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Expanding EV Charging Networks Considering Transportation Pattern and Power Supply Limit

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***Abstract*—For EV rollout, a charging network is needed right off the road. The charging station planning and deployment problem should consider the increasing penetration ratio of EVs over a long period of time, and the highly dynamic and location-dependent demands and power grid constraints. This paper focuses on the dynamic charging network design, i.e., how to optimize the charging station locations and the number of chargers in each station at different time stages with an increasing EV penetration ratio. For each candidate location, we first model its coverage area to estimate the dynamic EV charging requirements. Then, we formulate the problem at each time stage as profit maximization, which is a mixed-integer optimization problem. To make it tractable, we investigate the profitability of candidate locations and derive their upper and lower bounds on the expected profit. Then we take two steps to transform and relax the problem to convex optimization. A fast-converging search algorithm, named RMCL-E, is proposed. Using real vehicle traces, simulation results show that the proposed algorithm can make a good trade-off between the service blocking probability and the construction cost to maximize the total profit, which is attractive for charging service providers.**

***Index Terms*—Electric Vehicles, Charging Services, Charging Stations, Power Grid.**

NOMENCLATURE

k k-th time stage.
t t-th time slot.
$p_{ii,t}$ The probability that EVs in Φ_i will be served in charging station i during time slot t.
$p_{ij,t}$ The probability that EVs in Φ_i will move to neighboring zone j for charging service during time slot t.
$p_{il,t}$ The probability that EVs in Φ_i leave the charging system directly (being blocked) during time slot t.
$\lambda_{i,t}^k$ The expected number of EVs requesting charging services in Φ_i during time slot t at time stage k.
$\alpha_{ij,t}^k$ The upper bound on transfer probability $p_{ij,t}$ at time stage k.
$\alpha_{il,t}^k$ The upper bound on blocking probability $p_{il,t}$ at time stage k.
Φ_i The zone i.
$C_{1,i}$ The average daily cost of one charging station at location i.
$C_{2,i}$ The average daily cost of each charger at location i.
C_3 The expected revenue for charging one EV in charging stations.
$\hat{C}_i^{O,k}$ The average daily cost of operating the charging station i at time stage k.
$\hat{C}_i^{I,k}$ The expected revenue of charging station i during one day at time stage k.
$\hat{C}^k$ The total expected profit of the charging network during one day at time stage k.
$\overline{C}_i^k$ The upper bound on expected daily profit $\hat{C}^k$ at time stage k.
$\underline{C}_i^k$ The lower bound on expected daily profit $\hat{C}^k$ at time stage k.
E The charging rate of one charger.
$L_{i,t}^k$ The total number of EVs in Φ_i that leave the charging system without being charged at time stage k.
$M_{i,t}^k$ The expected number of EVs will be charged at charging station i during time slot t at time stage k.
$\check{N}_i^k$ The number of existing chargers in location i at time stage k.
$\tilde{N}_i^k$ The number of chargers to be installed in location i.
$\tilde{N}_i^k$ The upper bound on N_i^k in location i at time stage k.
$\hat{N}_{i,t}^k$ The number of the available chargers in charging station i during time slot t at time stage k.
$\underline{N}_i^k$ The optimal number of chargers when $\hat{C}^k = \underline{C}_i^k$.
$\overline{N}_i^k$ The optimal number of chargers when $\hat{C}^k = \overline{C}_i^k$.
$O(i)$ The set of the zones that are zone i's neighbors.
$P_{i,t}^B$ The blocking probability of location i during time slot t.
$\underline{P}_{i,t}^B$ The lower bound on the blocking probability $P_{i,t}^B$.
$\overline{P}_{i,t}^B$ The upper bound on the blocking probability $P_{i,t}^B$.
$Q_{i,t}$ The queue length at charging station i during time slot t.
$\bar{Q}_{i,t}$ The corresponding upper bound on the queue length $Q_{i,t}$.
R_i The expected charging requirement of one EV in Φ_i.
S_i^k Whether or not to build a charging station at location i.

