2^m}$ be the transition probability matrix that characterize the transition among different states, which is a row stochastic matrix. If we have $p_{ij}=p$ for all $(i,j)\in\mathcal{E}$, the transition probability matrix is row stochastic and all rows are the same, i.e., $\mathbf{Pr}\{\theta(k+1)=S_j|\theta(k)\}=\mathbf{Pr}\{\theta(k+1)=S_j\}=p^{m-c(S_j)}(1-p)^{c(S_j)}$, where $c(S_j)=\sum_{i=1}^{m}\{S_j\}_{ii}$ is the counting function of obtaining the number of ones in the diagonal of matrix S_j.

2) Markov Jump Linear System Modeling and Analysis: We consider that in each time period k, each car is able to obtain its own position, velocity, and acceleration deterministically, i.e., $q_i(k)$ since these measurements can be transmitted through the car's internal wired network. As a result, one obtains

$$\begin{aligned}u_i(k)=&\sum_{j=1}^{n}A_{ij}(\kappa_q(\bar{q}_{ij}(k)-q_i(k)+\sum_{\eta=j+1}^{i}\kappa_q h\bar{v}_{i\eta}(k))\\&+\kappa_v(\bar{v}_{ij}(k)-v_i(k))+\kappa_a(\bar{a}_{ij}(k)-a_i(k)))\\&+J_{ii}(\kappa_q(q_0(k)-q_i(k)-d_{i,0}(k))+\kappa_v(v_0(k)-v_i(t))\\&+\kappa_a(a_0(k)-a_i(k))).\end{aligned}\tag{21}$$

Let $\hat{\bar{q}}_{ij}(k)=\bar{q}_{ij}(k)-q_0(k)-\bar{d}_{j,0}(k)$, $\hat{\bar{v}}_{ij}(k)=\bar{v}_{ij}(k)-v_0(k)$, $\hat{\bar{a}}_{ij}(k)=\bar{a}_{ij}(k)-a_0(k)$, and $\hat{\bar{u}}_{ij}(k)=\bar{u}_{ij}(k)-u_0(k)$, where $\bar{d}_{j,0}(k)=\sum_{\eta=1}^{j}hv_\eta(k)=d_{j,0}(k)$. Then there holds,

$$\begin{aligned}u_i(k)==&\sum_{j=1}^{n}A_{ij}(\kappa_q(\hat{\bar{q}}_{ij}(k)-\hat{q}_i(k)+\sum_{\eta=j+1}^{i-1}h(-\hat{\bar{v}}_{i\eta}(k)+\hat{v}_\eta(k)))\\&+\kappa_v(\hat{\bar{v}}_{ij}(k)-\hat{v}_i(k))+\kappa_a(\hat{\bar{a}}_{ij}(k)-\hat{a}_i(k)))\\&-J_{ii}(\kappa_q\hat{q}_i(k)+\kappa_v\hat{v}_i(t)+\kappa_a\hat{a}_i(k))\end{aligned}\tag{22}$$

where $\hat{\bar{v}}_i(k)=v_i(k)$ is used when the communication information may be lost at time slot k.

Then, we denote the virtual state of the information available through the random communication link at time k by $\hat{\bar{q}}(k)\in\mathbb{R}^m$, $\hat{\bar{v}}(k)\in\mathbb{R}^m$, $\hat{\bar{a}}(k)\in\mathbb{R}^m$. Hence, we have

$$\begin{aligned}\hat{\bar{q}}(k)&=\theta(k)B\hat{q}(k)+(I-\theta(k))\hat{\bar{q}}(k-1),\\\hat{\bar{v}}(k)&=\theta(k)B\hat{v}(k)+(I-\theta(k))\hat{\bar{v}}(k-1),\\\hat{\bar{a}}(k)&=\theta(k)B\hat{a}(k)+(I-\theta(k))\hat{\bar{a}}(k-1),\end{aligned}$$

where $B\in\mathbb{R}^{m\times n}$ with $B_{ij}=1$ iff vehicle j is the origin of link i, and $B_{ij}=0$ otherwise. Because the leader vehicle moves at a constant speed and all variables are relative to the leader, the communicated information between the leader and the corresponding following ones can be omitted in the formulation. Then the platoon system dynamics can be described by (25). Let $C\in\mathbb{R}^{n\times m}$ be the matrix corresponding communicated information on links to the information received node. If vehicle i is receiver of link j, we have $C_{ij}=1$, and $C_{ij}=0$ otherwise. For $r=2$, $n=4$, we give the example of B and C as follows

$$B=\begin{bmatrix}1&0&0&0\\0&1&0&0\\1&0&0&0\\0&0&1&0\\0&1&0&0\end{bmatrix},C=\begin{bmatrix}0&0&0&0&0\\1&0&0&0&0\\0&1&1&0&0\\0&0&0&1&1\end{bmatrix}.$$

To characterize the control input part related to $\sum_{\eta=j+1}^{i-1}h(-\hat{\bar{v}}_{i\eta}(k)+\hat{v}_\eta(k))$, we define the matrix $\hat{C}\in\mathbb{R}^{n\times m}$, i.e.,

$$\hat{C}=\begin{bmatrix}0&0&0&0&0&0&0&\cdots&0\\0&0&0&0&0&0&0&\cdots&0\\0&1&0&0&0&0&0&\cdots&0\\0&0&2&1&0&0&0&\cdots&0\\0&0&0&0&3&2&1&\cdots&0\\\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&\vdots\end{bmatrix}.$$

Based on (22), we have the input in the compact form as

$$\begin{aligned}u(k)=&\begin{bmatrix}\kappa_qC&\kappa_vC-\kappa_qh\hat{C}&\kappa_aC\end{bmatrix}x^c(k)\\&+\begin{bmatrix}-\kappa_qD_J&-\kappa_vD_J+\kappa_qhS&-\kappa_aD_J\end{bmatrix}x^o(k),\end{aligned}\tag{23}$$

where $D_J=D+J$ and $x^c(k)=\begin{bmatrix}\hat{\bar{q}}^\top(k)&\hat{\bar{v}}^\top(k)&\hat{\bar{a}}^\top(k)\end{bmatrix}^\top\in\mathbb{R}^{3m}$. We denote the system state as $x(k)=\begin{bmatrix}x^{o\top}(k)&x^{c\top}(k-1)\end{bmatrix}^\top\in\mathbb{R}^{3(n+m)}$. Then, one obtains

$$x^c(k)=Ox^o(k)+\bar{O}x^c(k-1),\tag{25}$$

$$\Gamma_{\theta(k)} = \begin{bmatrix} I_n & \eta I_n & \frac{\eta^2}{2}I_n - \eta H & 0 & 0 & 0 \\ 0 & I_n & \eta I_n & 0 & 0 & 0 \\ -\kappa_q D_J & -\kappa_v D_J & (1-\frac{\eta}{\tau})I_n - \kappa_a D_J & \kappa_q C & \kappa_v C - \kappa_q h\hat{C} & \kappa_a C \\ \theta(k)B & 0 & 0 & I_m - \theta(k) & 0 & 0 \\ 0 & \theta(k)B & 0 & 0 & I_m - \theta(k) & 0 \\ 0 & 0 & \theta(k)B & 0 & 0 & I_m - \theta(k) \end{bmatrix}. \quad (24)$$

where $O = \begin{bmatrix} \theta(k)B & 0 & 0 \\ 0 & \theta(k)B & 0 \\ 0 & 0 & \theta(k)B \end{bmatrix} \in \mathbb{R}^{3m\times 3n}$ and $\bar{O} = \begin{bmatrix} I_m - \theta(k) & 0 & 0 \\ 0 & I_m - \theta(k) & 0 \\ 0 & 0 & I_m - \theta(k) \end{bmatrix} \in \mathbb{R}^{3m\times 3m}$. Combining (11), (23), and (25), one obtains the Markov jump linear system as

$$x(k+1) = \Gamma_{\theta(k)}x(k), \quad (26)$$

where $\Gamma_{\theta(k)} \in \mathbb{R}^{3(n+m)\times 3(n+m)}$ satisfies (24). By the stability of the above Markov jump linear system, we provide the following theorem to guarantee the internal stability of the vehicle platoon system.

