A Distributed Demand Response Control Strategy Using Lyapunov Optimization

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***Abstract*—Motivated by the potential ability of heating ventilation and air-conditioning (HVAC) systems in demand response (DR), we propose a distributed DR control strategy to dispatch the HVAC loads considering the current aggregated power supply (including the intermittent renewable power supply). The control objective is to reduce the variation of nonrenewable power demand without affecting the user-perceived quality of experience. To solve the problem, first, a queueing model is built for the thermal dynamics of the HVAC unit based on the equivalent thermal parameters (ETP) model. Second, optimization problems are formulated. Based on an extended Lyapunov optimization approach, a control algorithm is proposed to approximately solve the problems. Third, a DR control strategy with a low communication requirement is proposed to implement the control algorithm in a distributed way. Finally, practical data sets are used to evaluate and demonstrate the effectiveness and efficiency of the proposed control algorithm.**

***Index Terms*—Demand response (DR), heating ventilation and air-conditioning (HVAC), Lyapunov optimization, power variation, renewable power integration, smart grid, thermal dynamic queue.**

I. Introduction

DEMAND response (DR) is anticipated to be a critical application in smart grid. Aided by the advanced metering infrastructure (AMI), the power usage of different appliances in the customer premises can be adjusted either directly, such as operational parameters/states changing requested by grid operators; or indirectly, such as real-time pricing. By smoothing out the system power demand over time, DR is capable of providing peak shaving, load shifting and ancillary services to maintain the system reliability and stability.

On the power supply side, a growing number of renewable energy sources are introduced into the power grid. The renewable energy can reduce congestion in the grid and decrease the need for new generation or transmission capacity. However, the intermittent nature of renewable energy brings new challenges, which can be inimical to the power grid stability, and requires extra energy storage or local generation to balance the generated power with the demand. Thus, the potential positive environmental and economic benefits may be offset by these new problems and costs [1].

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On the customers side, the customer power demand can typically be divided into three categories, inelastic load and two types (*Type-I* and *Type-II*) of elastic load. The inelastic load must be satisfied immediately when needed, e.g., lighting. Hence, the inelastic load is not suitable for DR. The Type-I elastic load includes the power demand of the devices whose operation can be delayed but not interrupted, such as washers. For DR, this type of demand is mostly interested in providing peak shaving and load shifting services. The Type-II elastic load denotes the most flexible power demand, such as heating ventilation and air-conditioning (HVAC) systems. Considering the thermal capacity of the building, which introduces correlation of the temperature across time and is similar to a queueing system, the control of HVAC units can align well with the needs to smooth the energy demand variation in the time scale of minute-level. The potential of HVAC devices for load balancing/regulation service has been evaluated in [2].

In the literature, there have been many works on how to use DR to shave demand peaks or to shift the peak [3]–[8]. While both of the power peak and the power variation are important to the stability of the power systems, the later one fluctuates in a much smaller time scale (minute-level) with a relatively low amplitude comparing to the demand peak. In this paper, motivated by the HVAC units' potential in demand response service, our focus is to explore how to utilize in-house HVAC units to reduce the power demand variation, which has not attract enough attention previously. By smoothing the energy demanding in the minute-level, the total cost for the power generation can be reduced, as we can reduce the needs for online regulation services [9], [10].

The main contributions in this paper are fourfold. First, we build a queueing model for the thermal dynamics of HVAC units, a representative source of the Type-II elastic load. With such a queueing model, the controlled room temperature is similar to the power in a battery, which is increased (filled) when the HVAC unit is on (when the battery is charged) and is decreased (emptied) when the HVAC unit is off (when the battery is discharged). Second, optimization problems are formulated to minimize the average variation of the nonrenewable power demand by controlling the on/off states of HVAC units. By extending the Lyapunov optimization techniques in [11], we can jointly optimize the objective value and guarantee the room temperatures staying in customers' desired regions. Third, to further reduce the communication cost and complexity, we propose a suboptimal control algorithm and a strategy to implement the algorithm in a distributed way. One more merit of the control algorithm is that it can be tuned to effectively reduce the average variation of the nonrenewable power demand without significantly increasing the frequency of HVAC units' on/off switching. At last, using practical data sets, simulation results demonstrate that our control algorithm can effectively reduce the variation of nonrenewable power demand and guarantee the customers' comfortable experiences.

The rest of this paper is organized as follows. In Section II, we present a summary of related work on DR control. Section III describes our system model. In Section IV, we present our queueing model for the HVAC units' thermal dynamics. In Section V, optimization problems are formulated to minimize the average variation of nonrenewable power demand and a suboptimal control algorithm is proposed. In Section VI, a control strategy is proposed to implement our control algorithm in a distributed way. Section VII presents the numerical results followed by the concluding remarks and future research issues in Section VIII.

II. Related Work

With customers' participation, DR enables more options to balance the power supply and demand. To attract customers' participation, one important strategy is to shape the power demand through time-dependent pricing (TDP) [4]. By monitoring the electricity consumptions and providing customers the real-time price information through the smart grid infrastructure, the power operators can manipulate the electricity price. Thus, customers' electricity usage may be restrained at the high power price period and stimulated at the low power price period. Note that the controllable power demand can be any of the three types discussed above, when the electricity usages are controlled by customers according to the TDP. In [5], game theory has been applied to reduce the peak load of the grid considering the consumers' reactions to the electricity prices, the subsequent changes in the demand pattern of the target day, and the resulting effect on observed prices. The benefits of using TDP are illustrated in [12], which used real data to illustrate that shifting usage physically can reduce the risk of overloading. However, one limitation of the pricing based DR strategy is that it relies on the assumption of reasonable customers' reaction to the electricity price, which may not always be true in a real environment. Besides, it also depends on the price prediction to achieve such benefit, which is also a challenging issue.