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I. INTRODUCTION

Electric Vehicles (EVs) have made rapid development recently thanks to their prominent advantages in reducing greenhouse gas emissions, and fuel consumption and maintenance costs. Currently, EV charging at limited public charging infrastructures may take hours to fully charge the batteries due to the low charging rate and sometimes long waiting time. As the EV penetration continues to grow, it is difficult for the public charging facilities to satisfy all the EV charging requirements, especially that of EVs on the road. Thus, commercial charging stations can be developed by charging service providers to offer convenient charging services.

EV charging infrastructure deployment in a city should consider several important factors: the candidate locations and their space limits, the transportation and the charging requirements of EVs, EV owners' behaviors, and the stability requirements and charging load limits in the power grid. There is a rich set of literature aiming at addressing the EV charging infrastructure deployment problem, and a lot of charging station placement problems have been formulated. Flow refueling location models have been proposed to satisfy the EV charging

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requirements as much as possible [2]–[4]. Several charging station deployment schemes under the limit of power grid have been proposed to minimize the total cost including power generation, power transmission loss and construction cost of charging stations [5]–[9], to maximize the social welfare under the constraints of transportation and power grid [10]–[14], or to minimize the charging station construction cost under the constraints of EV charging requirements and charging loads [15]–[17]. These works either aimed to satisfy all the EV charging requirements or to maximize the social welfare. However, these works did not consider the dynamic transfers of EV charging requirements among charging stations due to the transportation network, which may reduce the accuracy of EV charging requirement model and affect the design of EV charging networks. More importantly, these works formulated the deployment problem from the perspective of central urban planners rather than charging service providers who intend to maximize their profits by building or expanding their charging networks at the candidate locations in different time stages.

Comparing with all types of EV chargers, DC fast charger is suitable for charging EVs on the road in high-traffic commercial locations due to its high charging rate. However, DC fast chargers are more expensive than the other chargers for installation and operation. Consequently, how to determine the location of charging stations and the number of chargers in each charging station to maximize their profit is an important open issue for charging service providers. The fast charging stations deployment problem is somewhat similar to the gas station deployment for fueling vehicles on road, which has been widely studied [18]–[21]. However, these gas station deployment schemes cannot be directly applied to the charging station deployment problem since their features and constraints are different [1]. As charging service providers, they need to design EV charging networks considering the installation and operation cost, the dynamic EV charging requirements and the constraints of the power grid. Our previous work [1] addressed the one-time charging networks deployment problem which cannot deal with the expansion problem of charging networks over time given the growing EV penetration ratio.

The main objective of charging service providers is to maximize the total profit of the charging network at each time stage, considering the construction cost, the increasing EV charging requirements and the power constraint from the power grid. To address the issue, we first formulate a profit maximization problem at each time stage, which is a mixed-integer problem and difficult to solve directly. Thus, we investigate the profitability of these candidate locations to derive the upper and lower bounds on the number of chargers in each station at each time stage, and then obtain a necessary condition to build or expand charging stations. Given the profitability analysis, we transform and relax the problem to a convex optimization problem. Then, we propose a heuristic algorithm to build a new charging network or expand the existing charging network based on the profitability of each location. At last, the performance of the proposed scheme is investigated using simulations with real traffic data. The contributions of this paper can be summarized as follows:

- We formulate the charging network planning and deployment problem as profit maximization considering the long-term increasing and short-term highly dynamic EV charging requirement in each area, the construction cost and the limits of the space and charging load at each location.
- We analyze the profitability of all candidate locations at each time stage and classify them into three categories, such that one of the integer variables can be removed. Also, we prove that the transformed problem is a convex optimization problem.
- We propose a Removing and Merging Candidate Locations with Expansion (RMCL-E) algorithm to improve the total profit of the charging network by excluding or merging some unprofitable and less profitable locations.
- Simulations based on the real traffic data are conducted to demonstrate the efficiency of the proposed scheme.