Theorem 4.12: The vehicle platoon system with random packet drops, i.e., (26), can achieve mean-square stability iff

$$\max\{|\text{eig}\{(P\otimes I_{(3(m+n))^2})\text{blkdiag}(\{\Gamma_{S_i}^\top \otimes \Gamma_{S_i}\}_i)\}|\} < 1, \quad (27)$$

where $\text{blkdiag}(\{\Gamma_{S_i}^\top \otimes \Gamma_{S_i}\}_i)$ denotes the block diagonal matrix created by aligning the input matrices $\Gamma_{S_i}^\top \otimes \Gamma_{S_i}$ for all $i = 1, 2, \cdots, 2^m$ along the diagonal.

Proof: We postpone the proof to Appendix E. ■

Remark 4.13: Note that mean-square stability is different from MSC, i.e., the covariance matrix $\mathbf{E}(x(k)x'(k))$ of the system state will converge, meaning that the stability of the platoon system can always be achieved under random packet losses. Our model and results are also applicable for communication links with time-dependent packet loss rates. But in that case, the dimension of the system matrix grows drastically with the number of communication links, and it will cause high complexity to evaluate the platoon system stability. How to solve this problem remains a future research issue.

V. String Stability Analysis

Suppose that the leading vehicle's information can be received by its followers with higher reliability where packet loss effects can be neglected. We consider that the leader vehicle faces a bounded disturbance, which means that $\|u_0(k)\| \le c_u$ and $\|a_0(0)\| \le c_a$, where $c_u \ge 0$ and $a_u \ge 0$ are constants. As a result, we have that $a_0(k) \equiv 0$ does not hold for all k. According to (10), we have

$$\begin{bmatrix} \hat{q}(k+1) \\ \hat{v}(k+1) \\ \hat{a}(k+1) \end{bmatrix} = \begin{bmatrix} I_n & \eta I_n & \frac{\eta^2}{2}I_n - \eta H \\ 0 & I_n & \tau I_n \\ 0 & 0 & (1-\frac{\eta}{\tau})I_n \end{bmatrix} \begin{bmatrix} \hat{q}(k) \\ \hat{v}(k) \\ \hat{a}(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{\eta}{\tau}I_n \end{bmatrix} u(k) + \begin{bmatrix} -\eta H \\ 0 \\ 0 \end{bmatrix} 1_n a_0(k). \quad (28)$$

To investigate the string stability, we need to focus on spacing errors, velocity errors, and acceleration errors. Let $x_i(k) = [\hat{q}_i^\top(k)\ \hat{v}_i^\top(k)\ \hat{a}_i^\top(k)]^\top$ for all $i \in \{0, 1, \cdots, n\}$ and $\bar{x}_i(k) = 0$ be the constant equilibrium solution for system (28) for $a_0(k) \equiv 0$. Then, we can have the following definition, i.e., discrete ℓ_p-string stability by referring to [43]–[45].

Definition 5.1: The platoon system (28) is ℓ_p-string stable if there exists a $\mathcal{K}$ function α and constants $c > 0$, $c_\omega > 0$, $\kappa_\omega > 0$ such that for any initial disturbance $e_{q1}(0)$ and new disturbance $a_0(t)$ satisfying

$$|e_{q1}(0)| < c \text{ and } \|a_0(k)\|_{\ell_\infty} < c_\omega,$$

the solution $e_{qi}(k)$, $\forall i \in \mathcal{V}$, exists for all $k \ge 0$ and satisfies

$$\|e_{qi}(k)\|_{\ell_p} \le \alpha(|e_{q1}(0)|) + \kappa_\omega c_\omega.$$

Based on the above definition, we will utilize H_∞ Norm Markov jump linear system to characterize the string stability of the platoon system under packet drops and disturbances. For each vehicle i, we have the following Markov jump linear system model

$$\mathbb{S}1 \begin{cases} x(k+1) = \Gamma_{\theta(k)}x(k) + \bar{E}a_0(k), \\ y_i(k) = C_{y_i}x(k), \\ z_i(k) = C_{z_i}x(k), \end{cases} \quad (29)$$

where $z_i(k) = e_{q_i}(k) = \hat{q}_i(k) - \hat{q}_{i-1}(k)$, $\bar{E} = [(\eta H 1_n)^\top\ 0] \in \mathbb{R}^{3(n+m)}$, and $C_{z_i} \in \mathbb{R}^{1\times 3(n+m)}$ is a row vector with i-th element being 1, the $i-1$-th element being -1, and all other elements being zero. Note that $y_i(k)$ is the output measurement obtained by node i. Then, we provide the H_∞ norm of system (29) from input $u_{i-1}(k)$ to output $z_i(k)$ as

$$\|\mathbb{S}1\|_\infty^2 = \sup_{u_{i-1}(k)\neq 0} \frac{\|z_i\|_2^2}{\|a_0\|_2^2}, \quad (30)$$

where $\|z_i\|_2^2 = \sum_{k=0}^{\infty} \mathbf{E}(z_i^2(k))$ and $\|a_0\|_2^2 = \sum_{k=0}^{\infty} \mathbf{E}(a_0^2(k))$. The following lemma is provided to ensure the stability of the Markov jump linear system based Lemma 2.7 in [46].

Lemma 5.2: The vehicle platooning system (29) is stable and satisfies the norm constraints $\|\mathbb{S}1\|_\infty^2 < \gamma$ iff there exist matrices $G_{iS_j} = G_{iS_j}^\top \in \mathbb{R}^{3(n+m)\times 3(n+m)} \succ 0$ such that

$$\begin{bmatrix} \Gamma_{S_j} & \bar{E} \\ C_{zi} & 0 \end{bmatrix}^\top \begin{bmatrix} G_{ipS_j} & \bar{E} \\ C_{zi} & I \end{bmatrix} \begin{bmatrix} \Gamma_{S_j} & \bar{E} \\ C_{zi} & 0 \end{bmatrix} - \begin{bmatrix} G_{iS_j} & 0 \\ 0 & \gamma I \end{bmatrix} \prec 0, \quad (31)$$

where $G_{ipS_j} = \sum_{l=1}^N p_{il} G_{iS_l}$ for all $S_j \in \mathbb{K}$.

Then, we have the following theorem to guarantee the string stability of the platoon system.

Theorem 5.3: If the following optimization problem has feasible solutions,

$$\min_{G_{iS_j}, \forall S_j \in \mathbb{K}, i\in\mathcal{V}} \gamma \quad \text{s.t.} \quad (31), \forall i \in \mathcal{V}, \quad (32)$$

the platooning system with random packet drops can achieve ℓ_p-string stability.

Remark 5.4: From the above analysis, we note that the ℓ_p-string stability can ensure that the length of the vehicle platoon will not amplify by the disturbance. Although we only consider the case that leading vehicle has disturbances, the above results can be easily extended to more general disturbing cases. Note that the major problem faced by the above solution to evaluate the string stability of the platoon system is computational complexity. Since the number of states in the Markov decision process is 2^m, which means that for each car i, there will be 2^m variable matrices in (32). Hence, it can be costly to solve the feasibility of the problem (32). We also need to point out that whether the modeling of a Markov jump linear system is effective depends on its weak controllability. Nevertheless, Theorem 5.3 can be applied as a useful tool to verify whether a platoon control system can maintain string stability.

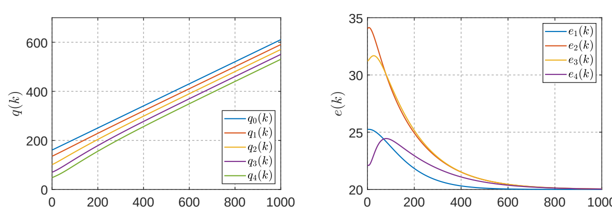
VI. Simulation Results

A. Internal Stability without Packet Losses

Here, we consider a vehicle platoon where parameters are given as S1 of Table I, where two predecessors' messages are used by each follower. The initial positions, velocities, and accelerations are selected from $q_i(0) = 30 * (n - i) + 10 * \mathrm{rand\ m}$ for all $i \in \mathcal{V}$ and 0, $v_0(0) = 30\mathrm{m/s}$, $v_i(0) = 25 + 5 * \mathrm{rand}$ for all $1 \leq i \leq n$, $a_0(0) = 0\mathrm{m/s}^2$, $a_i(0) = 2 + \mathrm{rand\ m/s}^2$ for all $1 \leq i \leq n$, where rand is a function to produce a number from the interval $[0, 1]$ uniformly. From Fig. 1, we observe that the stability can be achieved by all the following vehicles when the leading vehicle moves at a constant speed.