Another type of DR is to control the demand-side load directly by utilities or system operators. Considering the customers' requirements on comfortable experiences, most researches focus on utilizing the Type-I elastic load. Dividing the loads into real-time loads (inelastic load) and controllable loads (Type-I elastic load), [6] proposed an approach that attempts to produce a uniform load demand over time by scheduling the power usage lower than a preset target. In [3], a stochastic model was developed and two online demand scheduling policies were introduced to minimize the long-term average power grid operational cost. In the first one, the controller serves a new demand request immediately or postpones it to the end of its deadline, depending on the current power consumption. In the second one, a new power demand is activated immediately if power consumption is lower than a threshold; otherwise it is queued. A queued demand is activated when its deadline expires or when consumption drops below the threshold.

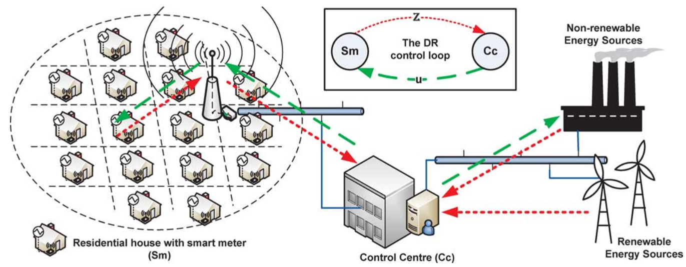

Fig. 1. Demand response in smart grid.

Comparing to the Type-I elastic load, Type-II elastic load can be more attractive for DR as the power can flow in two directions, not only help reduce the load demand in a period, but also compensate the power shortage by being "discharged" without requiring energy storage devices, i.e., functioning as battery. In the literature, there are also several strategies taking the advantage of the Type-II elastic load for the direct DR. [13] presented a strategy for that using water heaters as regulation resources. In [14], a direct control strategy was proposed to manage the large population of HVAC units using the system identification approach. A centralized optimal control algorithm with comfortable room temperature consideration was proposed in [15] by controlling the operational set-point of HVAC units. However, the control algorithm in [15] relies on the population information of the room temperature, which makes it vulnerable if the data packet is lost due to communication impairments [16]. Nevertheless, utilizing the power-storage feature of Type-II elastic load is a promising approach to provide ancillary service in smart grid, which motivates our work in this paper.

III. System Model

In this paper, we aim to reduce the variation of nonrenewable power demand, which is typically supplied by traditional power plants, by controlling the "ON/OFF" states of HVAC units.

Fig. 1 shows a typical demand response scenario in smart grid. In the service community, there are N distributed residential houses, assuming each of the house is equipped with an HVAC unit and a smart meter. A control center connects the customers to the renewable and nonrenewable power sources through communication networks, and directly controls customers' HVAC working states through communicating with the smart meters.

To satisfy customers' requirements on comfortable living environments, a comfortable temperature region is set for each residential house. Let T_i be the room temperature in the ith house ($i = 1, 2, \ldots, N$.) and $[T_{i,l},\ T_{i,h}]$ denote the comfortable temperature region, such that $T_i \in [T_{i,l},\ T_{i,h}]$. In each house, T_i is determined by the environment temperature (T_0), the previous room temperature and the working state u_i of the HVAC unit.

Let $u_i(t)$ be the state of an HVAC unit in slot t, $C_{i,h}$ be the equivalent heat capacity ($J/°$C), R_i be the equivalent thermal resistance ($°\mathrm{C}/W$) of the residential house, and $Q_{i,h}$ be the equivalent heat rate (W) of the HVAC unit. According to the ETP model in [2], the room temperature (°C) evolves as follows:

$$T_i(t+1) = \begin{cases} T_0(t) - \big[T_0(t) - T_i(t)\big]\eta, & \text{if } u_i(t) = 0 \\ \big(T_0(t) + Q_{i,h}R_i\big)(1-\eta) + T_i(t)\eta, & \text{o.w.,} \end{cases} \quad (1)$$

where $\eta = e^{-\Delta_t/R_i C_{i,h}}$. An HVAC unit only switches its working state when the room temperature reaches one of the region bounds, i.e., switching from "ON" ($u_i = 1$) to "OFF" ($u_i = 0$) if $T_i \geq T_{i,h}$, switching from "OFF" to "ON" if $T_i \leq T_{i,l}$, and keeping its working state when $T_{i,l} \leq T_i \leq T_{i,h}$. In this paper, it is assumed that the environment temperature T_0 is the same for the N residential houses but changes over time as a random variable and $T_0 \leq T_{0,\max}$. For the HVAC thermal dynamics related parameters, including $T_{i,l}$, $T_{i,h}$, $Q_{i,h}$, R_i, and $C_{i,h}$, they can be different for different customers and their houses.

To implement the DR control, time is divided into time slots with slot duration Δ_t. Instead of letting HVAC units work automatically, control decisions are made in each time slot to designate the working states of HVAC units.

On the customer side, at each time slot, we divide the load demand into two parts: HVAC load and non-HVAC load ($D_n(t)$) that includes the inelastic load and *Type-I* elastic load. On the supply side, the power is supplied by two kinds of sources, the traditional power grid and the renewable power sources, e.g., wind power, denoted as $P_r(t)$ and $P_c(t)$, respectively. We assume that there exists a peak power supply $P_{C,\max}$ and a peak load demand $D_{n,\max}$, so that $P_c(t) \leq P_{c,\max}$ and $D_n(t) \leq D_{n,\max}$.

In time slot t, the power supplies should be equal to the N customers' total loads, which requires that

$$P_c(t) + P_r(t) = D_n(t) + Q_p \sum_{i=1}^{N} u_i(t) \tag{2}$$

where Q_p is the power consumed by an HVAC unit if it is turned on.

However, as the renewable power ($P_r(t)$) is time-varying and noncontrollable, the power supplied by the traditional power grid ($P_c(t)$) has to vary timely, which causes great challenges to the power generation and the grid stability. To minimize the variation of the demand on the traditional power supply ($P_c(t)$) caused by these time-varying power supplies, we propose an approach to balance the load demand and power supply through tuning the load demand by directly controlling the HVAC units' "ON/OFF" states but not disturbing customers' comfortable experiences. We assume that customers have the incentive to participate in the direct DR control, as they will be compensated by the power company accordingly.