The rest of the paper is organized as follows. Section II presents the system model and the problem formulation. A profit-based classification method has been proposed to classify the candidate locations and exclude the unprofitable locations, such that the primal problem can be simplified, and then an RMCL-E algorithm has been designed to increase the total profit of the charging network in Section III. Section IV demonstrates the operational performance analysis based on simulation results. Finally, Section V concludes the work.

II. System Model and Problem Formulation

Considering a charging service provider, who plans to build several charging stations or expand the existing charging network at some given candidate locations in a city. Given urban planning factors and space limits, there exist several restrictions for each location, such as the upper bound on the number of chargers and the charging load constraint from the power grid. How to find the potential locations and how to negotiate with the power grid on the upper bound on the charging load are out of the scope of this paper. The key problem in this paper is to identify, among the candidate locations, where to build the charging stations, and how many chargers should be installed in each charging station.

In the following, we first model the expected profit of each candidate location, and then introduce the EV charging requirement model, which depends on the location and the transportation factors in its coverage area. Next, we introduce the operation model for charging stations and relationships among them. Note that, the operational model is used to analyze the profitability of each candidate location rather than the design of charging scheduling for the charging network. Based on the system model, we formulate the design of the charging network as a profit maximization problem.

A. Profit Model for Candidate Locations

To figure out the dynamic of the charging network, we used k to denote the k-th time stage and $k = \{1, 2, \cdots\}^1$. Denote by I the total number of all the candidate locations, and denote

[1] One time stage is a relatively long time duration, e.g., $1 \sim 2$ years when the EV charging requirement goes up substantially, while one time slot is a short time duration, e.g., an hour, in this paper.

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by i the i-th location. Let (S_i^k, N_i^k) denote the decision of the charging service provider at time stage k, where $S_i^k \in \{0, 1\}$ represents whether or not to build a charging station at location i, and $N_i^k \in \{1, 2, \cdots, \tilde{N}_i^k\}$ represents the number of chargers to be installed at location i. Let $\check{N}_i^k$ denote the number of existing chargers in location i and $\tilde{N}_i^k$ denote the upper bound on N_i^k in location i due to the space limits at time stage k, respectively. Thus, we have

$$\check{N}_i^k \le N_i^k \le \tilde{N}_i^k. \tag{1}$$

Note that, $S_i^k = 1$ when $\tilde{N}_i^k \neq 0$. Without loss of generality, we have $\check{N}_i^1 = 0$, $\check{N}_i^{k+1} = N_i^k$, and $\tilde{N}_i^{k+1} \ge \tilde{N}_i^k$.

Once the charging station has been built, its profit depends on the construction cost, energy cost, and the revenue for providing charging services to EVs. Generally, the construction cost of one candidate location depends on the location and the number of chargers, while the revenue depends on the number of EVs that it can serve.

The cost of each candidate location depends on several factors, such as the average daily cost, including the property cost and fee, the amortized construction cost, other initial construction costs, and the installation fee of each charger at the candidate location. To simplify the analysis, we use the average daily cost rather than the total cost. Let $C_{1,i}$ denote the average daily cost of one charging station at candidate location i and $C_{2,i}$ denote the average daily building and maintenance cost of each charger at candidate location i, respectively. The total average daily cost of operating the charging station at candidate location i at time stage k, denoted by $\hat{C}_i^{O,k}$, is

$$\hat{C}_i^{O,k} = S_i^k (C_{1,i} + C_{2,i} N_i^k), \quad \forall i. \tag{2}$$

Note that the cost $\hat{C}_i^{O,k}$ depends on whether the charging station i will be installed or not, S_i^k, and the number of chargers, N_i^k.