TABLE I
Parameters of different scenario

P	n	r	η	h	d	τ	κ_q	κ_v	κ_a
S1	5	2	0.015	0.4	8	0.01	−0.45	1	−0.40
S2	5	2	0.015	0.4	8	0.01	0.45	1	−0.2
S3	25	15	0.015	0.4	8	0.1	0.45	1	0.2
S4	4	2	0.005	0.4	8	0.1	0.45	1	0.2
S5	5	3	0.005	0.4	8	0.1	0.45	1	0.2



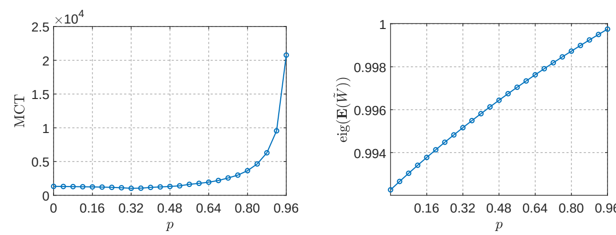
(a) Positions of all vehicles (b) Spacing errors variations

Fig. 1. All spacing errors converge to the same value $hv_0 + d$

B. Internal Stability with Packet Losses

In this part, we investigate how random packet losses affect the control performance of the vehicle platoon for two cases, i.e., without/with using historical information.

1) Cases without Using Historical Information: We consider the scenario with parameters as S2 of Table I and the initial positions, velocities, and accelerations are produced in the same way as before. We consider that p is from 0 to 0.96 with stepsize 0.04. For each p, we run the platooning process for 1000 times and then set the terminating error threshold as $\epsilon = 0.01$ for $\|e(k) - \bar{e}(k)\|$, $\|v(k) - \bar{v}(k)\|$, and $\|a(k) - \bar{a}(k)\|$, where $\bar{e}(k)$, $\bar{v}(k)$, and $\bar{a}(k)$ are vectors with all elements equal to the average of vector $e(k)$, $v(k)$, $a(k)$. Then, we obtain the mean convergence time (MCT) for different packet drop probabilities. It is observed from Fig. 2(b) that the maximum absolute value of all eigenvalues of the expected random system matrix increase with the packet loss rate of p, which illustrates our convergence in expectation result. From Fig. 2(a), we observe that when the probability of packet losses becomes larger, MCT increases at the same time. We also find that under homogeneous packet drop rate, internal stability can always be achieved but MCT for any given accuracy will increase with p. Note that each following vehicle is anticipated to receive two predecessors' messages. When p is small (e.g., < 0.3), the chance that both messages are lost is small. Given the leader remains moving at a constant velocity, the MCT performance is not affected significantly when p is small. The impact of p on platooning performance is more significant when the leader changes its velocity frequently.



(a) Mean convergence time (b) The maximum eigenvalue

Fig. 2. Homogeneous packet drop probability p

Then, by changing one parameter at a time and simulating 1000 times for each parameter, we draw curves of $\mathbf{E}(\rho(\tilde{W}))$ with packet loss rate p in Fig. 3. It can be observed that with the growth of the packet loss rate, $\mathbf{E}(\rho(\tilde{W}))$ increases, meaning that MCT decreases when p increases. Moreover, the increase of n, κ_a, and h will lead to a larger value of $\mathbf{E}(\rho(\tilde{W}))$.

Let $\tilde{\mathcal{E}}$ be the set of all communication links in the platoon system including the leading vehicle. We also investigate how heterogeneous packet loss rates affect the control performance of the platoon system, where three cases are considered, i.e., correlated1: $\sum_{(i,j)\in\tilde{\mathcal{E}}} p_{ij} = 1$; correlated2: $\sum_{(i,j)\in\tilde{\mathcal{E}}} (1 - p_{ij}) = 1$; independent: The packet loss rate on each link is produced randomly from uniform distribution between 0 and 1. We draw 1000 samples of $\mathbf{E}(\rho(\tilde{W}))$ and $\rho(\mathbf{E}(\tilde{W}))$ in Fig. 4. It is observed that the spatial channel correlation will have different impact on MCT.

We also consider a large size vehicle platoon whose parameters are set as S3 of Table I, and let $p = 0.6$. It can be observed from Fig. 5(a) that all inter-vehicle distances converge to $d + hv_0$ without any collisions, meaning that

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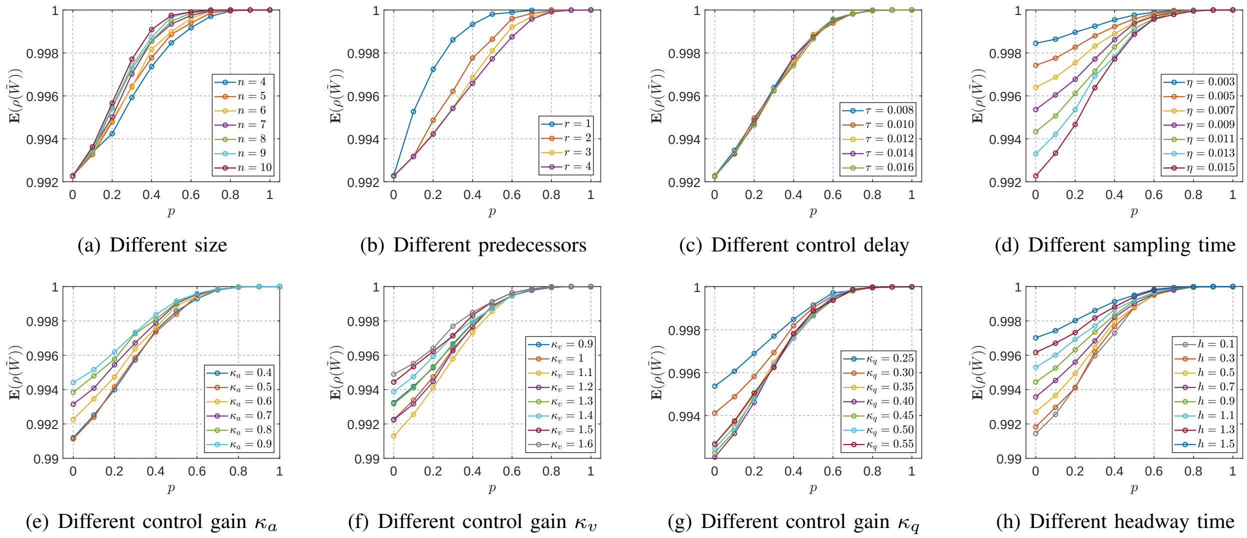
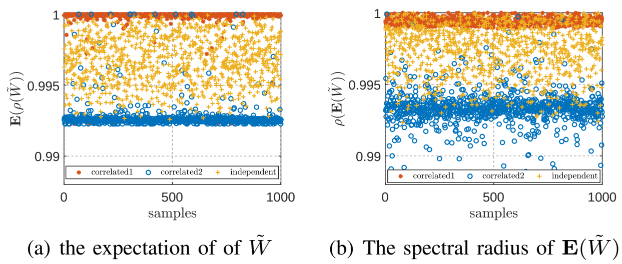


(a) Different size (b) Different predecessors (c) Different control delay (d) Different sampling time

(e) Different control gain κ_a (f) Different control gain κ_v (g) Different control gain κ_q (h) Different headway time

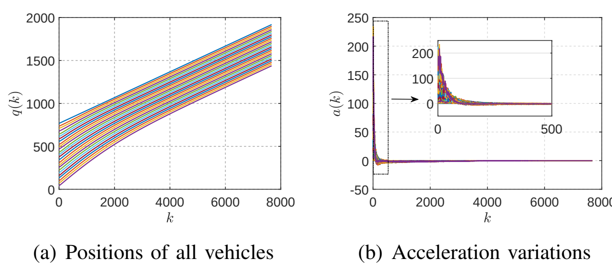
Fig. 3. The expectation of the spectral radius of matrix $\tilde{W}$ vs the packet loss rate for different scenarios



(a) the expectation of of $\tilde{W}$ (b) The spectral radius of $\mathbf{E}(\tilde{W})$

Fig. 4. Comparison among different time-independent cases

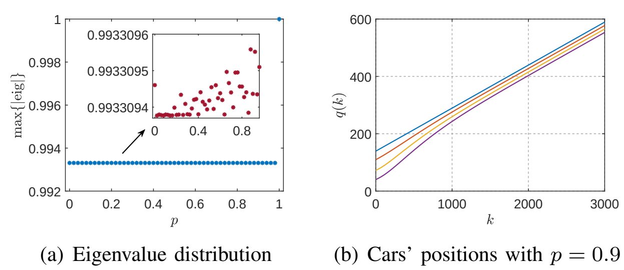
stability can be achieved with a homogeneous packet loss probability of $p = 0.6$, thanks to a high value of r. However, with packet losses, the curves of accelerations have drastic fluctuations shown in Fig. 5(b), which need to be avoided in practical scenarios.