IV. Queueing Model of HVAC Thermal Dynamics

In this work, considering that the thermal capacity of the building introduces correlation of the temperature across time, which is similar to a queueing system, we remodel the HVAC thermal dynamics in (1) using a queueing model. Let $\Delta T_{i,f}(t)$ be the temperature loss in each time slot, and $\Delta T_{i,o}(t)$ be the room temperature increased by the HVAC unit if it is on. Given the environment temperature $T_0(t)$, when $u_i(t) = 0$, the room temperature will decrease from $T_{i,h}(t)$ to $T_{i,l}(t)$ in $K_{i,f}(t)$ slots. Suppose

$$T_{i,l}(t) = T_{i,h}(t) - K_{i,f}(t)\Delta T_{i,f}(t) \tag{3}$$

and according to (1), we have

$$T_{i,l}(t) = T_0(t)\left[1 - \eta^{K_{i,f}(t)}\right] + T_{i,h}(t)\eta^{K_{i,f}(t)}. \tag{4}$$

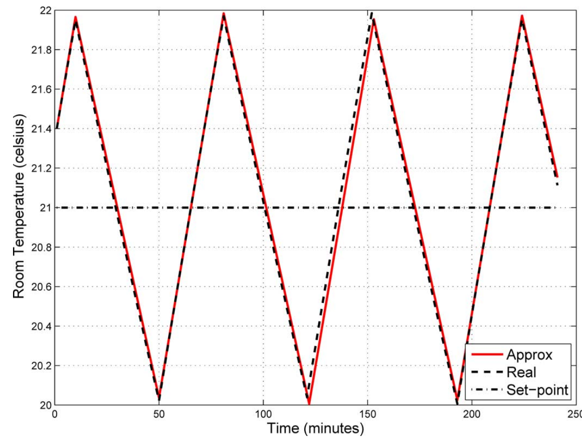

Fig. 2. Validation of the queueing model of HVAC thermal dynamics ($Q_{i,h} = 300$ W, $R_i = 0.1208°$ C/W, $C_{i,h} = 3599.3$ J/°C, $T_0 = 25°$ C).

Comparing (3) and (4), we can derive that

$$K_{i,f}(t) = \left\lceil \frac{R_i C_{i,h}}{\Delta_t} \log \frac{T_0(t) - T_{i,h}(t)}{T_0(t) - T_{i,l}(t)} \right\rceil \tag{5}$$

$$\Delta T_{i,f}(t) = \frac{[T_{i,h}(t) - T_0(t)]\left[1 - \eta^{K_{i,f}(t)}\right]}{K_{i,f}(t)}. \tag{6}$$

When $u_i(t) = 1$, the room temperature will increase from $T_{i,l}(t)$ to $T_{i,h}(t)$ in $K_{i,o}(t)$ slots. Suppose

$$T_{i,h}(t) = T_{i,l}(t) + K_{i,o}(t)[\Delta T_{i,o}(t) - \Delta T_{i,f}(t)]. \tag{7}$$

Similar to the case when $u_i(t) = 0$, we have

$$K_{i,o}(t) = \left\lfloor \frac{R_i C_{i,h}}{\Delta_t} \log \frac{T_0(t) + Q_{i,h}R_i - T_{i,l}(t)}{T_0(t) + Q_{i,h}R_i - T_{i,h}(t)} \right\rfloor \tag{8}$$

$$\Delta T_{i,o}(t) = \Delta T_{i,f}(t) + \frac{[T_0(t) + Q_{i,h}R_i - T_{i,l}(t)]\left[1 - \eta^{K_{i,o}(t)}\right]}{K_{i,o}(t)}. \tag{9}$$

By (1) and (3)–(9), we derive the queueing model of the HVAC thermal dynamics as follows:

$$T_i(t+1) = T_i(t) - \Delta T_{i,f}(t) + u_i(t)\Delta T_{i,o}(t). \tag{10}$$

To evaluate the accuracy of the queueing model, simulation has been run to compare the room temperature dynamics of an HVAC unit in 250 minutes based on the proposed model and the one in [2]. According to the results shown in Fig. 2, the proposed model matches the one in [2] quite well.

V. Optimal Demand Response Control

Based on the queueing model in Section IV, we study how to utilize HVAC units to reduce the average variation of nonrenewable power demand by using the mean square successive difference of $P_c(t)$ as our optimization objective.

A. Problem Formulation

It is assumed that the current $D_n(t)$ and $P_r(t)$ can be measured or estimated [17]. Let $\Delta_p(t) = D_n(t) - P_r(t) - P_c(t-1)$. $\Delta_p(t) + \sum_{i=1}^{N} u_i(t)Q_p$ represents the difference of the nonrenewable power demand in two successive time slot. Thus, our optimization problem is formulated as *P1*.

1) Problem I (P1):

$$\text{Min} \quad \lim_{t\to\infty} \frac{1}{t}\sum_{\tau=1}^{t}\left[\sum_{i=1}^{N} u_i(\tau)Q_p + \Delta_p(t)\right]^2 \tag{11}$$

$$\text{s.t.} \quad T_{i,l} \le T_i(t+1) - \Delta T_{i,f}(t) + u_i(t)\Delta T_{i,o}(t) \le T_{i,h} \tag{12}$$

$$u_i(t) \in \{0,\ 1\}, \quad \forall\, i = 1,2,\ldots,N \tag{13}$$

where (12) stands for each customer-desired room temperature requirement and (13) says that each HVAC unit can either be on or off.

The problem above is challenging mainly due to the time-coupling property brought by the first constraint. Previous methods handling similar problems are usually based on dynamic programming, requiring detailed knowledge of statistics of $P_r(t)$ and $P_c(t)$, and are vulnerable to the curse of dimensionality problem [18]. Moreover, these statistics may be unknown or difficult to obtain in practice.