The revenue of each candidate location depends on the total number of EVs being served in the charging station, EV charging requirements, and the electricity cost. To characterize the time-varying EV charging requirements, we divide a day into T time slots and t denotes the t-th time slot. Let C_3 denote the expected revenue for charging one EV in the charging station and $M_{i,t}^k$ denote the total expected number of EVs that are charged at charging station i during time slot t at time stage k, respectively. The total expected revenue of charging station i during the day at time stage k, denoted by $\hat{C}_i^{I,k}$, is

$$\hat{C}_i^{I,k} = \sum_t C_3 M_{i,t}^k. \tag{3}$$

Note that, the expected revenue of one charging station depends on the number of EVs that can be charged at the charging station successfully since $M_{i,t}^k$ not only depends on the EV charging requirements at time stage k, but also depends on the service capacity of the charging station under the constraints of the space and the power grid.

The total expected profit of the charging network during one day at time stage k, denoted by $\hat{C}^k$, can be given by

$$\hat{C}^k = \sum_i (\hat{C}_i^{I,k} - \hat{C}_i^{O,k}). \tag{4}$$

From the expression of $\hat{C}^k$, the total expected profit depends on the average daily cost and the revenue of each candidate location at time stage k. To maximize the total profit, the charging service provider could build the charging network based on the profit model of each candidate location.

B. EV Charging Requirement Model

Generally, the EV traffic is highly dynamic and the EV density changes with areas and time stages. In this part, we give the EV charging requirement model for each candidate location at each time stage.

As EVs prefer to be served in a nearby location, the whole service area (e.g., a city) is divided into several zones centered by the candidate locations as a Voronoi diagram [22]. Let Φ_i and $\lambda_{i,t}^k$ denote zone i and the expected number of EVs requesting charging services in Φ_i during time slot t at time stage k, respectively. According to the prediction of global EVs on International Energy Agency [23], the penetration of EVs will increase quickly over time. Thus, we assume that $\lambda_{i,t}^k > \lambda_{i,t}^{k-1}$ holds. Assuming that the arrival rate of EVs at candidate location i during time slot t follows a Poisson distribution with the average arrival rate of $\lambda_{i,t}^k$. At location i, the probability that n EVs arriving during time slot t at time stage k is given by

$$P\{n\} = \frac{e^{\lambda_{i,t}^k} (\lambda_{i,t}^k)^n}{n!}, \quad n = 0, 1, 2, \cdots.$$

Let R_i denote the expected charging requirement of one EV in Φ_i. The total amount of the expected charging requirements at location i during time slot t is $R_i \lambda_{i,t}^k$. Here, we assume that EV arrivals at different locations are independent. Generally, the EV charging requirements mainly depend on their traveling distances, which typically follows a log-normal distribution [24]. In different zones, EV arrivals are independent and their charging requirements may be different [5].

C. Charging System Model

In this section, we introduce the charging model of each charging station and define the transfer probabilities among the charging stations based on the queue length and the traffic among the candidate locations.

Due to the space limits of each candidate location and the utilization of the existing facility, the total number of chargers in candidate location i should satisfy (1). Furthermore, for the safety and stability consideration, the power grid can issue an upper bound on the charging load from one charging station based on its power supply capability. Let $\bar{P}_{i,t}^k$ denote the upper bound on the charging load from charging station i during time slot t at time stage k. Given this upper bound, the charging station may only allow a maximum number of the chargers to work during the time slot. The available chargers in charging station i during time slot t, denoted by $\hat{N}_{i,t}^k$, is

$$\hat{N}_{i,t}^k = \min\{N_i^k, \lfloor \frac{\bar{P}_{i,t}^k}{\alpha_{ji,t}} E \rfloor\}, \tag{5}$$

where E denotes the charging rate of one charger, depending on the type of the charger, and $\lfloor \cdot \rfloor$ denotes the floor function.

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In this paper, all the chargers are DC fast chargers with the similar charging rate. It can be found that the total number of available chargers in each charging station depends on not only the total number of chargers that are installed in the charging station, but also the upper bound on the charging load from the power grid during time slot t.