(a) Positions of all vehicles (b) Acceleration variations

Fig. 5. All spacing errors converge to the same value $hv_0 + d$

2) Cases Using Historical Information: We first investigate how the packet drop will affect the stability of the platoon system when all information is transmitted through communication networks. When the packet of vehicle i is lost, its position will be updated by using historical position, velocity, and acceleration. Since the computation complexity of the stability condition (27) is generally $O((3 * (m + n) * 2^m)^3)$, it is difficult to conduct extensive numerical result for large scale networks. Thus, we only consider the vehicle platoon with 4 cars and each car uses two predecessors' information for control, and the leading vehicle's information has no packet loss, which means that $m = 3$. We set the parameters as S4 of Table I. The probability of the packet loss at each communication link is set from $p = 0$ to 1 with a stepsize of 0.02. The distribution of eigenvalues of system matrix in (27) is shown in Fig. 6(a) and the positions of all the following vehicles are plotted in Fig. 6(b) when $p = 0.9$. From Fig. 6(a), as long as the packet loss rate is always independent and less that one, all absolute values of the eigenvalues (27) are strictly less than one, meaning that the mean-square stability can be maintained. Note that the convergence rate of the platoon system characterized by maximum eigenvalue has a tendency to decrease with the packet loss rate, but in some cases the convergence rate can also decrease with the growth of the packet loss rate. Fig. 6(b) shows that all vehicles will keep tracking its predecessor with a constant spacing after a period of control and, which illustrates the effectiveness of our theoretical results.



(a) Eigenvalue distribution (b) Cars' positions with $p = 0.9$

Fig. 6. Using historical information

Then, how the leading vehicle's speed change affects the control performance is investigated for S2 of Table I. We set the terminating error threshold as $\epsilon = 0.03$. The packet delivery loss rate is set as 0.9. We change the speed of the leading vehicle from iteration 10 to 190 with a stepsize of 20 and changing size is set from -14 to 4 with a stepsize of 2. We consider two initialization settings, i.e., large relative distance (LD) $q_i(0) = 30 * (n - i) + 10 * \mathrm{rand\ m}$ and small relative distance (SD) $q_i(0) = 10 * (n - i) + 10 * \mathrm{rand\ m}$. For initial iterations, each node i will only use the jth predecessor's information if all the 1th - $(j-1)$th predecessors'

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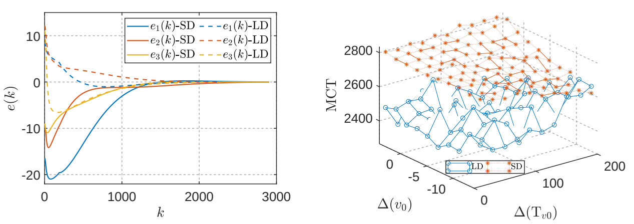
information is updated. We show the effect of the velocity change of the leading vehicle on the control performance of the platoon system in Fig. 7. Fig. 7(a) shows the variation of spacing errors with iterations when the speed change duration of the leading vehicle is 180 and the speed change size is -14. We observe that the internal stability can still be guaranteed when the leading vehicle changes its speed. Given the error threshold of 0.03, for each pair of speed changing size and duration, we simulate for 100 times. Then, we obtain MCT for the platoon system, which is shown in Fig. 7(b). It illustrates that the stability of the platoon system can always be achieved even when the leading vehicle changes its speed. The larger the speed change size (may be positive or negative directions) and duration are, the larger MCT is.



(a) Spacing error variation (b) MCT for different changes

Fig. 7. the control performance of vehicle platooning when speed change of the leading vehicle

For the scenario S5 of Table I, we also examine how different historical information affects the control performance of the platoon system. The initialization is set as $q_i(0) = 10 * (n + 1 - i) + (n + 1 - i)^2$ for $0 \le i \le n$, $a_i(0) = 3 + i$, $v_i(0) = 29 + i$ for all $i \in \mathcal{V}$ and the terminating error threshold is set as $\epsilon = 0.03$. For initial iterations, once the predecessor's information is received, it will be used and no information will be used, otherwise. We use MCT, the mean maximum spacing error, and the mean minimum spacing error to characterize the control performance of the platoon system. Fig. 8 plots the results for the scenario where the leading vehicle moves at a constant speed $v_0 = 30\mathrm{m/s}$ and that where the leading vehicle has disturbances ($u_0(k) = k$ for all $10 \le k \le 100$), respectively. We compare three cases: case0) not using historical information; case1) using the latest historical information; case2) using the one historical information before the latest one. When the leading vehicle moves at a constant speed, we observe from Fig. 8(a) that MCT increases slightly when using historical information and when the packet loss rate less than 0.6, MCT of case0 decreases with the packet loss rate. This is because that when $r = 3$, the probability that at least one packet is received from some predecessor is high and when the leader moves at a constant speed, the perturbation of p may decrease MCT when no historical information is used. When p is large, then using historical information is beneficial to decrease MCT. From Fig. 8(b), the minimum spacing error grows when historical information is used. It is because historical information may deviate from the true one. When the disturbance occurs in the leading vehicle, we find that MCT increases for all three cases and using no historical information can cause more time to ensure the stability of the platoon, which is shown in Fig. 8(c). Furthermore, without using historical information, the length of the platoon indicated by max-case0 shown in Fig. 8(d) can increase greatly with a large p, which means that the string stability cannot be ensured.

C. String Stability with Packet Losses

Consider that the probability of packet losses is from 0.01 to 1 and we set the disturbance on the acceleration of the leading vehicle as $1/(k)^{-1/4}*\mathrm{rand}$. Consider that the platoon system has 4 vehicles. First, using the Schur Complement, we obtain an LMI condition according to Lemma 4.3. To guarantee $\|\mathbb{S}1\|_\infty^2 = \inf \gamma$, there exist a positive definite matrix $G_{iS_j} \in \mathbb{R}^{3(n+m)\times 3(n+m)}$ and γ such that

$$\begin{bmatrix} G_{iS_j} & 0 & \Gamma_{S_j}^\top G_{ipS_j} & C_{zi}^\top \\ \bullet & \gamma I & \bar{E}^\top G_{ipS_j} & 0 \\ \bullet & \bullet & G_{ipS_j} & 0 \\ \bullet & \bullet & \bullet & I \end{bmatrix} \succ 0, \forall j \in \mathbb{K}.$$

whose dimension is $(6(n + m) + 2) \times (6(n + m) + 2)$. We find that the above LMI is infeasible. Through numerical simulations, by setting $p = 0.5$, we find that each inter-vehicle distance will converge to a stable value from Fig. 9. It is illustrated that LMI is infeasible since $\|\mathbb{S}1\|_\infty^2 = \sup_{u_{i-1}(k)\neq 0} \frac{\|z_i\|_2^2}{\|a_0\|_2^2}$ does not converge. However, the string stability can still be maintained since $|e_i(k)| < |e_{i-1}(k)|$ for all k. Then, we extend the scale of the platoon system as $n = 25$, $r = 5$ with the same packet loss rate $p = 0.5$, without changing other parameters. From Fig. 10(a) and Fig. 10(b), one notices that the string stability can still be maintained by the platoon system when independent and identical packet loss happens on all communication links, while all vehicles can maintain to be stable when the disturbance happens in the leading vehicle. Note that when all $e_i(k)$ for all $i \in V$ converge to zero, the platoon system achieve the stable state. We use the variance of the maximum relative spacing error $D(e(k)) = \max(e(k)) - \min(e(k))$ to characterize the convergence time. Changing the packet loss rate from 0 to 0.9 with stepsize 0.1, we investigate how the random packet losses affect the convergence time for the platoon system. To make sure that all spacing errors converge to zero, we set $a_0(k) = 1/(k)^{-1/4} \times \mathrm{rand}$ for $k < 3000$, and $a_0(k) = 0$ otherwise. As shown in Fig. 10(c), when packet loss rate p becomes larger, the maximum relative spacing error changes from $20\mathrm{m}$ for $p = 0$ to $70\mathrm{m}$ for $p = 0.9$. It takes almost 1000 iterations to make the maximum relative spacing error less than $10\mathrm{m}$ for $p = 0.9$. Moreover, the larger packet loss rate will incur vehicle collision if there are no collision avoidance strategies, which is illustrated in Fig. 10(d).