Another way to solve *P1* is to study its relaxed form by using the bounded average room temperature instead of constraint (12); thus:

2) Problem II (P2, Rhe Relaxed P1):

$$\text{Min} \quad (\text{the same as } (11))$$

$$\text{s.t.} \quad T_{i,l} \le \overline{T_i(t)} \le T_{i,h} \tag{14}$$

$$u_i(t) \in \{0,\ 1\}, \quad \forall\, i = 1,2,\ldots,N \tag{15}$$

where $\overline{T_i(t)}$ is the average room temperature.

The solution to *P2* is easy to be characterized based on the framework of Lyapunov optimization [11]. However, the solution for the relaxed problem may not be feasible for the original problem.

In this work, we introduce auxiliary parameters C_i $(i = 1,2,\ldots,N)$ for each HVAC unit and virtual temperature queues, $X_i(t)$, as a shift of $T_i(t)$. We have

$$X_i(t) = T_i(t) - C_i, \quad \forall\, i = 1,2,\ldots,N \tag{16}$$

where

$$C_i^l \le C_i \le C_i^h$$

$$\begin{cases} C_i^h = \frac{[V_m Q_p^2 + 2V_m Q_p I_x]}{\max_t\{\Delta T_{i,o}(t)\}} \\ +T_{i,h} - \max_t\{\Delta T_{i,o}(t) - \Delta T_{i,f}(t)\} \\ C_i^l = \frac{[(2N-1)V_m Q_p^2 + 2V_m Q_p A_x]}{\min_t\{\Delta T_{i,o}(t)\}} + T_{i,l} + \max_t\{\Delta T_{i,f}(t)\} \\ V_m = \frac{\min_t\{\Delta T_{i,o}(t)[T_{i,h} - T_{i,l} - \Delta T_{i,o}(t)]\}}{Q_p^2[\delta_1(2N-1) - \delta_2] + 2Q_p(\delta_1 A_x - \delta_2 I_x)} \\ A_x = \max_t\{D_n(t) - P_r(t) - P_c(t-1)\} \\ I_x = \min_t\{D_n(t) - P_r(t) - P_c(t-1)\}. \end{cases} \tag{17}$$

With (10) and (16), we obtain N virtual queues as

$$X_i(t+1) = X_i(t) - \Delta T_{i,f}(t) + u_i(t)\Delta T_{i,o}(t) \tag{18}$$

and then we can reformulate *P1* as a quadratic optimization problem based on the Lynapnuv optimization [11]. After some manipulations (Please refer to the Appendix.), we obtain:

3) Problem III (P3):

$$\text{Min} \quad V_m\left[\sum_{i=1}^{N} u_i(t)Q_p\right]^2 \tag{19}$$

$$+\sum_{i=1}^{N}\left[X_i(t)\Delta T_{i,o}(t) + 2V_m Q_p \Delta_p(t)\right]u_i(t)$$

$$\text{s.t.} \quad u_i(t) \in \{0,\ 1\},\ \forall\, i = 1,2,\ldots,N. \tag{20}$$

In each time slot, $P_c(t)$, $T_i(t)$ are updated according to (2) and (10), respectively, and the optimal control decision of $u_i^*(t)$ can be found by solving *P3*.

Theorem 1: The solution to *P3* is always feasible to *P1*.

Before we prove *Theorem 1*, we first present the solution to *P3*. Let $Y_i(t) = X_i(t)\Delta T_{i,o}(t)$. We first sort the N HVAC units according to $Y_i(t)$ in an ascending order. Let k be the indicator of the order of ith HVAC unit and i_k denote the HVAC unit be the kth one in the sorted sequence. We obtain the optimal solution to *P3* by finding the k^*th HVAC unit satisfying (21) and (22) as follows:

$$I(k^*) = Y(k^*) + 2V_m Q_p \Delta_p(t) + V_m Q_p^2(2k^*-1) \le 0 \tag{21}$$

$$I(k^*+1) = Y(k^*+1) + 2V_m Q_p \Delta_p(t) + V_m Q_p^2(2k^*+1) \ge 0 \tag{22}$$

where $I(k)$ denotes the increment of (11) when the kth HVAC unit is turned on.

Thus, the control decisions for all HVAC units are

$$u_i(t) = \begin{cases} 1, & i = 1,2,\ldots,k^* \\ 0, & i = k^*+1,\ldots,N \end{cases} \tag{23}$$

with $Y_1(t) \le Y_2(t) \le \ldots \le Y_N(t)$. In the following, we prove *Theorem 1*.

Proof: Assuming that $1 \le k \le N$, with (17), we obtain (24) when $T_{i_k}(t) \le T_{i_k,l} + \Delta T_{i_k,f}(t)$, and (25) when $T_{i_k}(t) \ge T_{i_k,h} - \Delta T_{i_k,o}(t) + \Delta T_{i_k,f}(t)$, respectively:

$$\begin{aligned} I(k) &= Y(k) + 2V_m Q_p \Delta_p(t) + V_m Q_p^2(2k-1) \\ &\le [C_{i_k}^l - C_{i_k}]\Delta T_{i_k,o}(t) \le 0 \end{aligned} \tag{24}$$

$$\begin{aligned} I(k) &= Y(k) + 2V_m Q_p \Delta_p(t) + V_m Q_p^2(2k-1) \\ &\ge [C_{i_k}^h - C_{i_k}]\Delta T_{i_k,o}(t) \ge 0. \end{aligned} \tag{25}$$