Given the charging rate of each charger, the expected charging time for each EV at charging station i is R_i/E, and the expected service rate of charging station i is $\hat{N}^k_{i,t}E/R_i$. Since $\hat{N}^k_{i,t}$ is limited, EVs need to wait in the queue when the queue is not full. Otherwise, they may either move to a neighboring zone, or leave the charging system directly without being served (being blocked). Let $Q_{i,t}$ denote the queue length at charging station i during time slot t and $\bar{Q}_{i,t}$ denote the corresponding upper bound on the queue length, respectively. An EV will leave zone i during time slot t when $Q_{i,t} > \bar{Q}_{i,t}$. In the following, we will derive the queue distribution in steady state.

Let $O(i)$ denote the set of the zones that are zone i's neighbors. Let $p_{ii,t}$ denote the probability that EVs in Φ_i will be served in charging station i during time slot t and $p_{ij,t}$ denote the probability that EVs in Φ_i will move to a candidate neighboring zone j, $j \in O(i)$, and enter the queue at charging station j during time slot t, respectively. Let $p_{il,t}$ denote the probability that EVs in Φ_i leave the charging system directly (being blocked) during time slot t. When the queue at charging station i is not full, i.e., $Q_{i,t} \le \bar{Q}_{i,t}$, all the EVs will be charged at charging station i and $p_{ii,t} = 1$ holds. Otherwise, part of EVs will leave charging station i without being charged and $0 \le p_{ii,t} < 1$ holds. Note that $p_{ii,t} = 0$ holds when $N^k_i = 0$, which means that charging station i has not been built. Thus, the values of $p_{ii,t}$, $p_{ij,t}$, and $p_{il,t}$ satisfy the following constraints:

$$\begin{cases} p_{ii,t} = 1, & \text{if } Q_{i,t} \le \bar{Q}_{i,t}; \\ 0 \le p_{ii,t} < 1, & \text{Otherwise,} \end{cases} \quad (6)$$

$$\begin{cases} p_{ij,t} = 0 \ \& \ p_{il,t} = 0, & \text{if } Q_{i,t} \le \bar{Q}_{i,t}; \\ 0 < p_{ij,t} \le \alpha^k_{ij,t} \ \& \ 0 < p_{il,t} \le \alpha^k_{il,t}, & \text{Otherwise,} \end{cases} \quad (7)$$

where $\alpha^k_{ij,t}$ and $\alpha^k_{il,t}$ denote the upper bounds on the transfer probabilities that an EV moves from charging station i to charging station j and that leaves the charging system directly at time stage k, respectively. The transfer probability $\alpha^k_{ij,t}$ satisfies the following properties:

- The value of $\alpha^k_{ij,t}$ decreases with the increase of travel distance d_{ij};
- The value of $\alpha^k_{ij,t}$ depends on the distribution of EVs' travel directions during time slot t at time stage k.

Furthermore, the transfer probabilities, $\{\alpha^k_{ij,t}, j \in O(i)\}$, of charging station i satisfy the following constraint:

$$\sum_{j\in O(i)} \alpha^k_{ij,t} + \alpha^k_{il,t} = 1. \quad (8)$$

In this paper, the value of $\alpha^k_{ij,t}$ is derived from the time-varying transportation data. Specially, based on the transportation data, at time slot t, the numbers of EVs in zone i and that moved from zone i to zone j can be counted. Then, the value of $\alpha^k_{ij,t}$ can be calculated according to their ratio. We define the probabilities $p_{ij,t}$ and $p_{il,t}$ as

$$p_{ij,t} = P^B_{i,t}\alpha^k_{ij,t} \text{ and } p_{il,t} = P^B_{i,t}\alpha^k_{il,t}, \quad (9)$$

where $P^B_{i,t}$ denotes the blocking probability $P\{Q_{i,t} \ge \bar{Q}_{i,t}\}$. Thus, probability $p_{ii,t}$ can be given by

$$p_{ii,t} = 1 - P^B_{i,t}. \quad (10)$$

For the relationships among $p_{ii,t}$, $p_{ij,t}$ and $p_{il,t}$, we have

$$p_{ii,t} + \sum_{j\in O(i)} p_{ij,t} + p_{il,t} = 1, \quad \forall i, t. \quad (11)$$

According to (9), the expected number of EVs in Φ_j moving to charging station i during time slot t is

$$\lambda^k_{j,t}p_{ji,t} = \lambda^k_{j,t}P^B_{j,t}\alpha^k_{ji,t}. \quad (12)$$

Thus, the total expected arrival rate at charging station i is $\lambda^k_{i,t} + \sum_{i\in O(j)} \lambda^k_{j,t}p_{ji,t}$. We assume that, if EVs in Φ_j are blocked at charging station i, they will leave the charging station system (being blocked) due to the long travel distance.