VII. Discussions

A. Controller Design

It is desirable to investigate the controller design problem for the platoon system when the string stability cannot be maintained. Here, we provide a method to design controller by Markov jump linear system theory. Considering that

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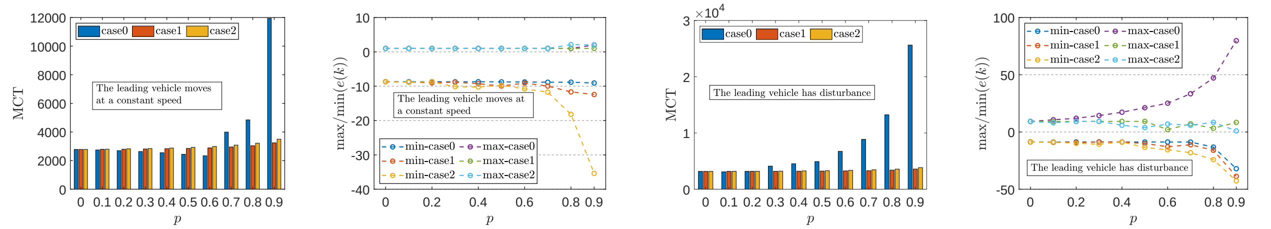
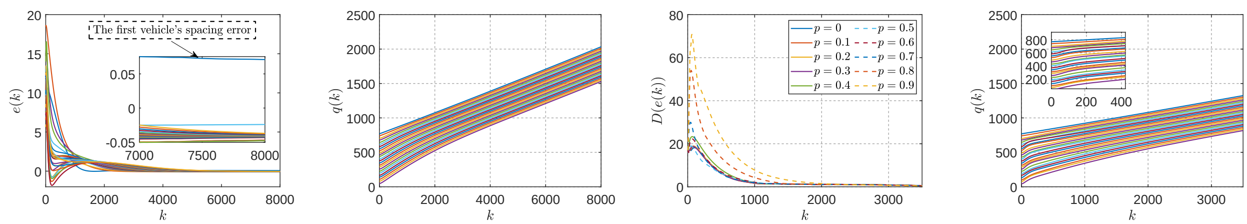


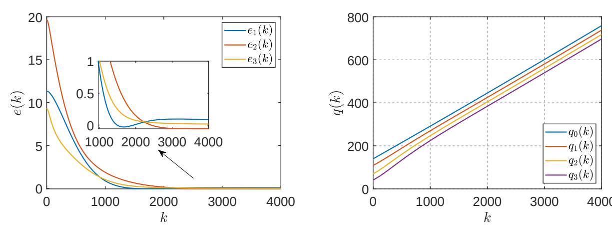
(a) Mean convergence time without disturbance (b) Max/min spacing errors without disturbance (c) Mean convergence time with disturbance (d) Max/min spacing errors with disturbance

Fig. 8. Comparison among case0, case1, and case2



(a) Spacing error dynamics-$p = 0.5$ (b) Position variations-$p = 0.5$ (c) Convergence-$p = 0 \sim 0.9$ (d) Position variations-$p = 0.9$

Fig. 10. $n = 25$, $r = 5$ for cases with $p = 0.5$, $p = 0 \sim 0.9$, and $p = 0.9$, respectively



(a) Spacing error dynamics (b) Position variations

Fig. 9. $n = 4$, $r = 2$, and $p = 0.5$

once packet drops happen and the string stability cannot be preserved, we will trigger other communication mechanism and channel to the control performance. We first consider the centralized controller design problem. The vehicle platooning system with new control input is denoted as

$$\mathbb{S}2 \begin{cases} x(k+1) = \Gamma_{\theta(k)}x(k) + \tilde{B}u(k) + Ea_0(k), \\ y_i(k) = C_{yi}x(k), \\ z_i(k) = C_{zi}x(k), \end{cases} \tag{33}$$

where $\tilde{B} = [0_n; 0_n; \frac{\eta}{\tau}I_n; 0_m; 0_m; 0_m]^\top$. Let $y(k) = [y_1(k), y_2(k), \cdots, y_n(k)]^\top$. Then, we denote the controller as the following form

$$\mathbb{C}1 \begin{cases} x_c(k+1) = A_{c\theta(k)}x_c(k) + B_{c\theta(k)}y(k), \\ u(k) = C_{c\theta(k)}x_c(k) + D_{c\theta(k)}y(k), \end{cases} \tag{34}$$

where $A_{c\theta(k)}$, $B_{c\theta(k)}$, $C_{c\theta(k)}$, $D_{c\theta(k)}$ are of compatible dimensions. The goal is to determine these matrices such that the string stability of the platoon system can be preserved. Connecting the controller (34) and (33), we thus obtain the following system

$$\mathbb{S}3 \begin{cases} \bar{x}(k+1) = \bar{A}_{\theta(k)}\bar{x}(k) + \bar{E}a_0(k), \\ z_i(k) = \bar{C}_{zi}\bar{x}(k), \end{cases} \tag{35}$$

where the indicated matrices are $\bar{A}_{\theta(k)} = \begin{bmatrix} \Gamma_{\theta(k)} + \tilde{B}D_{c\theta(k)}C_y & \tilde{B}C_{c\theta(k)} \\ B_{c\theta(k)}C_y & A_{c\theta(k)} \end{bmatrix}$ and $\bar{E} = [E; 0_{\dim(x_c)}]$ with $\dim(x_c)$ taking the dimension of vector $x_c(k)$. Then, we have the following ℓ_p string stability condition.

Lemma 7.1: The vehicle platooning system (35) is stable and satisfies the norm constraints $\|\mathbb{S}3\|_\infty^2 < \bar{\gamma}$ iff there exist matrices $\bar{G}_{iS_j} = \bar{G}_{iS_j}^\top \succ 0$ such that

$$\begin{bmatrix} \bar{A}_{S_j} & \bar{E} \\ \bar{C}_{zi} & 0 \end{bmatrix}^\top \begin{bmatrix} \bar{G}_{ipS_j} & \bar{E} \\ \bar{C}_{zi} & I \end{bmatrix} \begin{bmatrix} \bar{A}_{S_j} & \bar{E} \\ \bar{C}_{zi} & 0 \end{bmatrix} - \begin{bmatrix} \bar{G}_{iS_j} & 0 \\ 0 & \bar{\gamma}I \end{bmatrix} \prec 0, \tag{36}$$

for all $\theta_i(k)$, where $\bar{G}_{ipS_j} = \sum_{j=1}^{N} p_{ij}\bar{G}_{iS_j}$ for all $S_j \in \mathbb{K}$.

Then, the controller design problem become the following optimization problem,

$$\min_{\bar{\gamma},\ \bar{G}_{iS_j}, \forall S_j \in \mathbb{K}, i \in \mathcal{V}} \|\mathbb{S}3\|_\infty^2$$
$$\text{s.t.} \qquad (36), \forall i \in \mathcal{V}, \forall S_j \in \mathbb{K}.$$

The LMI toolbox in Matlab can be used to solve the above optimization problem. However, its high computational problem caused by the growth of the number of communication links is still the main difficulty.

Remark 7.2: When the number of states is 2^m, the number of the total decision variables is $2^m3(n+m)(3(n+m)+1)/2$ and the number of rows of the LMIs is $(6(n+m)+2)2^m$ for one vehicle string stability criterion. To deal with the challenge in solving large-scale LMI problems using interior-point methods, first-order methods (like proximal descent, projected gradient descent, and ADMM) and second-order algorithms have been investigated. In [47], a Newton-PCG algorithm has been proposed to solve large and sparse LMI feasibility problems, which converges in linear time and memory when the parameter matrices share a Cholesky factorization sparsity pattern. However, the obtained LMI feasibility based condition still faces the scalability problem when the number of communication links grows in the platoon system. But if the platoon system has a reasonably small number of communication links such as $2^m \leq n$, the complexity is still polynomial of the platoon size n and thus tolerable. One possible way is to view each vehicle and its predecessors whose information is used as a subsystem, where the number of communication links in each subsystem is small enough

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to obtain the solution for the given condition. By ensuring the stability of each subsystem, we can eventually ensure the stability of the whole platoon system.