As the function $Y(k) + 2V_m\Delta_p(t) + V_m Q_p^2(2k-1)$ is monotonically increasing, the control decision is $u_k(t) = 1$ when $T_{i_k}(t) \le T_{i_k,l} + \Delta T_{i_k,f}(t)$ and $u_k(t) = 0$ when $T_{i_k}(t) \ge T_{i_k,h} - \Delta T_{i_k,o}(t) + \Delta T_{i_k,f}(t)$, such that no room temperature will be out of the customer desired region, which satisfies the constraint in (12). Note that, for the HVAC units with room temperature between $T_{i,l} + \Delta T_{i,f}(t)$ and $T_{i,h} - \Delta T_{i,o}(t) + \Delta T_{i,f}(t)$, they can be either turned on or off. ■

Theorem 2: If $T_0(t)$ is i.i.d. over slots, then the expected $\overline{\Delta_c}$ under our algorithm is within a bound B/V_m of the optimal value $\overline{\Delta_c^*}$

$$\overline{\Delta_c} \le \overline{\Delta_c^*} + \frac{B}{V_m} \tag{26}$$

where

$$B = \frac{1}{2}\sum_{i=1}^{N} \max_t\{\Delta T_{i,o}^2(t) + \Delta T_{i,f}^2(t)\}. \quad (27)$$

Proof: First, *P1* is relaxed to be *P2* with the mean rate stable constraint only. Let $\overline{\Delta}_c^*$ be the optimal value of $c(t)$ of *P1* and $\overline{\Delta}_{c,rel}^*$ be the optimal value of $c(t)$ in *P2*. As all the solutions to *P1* will be feasible to the relaxed problem and there are looser constraints in *P2* than in *P1*, $\overline{\Delta}_{c,rel}^* \le \overline{\Delta}_c^*$.

Second, with (16), we can formulate a new optimization problem *P4* (as shown in the Appendix.) with the same objective as that in *P1*, but with constraints on mean rate stable of those virtual queues only. Let $\overline{\Delta}_{c,X}^*$ denote the optimal value of the objective in *P4*. To solve *P4*, we actually solve *P3*. It can be proved using Lyapunov theory [11] that there exist a constant B, $\overline{\Delta}_c \le \overline{\Delta}_{c,X}^* + B/V_m$.

Third, comparing *P2* and *P4*, they are of the same form except the different meaning of the queues. Thus, $\overline{\Delta}_{c,X}^* = \overline{\Delta}_{c,rel}^* < \overline{\Delta}_c^*$. We derive that the achievable objective is bounded by $\overline{\Delta}_c \le \overline{\Delta}_{c,X}^* + B/V_m \le \overline{\Delta}_c^* + B/V_m$. ■

B. DR Control Algorithm

Our control algorithm using the extended Lyapunov optimization above is summarized in *Algorithm 1*.

Algorithm 1 The DR control algorithm

1: Set C_i $(i = 1, 2, \ldots, N)$ satisfying **(17)**
2: In each time slot t
3: collect $T_i(t)$, $\Delta T_{i,o}(t)$, $\Delta T_{i,f}(t)$ $D_n(t)$ of each customer' house, $P_r(t)$ and $P_c(t-1)$ in the last slot
4: update the values of $X_k(t)$
5: sort the virtual queue according to $Y_i(t-1) = X_i(t)\Delta T_{i,o}(t)$
6: find k^* according to **(21)-(22)**
7: For each HVAC unit, $u_i(t) = 1$ if $i <= k^*$, otherwise, $u_i(t) = 0$

Note that, when different combinations of the auxiliary parameter C_i for the N queues are used, the control decisions made may be different, so as the room temperatures, which will be demonstrated in Section VII. However, when C_i satisfies (17), the result of the control objective will be almost the same.

VI. Distributed Demand Response Control Strategy

To implement *Algorithm 1*, one strategy is to use a centralized control by collecting all required information to the control center and then delivering the $u_i(t)$ to each customer every slot after decision-making. However, such a strategy is not efficient as these information (line 3 in *Algorithm 1*) has to be reported to the control center frequently, which introduces large communication cost and time delay for decision-making. In addition, such a strategy is vulnerable to security and privacy problems as both customers' private information and control decisions may be intercepted during the communication process.

Observing the solution to *P3* in (21)–(23), it is found that the HVAC unit's state in a time slot depends on the sorted $Y(k)$ and corresponding order k. While $X_i(t)$, $\Delta T_{i,o}(t)$, $\Delta T_{i,f}(t)$, and $V_m Q_p$ can be known by each customer, the only information required for customers to make their own decisions on the HVAC units' states is the value of $\Delta_p(t)$ and k. Thus, in this paper, we propose a distributed DR control strategy, as shown in *Strategy 1*.

In a control slot, the control center distributes a summary of power consumption $\Delta_p(t)$ and the virtual queue sequence (k) to all customers. Accordingly, each customer makes the decision independently by checking whether $I(k)$ is positive or not according to its own virtual queue sequence k. On the other hand, the control center can also predict customers' decisions by (21)–(23), the room temperatures by (10), and the virtual queue length by (16).

With such a distributed implementation, the benefits of such a strategy are three-folds. First, the communication cost can be reduced with fewer customer reports than that in centralized control; second, the control can be more reliable as there is no control-error due to the communication error in delivering the control decisions from the control center to customers; third, the system can be more secure with a lower frequency for customers to report their private information and no control decision is delivered over communication networks.

Strategy 1 The distributed DR control strategy

1: In each time slot t,
2: the control center calculates the summary of power consumption $(\Delta_p(t))$ and the virtual queue sequence (k)
3: the control center delivers $\Delta_p(t)$ and k to customers
4: each customer calculate its own $I(k)$
5: **if** $I(k) < 0$ **then**
6: the customer turns on its HVAC unit during the slot t
7: **else**
8: the customer turns off its HVAC unit during the slot t
9: **end if**

Note that it is possible to have an inaccurate prediction of the household's room temperature as we use a queueing model for approximation. However, the difference will be quite small within a slot when the control slot is short, e.g., one minute as shown in Fig. 2. To avoid the inaccurate prediction to accumulate to the point of causing a negative effect, one approach is to increase the frequency of customers reporting their room temperatures so that the control center can limit the error. How to quantify the inaccurate prediction and design an optimal reporting interval to balance the control benefit and communication cost [19] is left for future study.