Generally, the blocking probability $P^B_{i,t}$ depends on the EV charging requirements and the service ability of each charging station. To obtain the blocking probability $P^B_{i,t}$, we formulate the charging processes of each charging station as a first-in-first-out M/M/c/N model in queuing theory[2]. The value of blocking probability $P^B_{i,t}$ can be found in Appendix A.

Since the total expected arrival rate at charging station i is $\lambda^k_{i,t} + \sum_{i\in O(j)} \lambda^k_{j,t}p_{ji,t}$, the blocking probabilities and arrival rate for two neighboring charging stations may affect each other's. It is very complicated to calculate the blocking probabilities of the charging stations considering the transfer probability since they are coupled. Let $\underline{P}^B_{i,t}$ and $\overline{P}^B_{i,t}$ denote the lower and upper bounds on the blocking probability $P^B_{i,t}$. $\underline{P}^B_{i,t}$ is the optimal blocking probability for charging station i when the arrival rate is $\lambda^k_{i,t}$, while $\overline{P}^B_{i,t}$ is the optimal blocking probabilities when the arrival rate is $\lambda^k_{i,t} + \sum_{j\in O(i)} \lambda^k_{j,t}\alpha^k_{ji,t}$. Since the blocking probability $P^B_{i,t}$ is an increasing and convex function of service rate $\frac{\lambda^k_{i,t}}{N^k_i E/R_i}$ [25], $P^B_{i,t}$ is bounded by $\underline{P}^B_{i,t}$ and $\overline{P}^B_{i,t}$. Thus, we estimate the blocking probability $P^B_{i,t}$ by

$$P^B_{i,t} = \underline{P}^B_{i,t} + a^k_{i,t}(\overline{P}^B_{i,t} - \underline{P}^B_{i,t}), \quad (13)$$

where $a^k_{i,t}$ is a weight depending on the congestion level of its neighboring locations at time stage k. Generally, $[\lambda^k_{j,t} - N^k_j E/R_i]^+$ is used to estimate the blocking probability of zone j and $a^k_{i,t} = \sum_{j\in O(i)} \alpha^k_{ji,t}[\lambda^k_{j,t} - N^k_j E/R_i]^+ / \sum_{j\in O(i)} \alpha^k_{ji,t}\lambda^k_{j,t}$ to estimate the average blocking probability of charging station i's neighboring zones.

Since the EVs from another charging station j, $j \in O(i)$, may be blocked by charging station i, only part of these

[2]As the queueing model is not the main focus of this work, we adopt the simple M/M/c/N model, where the service time of EV charging here is modeled as an exponential R.V. If the service time fits other R.V., we can extend the work by applying the M/G/c/N model to obtain the blocking probability. From simulation results, we note that using other service time distributions such as Gaussian distribution does not have an obvious impact on the charging station deployment.

transferring EVs will be charged at charging station i. The total number of EVs that can be charged at charging station i during time slot t is $(\lambda_{i,t}^k + \sum_{i \in O(j)} \lambda_{j,t}^k p_{ji,t}) P_{ii,t}$, where $P_{ii,t} = 1 - P_{i,t}^B$. Note that the EVs from the other charging stations will not affect λ_i^k and have the similar blocking probability as the EVs in Φ_i. Furthermore, the EVs from another neighboring zone j, $j \in O(i)$, and being blocked at charging station i will leave the charging system. Thus, the expected number of EVs will be charged at charging station i during time slot t at time stage k, $M_{i,t}^k$, can be given by

$$M_{i,t}^k = (\lambda_{i,t}^k + \sum_{i \in O(j)} \lambda_{j,t}^k p_{ji,t}) p_{ii,t}, \quad \forall i, t. \tag{14}$$