B. Modeling and Performance Optimization

The discretization of the car may incur the inaccuracy to the whole platoon system. It is more desirable to model the communication intervals as $\bar{k} = \lfloor \eta k/\phi \rfloor$, where ϕ is a constant number characterizing the proportion between the communication interval and sampling time and $\lfloor \cdot \rfloor$ is the flooring operator. After receiving the information from neighboring cars, each car will use that information to conduct local control by updating its local state in the input part. As long as η is small enough, the discrete-time system can model the practical continuous-time system accurately and the communication interval does not need to be the same as η. Note that more parameters are shown in the platoon system, which means that the analysis can be more complex and difficult. However, we can view it as an event-based control system, where the event is based on a constant time interval, which is left as our future works.

From numerical results, although random packet losses may not affect the spacing error dynamics too much, it will definitely severely degrade the acceleration input curve by causing fluctuations. The larger random packet loss rate is, the larger the fluctuations of the acceleration curve are. We notice that random packet losses can drastically affect the transient dynamic of the platoon. Moreover, the collision avoidance needs to be considered to guarantee the safety of the platoon system since a larger packet loss rate can cause the spacing error to be less than zero.

VIII. Conclusions and Future Work

In this paper, we investigated how random packet losses affect the control performance of the vehicle platoon with a multiple-predecessors IFT. Then, a systematic way is given to analyze internal stability and string stability for different packet loss scenarios. We first considered independently and identically distributed random packet losses and no historical information will be used if packet losses happen. Then, we used the matrix perturbation method to obtain an analytical upper on the packet loss rate to guarantee all cars in the platoon can form a queue with a velocity-based inter-vehicle distance in expectation. We found that mean-square convergence can be achieved if the expectation of the spectral radius of the random system matrix is always less than 1. Extensive numerical results show that mean convergence time increase with the random packet losses rate. For the case, that packet loss rates are temporally corrected or historical information will be used when packet losses happen, we modeled the lossy network as a Markov chain process and obtained the sufficient and necessary condition to guarantee that the length of the platoon can be finitely limited. The condition is formulated as the feasibility of an LMI problem, and the computation cost and dimension increase with the network scale and the number of communication links.

For future work, we will further analyze the effect of random packet losses on the stability of general IFT-based platooning. Moreover, how to solve the high computational issue when applying Markov jump linear system theory to random communication errors is another important direction. Other important research issues are to deal with more accurate system modeling, distributed collision avoidance controller design given communication errors, and communication resource optimization, which reckon further investigation.

References

[1] M. Maurer, J. C. Gerdes, B. Lenz, and H. Winner. Autonomous driving. *Berlin, Germany: Springer Berlin Heidelberg*, 10:978–3, 2016.

[2] D. Jia, K. Lu, J. Wang, X. Zhang, and X. Shen. A survey on platoon-based vehicular cyber-physical systems. *IEEE Communications Surveys & Tutorials*, 18(1):263–284, 2015.

[3] N. Lu, N. Cheng, N. Zhang, X. Shen, and J. M. Mark. Connected vehicles: Solutions and challenges. *IEEE Internet of Things Journal*, 1(4):289–299, 2014.

[4] S. Feng, Y. Zhang, S. E. Li, Z. Cao, H. X. Liu, and L. Li. String stability for vehicular platoon control: Definitions and analysis methods. *Annual Reviews in Control*, 2019.

[5] A. Alam, B. Besselink, V. Turri, J. Martensson, and K. H. Johansson. Heavy-duty vehicle platooning for sustainable freight transportation: A cooperative method to enhance safety and efficiency. *IEEE Control Systems Magazine*, 35(6):34–56, 2015.

[6] Z. Ju, H. Zhang, and Y. Tan. Deception attack detection and estimation for a local vehicle in vehicle platooning based on a modified UFIR estimator. *IEEE Internet of Things Journal*, 7(5):3693–3705, 2020.

[7] X. Yang, L. Liu, N. H. Vaidya, and F. Zhao. A vehicle-to-vehicle communication protocol for cooperative collision warning. In *Proc. MOBIQUITOUS*, pages 114–123. IEEE, 2004.

[8] G. Guo and S. Wen. Communication scheduling and control of a platoon of vehicles in VANETs. *IEEE Transactions on Intelligent Transportation Systems*, 17(6):1551–1563, 2015.

[9] Y. A. Harfouch, S. Yuan, and S. Baldi. An adaptive switched control approach to heterogeneous platooning with intervehicle communication losses. *IEEE Transactions on Control of Network Systems*, 5(3):1434–1444, 2017.

[10] F. Acciani, P. Frasca, G. Heijenk, and A. Stoorvogel. Stochastic string stability of vehicle platoons via cooperative adaptive cruise control with lossy communication. *arXiv preprint arXiv:1905.04779*, 2019.

[11] S. Darbha, S. Konduri, and P. Pagilla. Effects of V2Vcommunication on time headway for autonomous vehicles. In *Proc. ACC*, pages 2002–2007. IEEE, 2017.

[12] P. Seiler and R. Sengupta. Analysis of communication losses in vehicle control problems. In *Proc. ACC*, volume 2, pages 1491–1496. IEEE, 2001.

[13] P. Seiler and R. Sengupta. An H/sub/spl infin//approach to networked control. *IEEE Transactions on Automatic control*, 50(3):356–364, 2005.

[14] R. Teo, D. M. Stipanovic, and C. J. Tomlin. Decentralized spacing control of a string of multiple vehicles over lossy datalinks. *IEEE Transactions on Control Systems Technology*, 18(2):469–473, 2009.

[15] J. Ploeg, E. Semsar-Kazerooni, G. Lijster, N. van de Wouw, and H. Nijmeijer. Graceful degradation of cooperative adaptive cruise control. *IEEE Transactions on Intelligent Transportation Systems*, 16(1):488–497, 2014.

[16] R. H. Middleton and J. H. Braslavsky. String instability in classes of linear time invariant formation control with limited communication range. *IEEE Transactions on Automatic Control*, 55(7):1519–1530, 2010.

[17] Y. Zheng, S. E. Li, J. Wang, D. Cao, and K. Li. Stability and scalability of homogeneous vehicular platoon: Study on the influence of information flow topologies. *IEEE Transactions on Intelligent Transportation Systems*, 17(1):14–26, 2015.

[18] S. E. Li, X. Qin, Y. Zheng, J. Wang, K. Li, and H. Zhang. Distributed platoon control under topologies with complex eigenvalues: Stability analysis and controller synthesis. *IEEE Transactions on Control Systems Technology*, 27(1):206–220, 2017.

[19] S. Stüdli, M. M. Seron, and R. H. Middleton. Vehicular platoons in cyclic interconnections. *Automatica*, 94:283–293, 2018.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JIOT.2020.3004573, IEEE Internet of Things Journal

[20] S-E. Li, Y. Zheng, K. Li, Y. Wu, J. K. Hedrick, F. Gao, and H. Zhang. Dynamical modeling and distributed control of connected and automated vehicles: Challenges and opportunities. *IEEE Intelligent Transportation Systems Magazine*, 9(3):46–58, 2017.

[21] F. Gao, X. Hu, S. E. Li, K. Li, and Q. Sun. Distributed adaptive sliding mode control of vehicular platoon with uncertain interaction topology. *IEEE Transactions on Industrial Electronics*, 65(8):6352–6361, 2018.

[22] Y. Bian, Y. Zheng, W. Ren, S. E. Li, J. Wang, and K. Li. Reducing time headway for platooning of connected vehicles via V2V communication. *Transportation Research Part C: Emerging Technologies*, 102:87–105, 2019.

[23] X. Liu, A. Goldsmith, S. S. Mahal, and J. K. Hedrick. Effects of communication delay on string stability in vehicle platoons. In *Proc. ITSC*, pages 625–630. IEEE, 2001.

[24] A. Peters, R. Middleton, and O. Mason. Leader tracking in homogeneous vehicle platoons with broadcast delays. *Automatica*, 50(1):64–74, 2014.