VII. Performance Evaluation

A. Simulation Settings

We evaluate the proposed DR control algorithm in Section V in a community with 2000 residential houses and a 1-MW wind turbine providing renewable energy using practical data. For the power supply, the renewable power data are generated with a typical turbine power-curve using the wind speed data during Apr. 10–12, 2012 taken from Canada climate website, http://climate.weather.gc.ca. On the customer side, we used typical res-

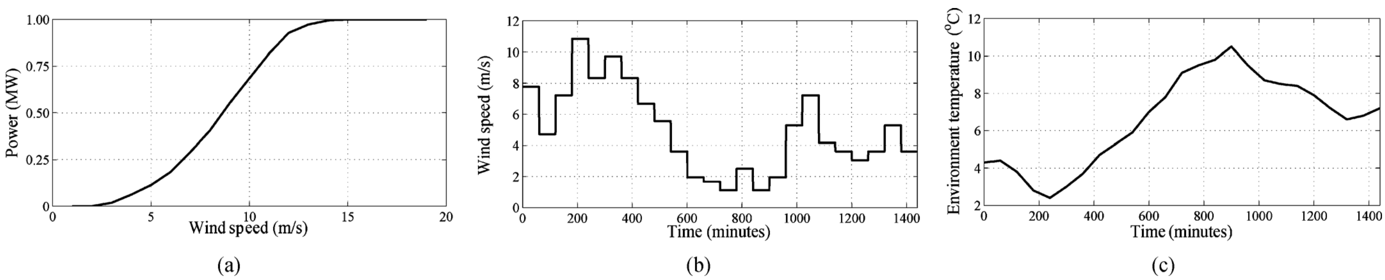

Fig. 3. Environment data. (a) Wind turbine power-curve. (b) 24-hour wind speed. (c) 24-hour environment temperature.

TABLE I
SIMULATION PARAMETERS

Variables	Values	Variables	Values
$Q_{i,h}$	(300, 800) W	Q_p	600 W
R_i	(0.1, 0.15) °C/W	$(T_{i,h}-T_{i,l})/2$	(18, 23) °C
$C_{i,h}$	(3,000, 4,000) J/°C	$T_{i,h}-T_{i,l}$	(1, 4) °C

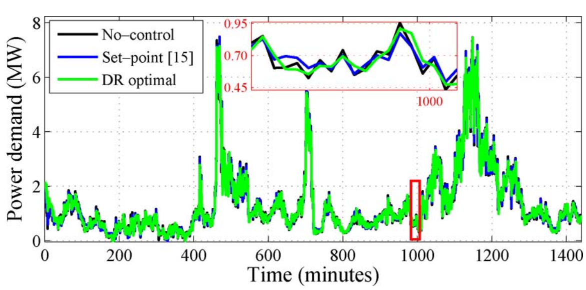

Fig. 4. Nonrenewable power demand.

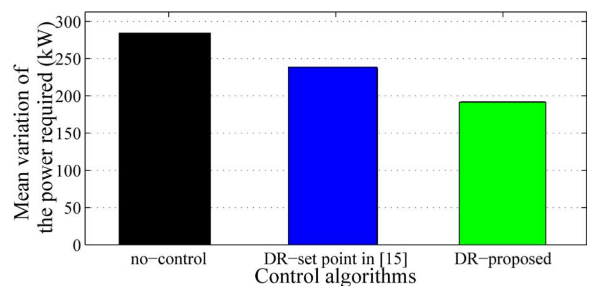

Fig. 5. Mean variation of the nonrenewable power demand ($C_i = C_i^h$).

idential house non-HVAC loads [20]. While the data from [20] are the discrete average load per 5 minutes, we interpolated it into per minute loads with Gaussian fluctuation, which is 10% of the load on average. The environment temperature data, which are required for the proposed control algorithm, are also from Canada climate website. Fig. 3 shows a) the wind turbine power-curve (cut-in speed: 3 m/s, cut-out speed: 20 m/s, rated power output: 1 MW), b) the 24-hour wind speed and c) the 24-hour environment temperatures used in the simulation.

For DR control, we set the control slot, Δ_t, as one minute, which is short enough that the customers' load demand and the renewable energy supply are assumed static. Table I presents the HVAC units related parameters. For the HVAC thermal dynamics related parameters, including $Q_{i,h}$, R_i, $C_{i,h}$, $T_{i,l}$, and $T_{i,h}$, we assume they are uniformly distributed in the range shown in Table I. For the HVAC unit power load, it is assumed to be 600 W.

B. Simulation Results

In this section, the proposed DR control algorithm is first compared with two other schemes: one without DR control and one using the algorithm in [15], in which the HVAC units are controlled by adjusting the customers' set-point of the room temperatures within $[(T_{i,h}+3T_{i,l})/4,\ (3T_{i,h}+T_{i,l})/4]$.

1) Control Effectiveness: Fig. 4[1] shows the power demand for the nonrenewable energy. It is observed that the power demand without DR control in Fig. 4 fluctuates seriously due to the load variation and the intermittent renewable power supply. By contrast, both the proposed algorithm and that in [15] effectively reduce the fluctuations, which brings down the risk of power outage and reduces the need for activating high cost supplementary power generation sources for load balancing/regulation.

Fig. 5 presents the average nonrenewable power demand differences in consecutive slots. Although the variation of the nonrenewable power supply seems larger using the proposed scheme than that in [15] occasionally in Fig. 4, the overall performance of the control algorithm outperforms that in [15] substantially with a 19% gain. When compared to that without DR control, a 32% gain is achieved by the proposed algorithm in the simulation. This is because the proposed scheme directly control the HVAC units' states instead of attempting to affect the states through any intermediate variable, i.e., the room temperature set-point, which enables a finer granularity to tune the loads.