[25] M. Di Bernardo, A. Salvi, and S. Santini. Distributed consensus strategy for platooning of vehicles in the presence of time-varying heterogeneous communication delays. *IEEE Transactions on Intelligent Transportation Systems*, 16(1):102–112, 2014.

[26] S. Santini, A. Salvi, A. S. Valente, A. Pescapè, M. Segata, and R. Cigno. A consensus-based approach for platooning with intervehicular communications. In *Proc. INFOCOM*, pages 1158–1166. IEEE, 2015.

[27] S. Santini, A. Salvi, A. Valente, A. Pescapé, M. Segata, and R. Cigno. A consensus-based approach for platooning with intervehicular communications and its validation in realistic scenarios. *IEEE Transactions on Vehicular Technology*, 66(3):1985–1999, 2016.

[28] I. G. Jin and G. Orosz. Dynamics of connected vehicle systems with delayed acceleration feedback. *Transportation Research Part C: Emerging Technologies*, 46:46–64, 2014.

[29] L. Zhang and G. Orosz. Motif-based design for connected vehicle systems in presence of heterogeneous connectivity structures and time delays. *IEEE Transactions on Intelligent Transportation Systems*, 17(6):1638–1651, 2016.

[30] I. G. Jin and G. Orosz. Optimal control of connected vehicle systems with communication delay and driver reaction time. *IEEE Transactions on Intelligent Transportation Systems*, 18(8):2056–2070, 2016.

[31] W. Qin, M. Gomez, and G. Orosz. Moment-based plant and string stability analysis of connected cruise control with stochastic delays. In *Proc. ACC*, pages 2747–2752. IEEE, 2015.

[32] Y. Li, C. Tang, K. Li, X. He, S. Peeta, and Y. Wang. Consensus-based cooperative control for multi-platoon under the connected vehicles environment. *IEEE Transactions on Intelligent Transportation Systems*, 20(6):2220–2229, 2018.

[33] A. Al-Jhayyish and K. Schmidt. Feedforward strategies for cooperative adaptive cruise control in heterogeneous vehicle strings. *IEEE Transactions on Intelligent Transportation Systems*, 19(1):113–122, 2017.

[34] C. Bergenhem, K. Meinke, and F. Ström. Quantitative safety analysis of a coordinated emergency brake protocol for vehicle platoons. In *International Symposium on Leveraging Applications of Formal Methods*, pages 386–404. Springer, 2018.

[35] C. Wu, Y. Lin, and A. Eskandarian. Cooperative adaptive cruise control with adaptive Kalman filter subject to temporary communication loss. *IEEE Access*, 7:93558–93568, 2019.

[36] S. Sheikholeslam and C. A. Desoer. Longitudinal control of a platoon of vehicles with no communication of lead vehicle information: A system level study. *IEEE Transactions on Vehicular Technology*, 42(4):546–554, 1993.

[37] S. S. Stankovic, M. J. Stanojevic, and D. D. Siljak. Decentralized overlapping control of a platoon of vehicles. *IEEE Transactions on Control Systems Technology*, 8(5):816–832, 2000.

[38] C. Campolo, A. Vinel, A. Molinaro, and Y. Koucheryavy. Modeling broadcasting in IEEE 802.11 p/WAVE vehicular networks. *IEEE Communications Letters*, 15(2):199–201, 2010.

[39] E. O. Elliott. Estimates of error rates for codes on burst-noise channels. *The Bell System Technical Journal*, 42(5):1977–1997, 1963.

[40] E. A. Grove and G. Ladas. *Periodicities in nonlinear difference equations*. Chapman and Hall/CRC, 2004.

[41] G. W. Stewart. Matrix perturbation theory. 1990.

[42] G. M. Krause. Bounds for the variation of matrix eigenvalues and polynomial roots. *Linear algebra and its applications*, 208:73–82, 1994.

[43] S. Feng, H. Sun, Y. Zhang, J. Zheng, H. Liu, and L. Li. Tube-based discrete controller design for vehicle platoons subject to disturbances and saturation constraints. *IEEE Transactions on Control Systems Technology*, 28(3):1066–1073, 2020.

[44] B. Besselink and K. H. Johansson. String stability and a delay-based spacing policy for vehicle platoons subject to disturbances. *IEEE Transactions on Automatic Control*, 62(9):4376–4391, 2017.

[45] J. Ploeg, D. W. Vanouw, and H. Nijmeijer. Lp string stability of cascaded systems: Application to vehicle platooning. *IEEE Transactions on Control Systems Technology*, 22(2):786–793, 2013.

[46] A. PC. Gonçalves, A. Fioravanti, and J. C. Geromel. Dynamic output feedback H_∞ control of discrete-time Markov jump linear systems through linear matrix inequalities. In *Proc. CDC*, pages 4787–4792. IEEE, 2008.

[47] R. Y. Zhang and J. Lavaei. Efficient algorithm for large-and-sparse LMI feasibility problems. In *Proc. CDC*, pages 6868–6875. IEEE, 2018.

[48] S. Kar and J. M. F. Moura. Sensor networks with random links: Topology design for distributed consensus. *IEEE Transactions on Signal Processing*, 56(7):3315–3326, 2008.

APPENDIX A
PROOF OF THEOREM 4.3

Proof: To obtain the parameter range to guarantee the system stability, we have

$$\begin{aligned}
&\mathbf{det}(\lambda I_{3n} - W)\\
&= \begin{vmatrix} (\lambda-1)I_n & -\eta I_n & -\frac{\eta^2}{2}I_n + \eta H \\ 0 & (\lambda-1)I_n & -\eta I_n \\ \frac{\eta}{\tau}\kappa_q L_J & \frac{\eta}{\tau}\kappa_v L_J & (\lambda - 1 + \frac{\eta}{\tau})I_n + \frac{\eta}{\tau}\kappa_a L_J \end{vmatrix}\\
&= \begin{vmatrix} (\lambda-1)I_n & -\eta I_n & -\frac{\eta^2}{2}I_n + \eta H \\ 0 & (\lambda-1)I_n & -\eta I_n \\ \frac{\eta}{\tau}\kappa_q L_J & \frac{\eta}{\tau}\kappa_v L_J & (\lambda - 1 + \frac{\eta}{\tau})I_n + \frac{\eta}{\tau}\kappa_a L_J \end{vmatrix} \\
&\times \begin{vmatrix} I_n & \frac{\eta}{(\lambda-1)}I_n & \frac{\eta^2(1+\lambda)}{2(1-\lambda)^2}I_n - \frac{\eta}{(\lambda-1)}H \\ 0 & I_n & \frac{\eta}{(\lambda-1)}I_n \\ 0 & 0 & I_n \end{vmatrix}.
\end{aligned} \tag{37}$$

And then there holds

$$\begin{aligned}
&\mathbf{det}(\lambda I_{3n} - W)\\
&= \begin{vmatrix} (\lambda-1)I_n & 0 & 0 \\ 0 & (\lambda-1)I_n & 0 \\ \frac{\eta}{\tau}\kappa_q L_J & \frac{\eta}{\tau}\kappa_q L_J \frac{\eta}{(\lambda-1)}I_n + \frac{\eta}{\tau}\kappa_v L_J & \det_{33} \end{vmatrix}\\
&= |(\lambda-1)I_n(\lambda-1)I_n(\frac{\eta}{\tau}\kappa_q L_J(\frac{\eta^2(1+\lambda)}{2(1-\lambda)^2}I_n - \frac{\eta}{(\lambda-1)}H)\\
&\quad + \frac{\eta}{\tau}\kappa_v L_J\frac{\eta}{(\lambda-1)}I_n + (\lambda - 1 + \frac{\eta}{\tau})I_n + \frac{\eta}{\tau}\kappa_a L_J)|\\
&= |(\frac{\eta\kappa_q}{\tau}(\frac{\eta^2(1+\lambda)}{2}L_J - \eta(\lambda-1)HL_J) + \frac{\eta^2\kappa_v}{\tau}(\lambda-1)L_J\\
&\quad + ((\lambda-1)^3 + \frac{\eta}{\tau}(\lambda-1)^2)I_n + (\lambda-1)^2\frac{\eta\kappa_a}{\tau}L_J)|\\
&= \prod_{i=1}^{n}(\lambda^3 + (\frac{\eta(\kappa_a r_i + 1)}{\tau} - 3)\lambda^2\\
&\quad + (\frac{\eta^3\kappa_q r_i}{2\tau} - \frac{\eta^2\kappa_q h r_i}{\tau} + \frac{\eta^2\kappa_v r_i}{\tau} - \frac{2\eta\kappa_a r_i}{\tau} - \frac{2\eta}{\tau} + 3)\lambda\\
&\quad + (1 + \frac{\eta}{\tau} + \frac{\eta\kappa_a r_i}{\tau} - \frac{\eta^2\kappa_v r_i}{\tau} + \frac{\eta^3\kappa_q r_i}{2\tau} + \frac{\eta^2\kappa_q h r_i}{\tau})),
\end{aligned} \tag{38}$$

where $\det_{33} = \frac{\eta}{\tau}\kappa_q L_J(\frac{\mathrm{T}_s^2(1+\lambda)}{2(1-\lambda)^2}I_n - \frac{\eta}{(\lambda-1)}H) + \frac{\eta}{\tau}\kappa_v L_J\frac{\eta}{(\lambda-1)}I_n + (\lambda - 1 + \frac{\eta}{\tau})I_n + \frac{\eta}{\tau}\kappa_a L_J$. Let

$$\begin{aligned}
\iota_2 &= \frac{2\eta(\kappa_a r_i + 1) - 6\tau}{2\tau},\\
\iota_1 &= \frac{\mathrm{T}_s^3\kappa_q r_i - 2\mathrm{T}_s^2\kappa_q h r_i + 2\mathrm{T}_s^2\kappa_v r_i - 4\eta\kappa_a r_i - 4\eta + 6\tau}{2\tau},\\
\iota_0 &= \frac{2\tau + 2\eta + 2\eta\kappa_a r_i - 2\mathrm{T}_s^2\kappa_v r_i + \mathrm{T}_s^3\kappa_q r_i + 2\mathrm{T}_s^2\kappa_q h r_i}{2\tau}.
\end{aligned}$$

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Based on Lemma 4.2, we obtain (16) and thus complete the proof. ■

APPENDIX B
PROOF OF THEOREM 4.7

Proof: Let $y(k) = [\hat{q}(k); \hat{v}(k); \hat{a}(k)]$. Since the connectivity probability of all communication links is p, we have

$$\begin{aligned}\mathbf{E}(L_J) &= \mathbf{E}(J) + \mathbf{E}(D) - \mathbf{E}(A)\\ &= (1-p)L_J\\ &= L_J - pL_J\end{aligned} \tag{39}$$

Thus, from (17) and (39), it follows

$$\mathbf{E}(W) = \tilde{W} = W + p\Delta. \tag{40}$$

Then, we have

$$\mathbf{E}(y(k)) = \tilde{W}^k y(0). \tag{41}$$

Since(16) holds, one infers $\max(|\mathrm{eig}(W)|) < 1$, i.e., $|\lambda_i| < 1$ for all $1 \le i \le 3n$. Meanwhile, one has

$$\begin{aligned}\|W\|_\infty = \max\{&1 + \eta + \eta(n-1)h + |\eta^2 - \eta h|,\\ &2r\eta/\tau(\kappa_q + \kappa_v + \kappa_a/2)\\ &+ |1 - \eta\tau - \eta\kappa_a\{L_J|\}_{ii}/\tau\}\end{aligned}$$

and

$$\begin{aligned}\|\tilde{W}\|_\infty = \max\{&1 + \eta + \eta(n-1)h + |\eta^2 - \eta h|,\\ &2(1-p)r\eta/\tau(\kappa_q + \kappa_v + \kappa_a/2)\\ &+ |1 - \eta\tau - (1-p)\eta\kappa_a\{L_J|\}_{ii}/\tau\}.\end{aligned}$$

Combining with $1+\eta+\eta(n-1)h+|\eta^2/2-\eta h| \ge 2r\eta(\kappa_q + \kappa_v + \kappa_a)/\tau + |1 - \eta/\tau|$, we obtain

$$\|W\|_\infty = \|\tilde{W}\|_\infty = 1 + \eta + \eta(n-1)h + |\eta^2 - \eta h|.$$

From (18), it follows

$$4(\|W\|_\infty + \|\tilde{W}\|_\infty)^{1-1/3n} \|p\Delta\|_\infty^{1/3n} < 1 - |\lambda_{3n}|.$$

It means that the perturbation of $p\Delta$ can still guarantee that the maximum magnitude of all eigenvalues of $\tilde{W}$ is strictly less than 1, where the fact of Lemma 4.6 is used. As a result, we have $\lim_{k\to\infty} \tilde{W}^k y(0) = 0$, i.e., $\lim_{k\to\infty} \mathbf{E}(y(k)) = 0$. Hence, we have completed the proof. ■

APPENDIX C
PROOF OF THEOREM 4.9

Proof: Let $\tilde{W}(j)$ be the random matrix of the system matrix at iteration j produced form an independent and identical distribution. For any initial condition $y(0) \in \mathbb{R}^{3n}$, one has

$$\begin{aligned}\|y(k)\| &= \|\prod_{j=0}^{k-1} \tilde{W}(j)y(0)\|\\ &\le \rho(\tilde{W}(k-1))\|\prod_{j=0}^{k-2} \tilde{W}(j)y(0)\|\\ &\le \prod_{j=0}^{k-2} \rho(\tilde{W}(j))\|y(0)\|\end{aligned} \tag{42}$$

where lemma 7 in [48] is used. Under (42), by taking the expectation on both sides of (42), it follows

$$\begin{aligned}\mathbf{E}(\|y(k)\|) &\le \mathbf{E}(\prod_{j=0}^{k-2} \rho(\tilde{W}(j)))\|y(0)\|\\ &\le (\mathbf{E}(\rho(\tilde{W})))^k \|y(0)\|.\end{aligned} \tag{43}$$

Taking the limitation of k on both sides, we have completed the proof. ■

APPENDIX D
PROOF OF THEOREM 4.10

Proof: Due to the Markov's inequality, we have $\mathbf{Pr}\{\|y(K)\| \ge \epsilon\} \le \mathbf{E}(\|y(K)\|)/\epsilon$ given $\epsilon > 0$. Combining with Lemma 4.9, one can obtain

$$\begin{aligned}\mathbf{Pr}\{\|y(K)\| < \epsilon\} &= 1 - \mathbf{Pr}\{\|y(K)\| \ge \epsilon\}\\ &> 1 - \mathbf{E}(\|y(K)\|)/\epsilon\\ &> 1 - (\mathbf{E}(\rho(\tilde{W})))^K \|y(0)\|/\epsilon.\end{aligned} \tag{44}$$

As $\mathbf{Pr}\{\|y(K)\| < \epsilon\} = \delta$ and $\|y(0)\| \neq 0$, we have $K > \log(\frac{\epsilon(1-\delta)}{\|y(0)\|})/\log(\mathbf{E}(\rho(\tilde{W})))$ from (44). Thus, the proof is completed. ■

APPENDIX E
PROOF OF THEOREM 4.12

Proof: To characterize the stability of the random system, we use the expectation of the system state and the expectation of the covariance matrix, i.e., $\mathbf{E}(x(k))$ and $\mathbf{E}(x(k)x^\top(k))$. Meanwhile, it is defined,

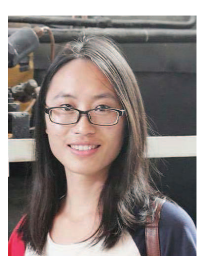
$$\begin{aligned}\psi_i(k) &= \mathbf{E}(x(k)_{\chi_{\{\theta(k)=S_i\}}}),\\ Q_i(k) &= \mathbf{E}(x(k)x^\top(k)_{\chi_{\{\theta(k)=S_i\}}}).\end{aligned}$$

As a result, we have

$$\nu_i(k) = \mathbf{E}(x(k)) = \sum_{i=1}^{n} \psi_i(k),$$

$$\Sigma_i(k) = \mathbf{E}(x(k)x^\top(k)) = \sum_{i=1}^{n} Q_i(k).$$

Then, by using the Markov decision process of $\{x(k), \theta(k)\}$, we can obtain the linear operators $\mathrm{B} : \psi(k) \to \psi(k+1)$ and $\mathcal{T} : Q(k) \to Q(k+1)$. Through transforming the linear operator as the matrix form, we can obtain the sufficient and necessary conditions to guarantee the mean square stability of the platoon system. ■



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