2) Cost of the Control Algorithm: While the HVAC units are controlled to reduce the variability of power production, one potential cost is the increase of the frequency of the HVAC on/off switching. In Fig. 6, such impact is evaluated by the PMF of the number of HVAC on/off cycles per hour. It is found that the number of on/off cycles increased from about 0–3 cycles per hour without DR control to about 1–5 cycles per hour using the one in [15], and to about 5–20 cycles per hour using the proposed control algorithm.

For the control algorithm in [15], as the set-point is controlled, only the HVAC units, with which the room temperatures are close to T_i^l or T_i^h, will toggle their states and others will keep their states. Thus, its impact on the frequency of HVAC on/off

[1]In the figure, we enlarge the result between 964–1002 minutes. Please refer to [21] for the enlarged version of the figure.

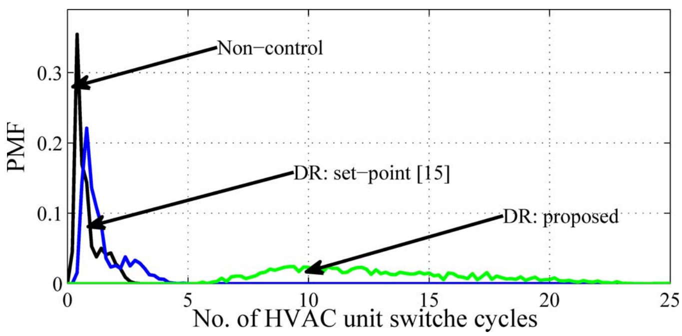

Fig. 6. PMF of the HVAC on/off cycles per hour ($C_i = C_i^h$).

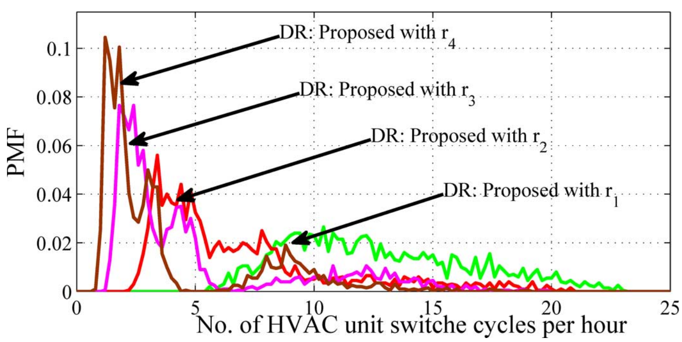

Fig. 7. PMF of the HVAC on/off cycles per hour with the adaptive $C_i(t)$ ($r_1 = 1, r_2 = 0.9, r_3 = 0.8, r_4 = 0.7$).

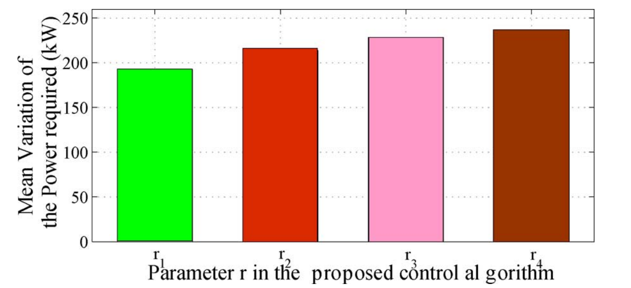

Fig. 8. Mean variation of the nonrenewable power demand with the adaptive $C_i(t)$ ($r_1 = 1, r_2 = 0.9, r_3 = 0.8, r_4 = 0.7$).

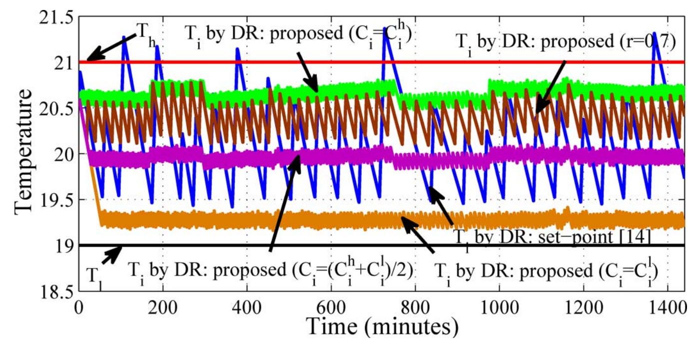

Fig. 9. Residential house room temperature sample ($T_{i,l} = 19^\circ$C, $T_{i,h} = 21^\circ$C).

switching is limited. Comparing to [15], the proposed algorithm directly controls the HVAC units' states. In every slot, both of the previous on and off HVAC units may change their states, which causes more frequent HVAC on/off switching.

To avoid overusing the HVAC units, we can try to keep the HVAC units' states as much as possible. To do so, instead of using the constant C_i in (16), we can use a time varying $C_i(t)$, which is related to the HVAC unit' previous state $u_i(t-1)$, and

$$\begin{cases} C_i(t) = u_i(t-1)C_i^h + [1 - u_i(t-1)]C_i^r \\ C_i^r = rC_i^h + (1-r)C_i^l, \ (0 \le r \le 1) \\ X_i'(t+1) = X_i'(t) + C_i(t) \\ \quad +C_i(t+1) - \Delta T_{i,f}(t) + u_i(t)\Delta T_{i,o}(t). \end{cases}$$

When $r = 1$, it is the same as using C_i. When $r < 1$, if an HVAC unit is previously turned on, it will get a large shift for its $T_i(t)$ to have a low order in S; otherwise, it is likely to gain a high order. In this way, HVAC units will be more likely to keep their states to avoid frequent on/off switching. As shown in Fig. 7, with a smaller r, there is high probability to have the HVAC on/off switch less than 5 cycles per hour.

On the other hand, with the adaptive $C_i(t)$, the control effectiveness may also be affected. Fig. 8 shows the power variation with different values of r. As it is shown, the mean variation of nonrenewable power demand increases as the value of r decreases. Thus, an adaptive $C_i(t)$ may help balance the cost and the effectiveness of the proposed control algorithm. Also, it is possible to devise an incentive mechanism to encourage users tolerating a larger value of r to provide more DR, which is left for future research.

Note that, with the adaptive $C_i(t)$, $T_i(t)$ may changes with a larger fluctuation but still bounded (shown later in Fig. 9). The proof for this can be found in [21].

3) Impact on Customers' Comfort Requirements: At last, we evaluate the impact of the DR control algorithm on the customer's comfort requirements by showing the room temperature of a sample residential house in Fig. 9. As it is shown, the proposed algorithm guarantees that the desired temperature requirements. However, the algorithm in [15] violates the desired room temperature setting sometimes. It is also found that the proposed scheme can provide more comfortable experience for a customer as the room temperatures is more stable with a smaller gap between the desired region. Meanwhile, when the adaptive $C_i(t)$ used, the room temperature controlled by the proposed DR control algorithm fluctuates in a larger range with a small r.

VIII. Conclusion and Future Work

In this paper, we have studied the DR control using HVAC units. A DR control algorithm based on the Lyapunov optimization has been proposed. Simulations with practical data sets have showed that the proposed control algorithm is effective in reducing the variation of the nonrenewable power demand and guaranteeing customers' comfortable experiences. Besides, a distributed strategy to implement the control algorithm has been proposed, which has fewer communication cost and more secure. Moreover, simulation results demonstrates that the proposed algorithm can be tuned to balance the control cost and its effectiveness.

Several research issues beckon for further investigation. First, the DR problem discussed may be extended to include the Type-I elastic load for peak shifting. As the Type-I elastic load is delay-tolerant, a maximum queueing delay constraint can be built in each residential house. Thus, the current system model and problem formulation in *P1* can be extended to include two types of queues, the thermal dynamics queues and the Type-I load queues, and the method developed in this paper may also be applicable. Such a method may also help further reduce the control cost in frequent HVAC on/off switches by providing more controllable demand. Second, the proposed DR control strategy assumes that the HVAC units are of the same power. This assumption is reasonable in some scenarios, such as in department buildings and university dormitories, where the administrators are likely to install similar HVAC units for each unit. In the scenario with HVAC units of heterogeneous power, our algorithm may still be applicable by grouping these HVAC units according to their power, assigning the renewable power demand to groups in proportion to the group sizes, and then applying the algorithm in each group separately. Third, the effectiveness of the proposed distributed DR strategy may be influenced by two factors, including the accuracy of the control center's prediction on the sequence of virtual temperature queues and the potential communication errors. To improve the robustness of the distributed DR control strategy, one approach is to increase the frequency of customers' room temperatures reports to mitigate the error. How to quantify the impact of inaccurate prediction and design an optimal reporting interval to balance the control benefit and communication cost is still an open topic.

APPENDIX
THE FORMULATION OF *P3*

With (16), we can formulate a new optimization problem:

Problem IV (P4):

$$\begin{aligned} \text{Min} \quad & (\text{the same as } (11)) \\ \text{s.t.} \quad & X_{i,l} \le \overline{X_i(t)} \le X_{i,h} \\ & u_i(t) \in \{0,\ 1\},\ \forall\, i = 1, 2, \ldots, N \end{aligned}$$

where $X_{i,l} = T_{i,l} - C_i$ and $X_{i,h} = T_{i,h} - C_i$.

Different from *P1*, in which each queue is bounded, the virtual queues are with mean rate stable constraints only. Based on this new optimization problem, we make use of the Lyapunov optimization techniques. Define the Lyapunov function as $L(\Theta(t)) = (1/2)\sum_{i=1}^{N} X_i(t)^2$ and the conditional 1-slot Lyapunov drift as $\Delta(\Theta(t)) \triangleq \mathbb{E}\{L(\Theta(t+1)) - L(\Theta(t))|\Theta(t)\}$.

Following the *MIN DRIFT-PLUS-PENALTY* algorithm in [11], to solve *P4* is to make decisions $u_i(t)$ in each slot t by minimizing $\Delta(\Theta(t)) + V_m \mathbb{E}\{Z(t)|\Theta(t)\}$, where $Z(t) = \left[\Delta_p(t) + \sum_{i=1}^{N} Q_p u_i(t)\right]^2$. Similar to that in [11], it is easy to prove that $\Delta(\Theta(t)) + V_m \mathbb{E}\{Z(t)|\Theta(t)\}$ is upper bounded by

$$B + V_m \mathbb{E}\{Z(t)|\Theta(t)\} + \sum_{i=1}^{N} X_i(t) \mathbb{E}[u_i(t)\Delta T_{i,o}(t) - \Delta T_{i,f}(t)|\Theta(t)].$$

Rather than directly minimizing the drift-plus-penalty every slot, one approach is to minimize its upper bound as follows:

Problem V (P5):

$$\begin{aligned} \text{Min} \quad & V_m Z(t) + \sum_{i=1}^{N} X_i(t)[u_i(t)\Delta T_{i,o}(t) - \Delta T_{i,f}(t)], \\ \text{s.t.} \quad & u_i(t) \in \{0,\ 1\}, \quad \forall\, i = 1, 2, \ldots, N. \end{aligned}$$

As $X_i(t)$, $\Delta T_{i,f}(t)$, and $\Delta_p(t)$ are given in every slot, *P5* equivalent to *P3*. Besides, let $\overline{\Delta}_{c,X}^{*}$ denote the optimal objective value of *P4* and $\overline{\Delta}_c$ be the achievable objective of *P3*, it can be proved that $\overline{\Delta}_c \le \overline{\Delta}_{c,X}^{*} + B/V_m$. The proof follows directly from the framework of Lyapunov optimization in [11], which is omitted here for brevity.

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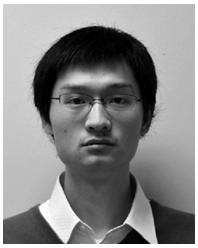

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