The total number of EVs in Φ_i that leave the charging system without being charged at time stage k, denoted by $L_{i,t}^k$, is given by

$$L_{i,t}^k = \lambda_{i,t}^k (p_{il,t} + \sum_{j \in O(i)} p_{ij,t} P_{j,t}^B). \tag{15}$$

Since the detour time of EVs is small comparing with each time slot, its impact on the charging system is omitted.

D. Problem Formulation

In this paper, we aim at maximizing the total profit of the charging service provider at time stage k by dynamically building or expanding the charging network based on the increasing EV charging requirements. Based on the above system model, we formulate the design of the charging network as the following profit maximization problem:

$$\mathbf{P0}: \quad \max_{S_i^k, N_i^k} \hat{C}^k \tag{16}$$

$$S.t.: \quad M_{i,t}^k = (\lambda_{i,t}^k + \sum_{i \in O(j)} \lambda_{j,t}^k p_{ji,t}) p_{ii,t}, \quad \forall i, t, \tag{17}$$

$$\hat{N}_{i,t}^k = \min\{N_i^k, \lfloor \frac{\bar{P}_{i,t}^k}{E} \rfloor\}, \quad \forall i, t, \tag{18}$$

$$\check{N}_i^k \le N_i^k \le \tilde{N}_i^k, \quad \forall i, \tag{19}$$

$$S_i^k = \{0, 1\} | \check{N}_i^k = 0, \quad \forall i. \tag{20}$$

The objective function is to maximize the total profit for the charging service provider at time stage k by building a new charging network or expanding the existing one. The first constraint shows the expected number of EVs that will be charged at each charging station. The second equation determines the number of available chargers in each charging station. The third and the fourth constraints define the limits of each charging station.

From the problem formulation, the decision variables, e.g., $\{S_i^k, N_i^k, i \in I\}$, are integers. Furthermore, the decision variables $\{N_i^k, i \in I\}$ are coupled by the objective function and the first constraint. Thus, the optimization problem is a mixed-integer programing problem, which is difficult to solve directly by the existing tools. To solve this problem, we analyze the profitability of each candidate location and exclude the unprofitable locations, such that the profit maximization problem can be simplified.

III. Problem Transformation and Solution

In this section, we first propose a profit-based classification method to classify the candidate locations into three categories according to their ranges of possible profits. Since the charging service provider only builds the charging network at the profitable locations, the problem can be transformed into another optimization problem. Then, we design a heuristic RMCL-E algorithm to increase the total profit by removing the unprofitable locations and merging the low profitable candidate locations.

A. Classification of Charging Stations

Considering the imbalanced and time-varying EV charging requirements in different areas, some candidate locations may not generate profit due to low EV charging requirements in their coverage area and/or high construction cost. Thus, we can analyze the profitability of each candidate location and identify the unprofitable locations.

Given a candidate location i, let $\overline{C}_i^k$ and $\underline{C}_i^k$ denote the upper bound and the lower bound on its expected daily profit at time stage k, respectively. To obtain the values of $\overline{C}_i^k$ and $\underline{C}_i^k$, we make the following two assumptions:

Assumption I: Assume that all the neighboring zones of i except the existing charging stations are closed (no more charging stations have been built). Thus, the EVs in $\Phi(j)$, $\{j, j \in O(i) | \check{N}_j^k = 0\}$, move to charging station i for charging service with a probability $p_{ji,t} = \alpha_{ji,t}^k$ while the EVs in $\Phi(j)$, $\{j, j \in O(i) | \check{N}_j^k \ne 0\}$, move to charging station i for charging service with a probability $p_{ji,t} = \underline{P}_{j,t}^B \alpha_{ji,t}^k$. Hence, the maximal number of EVs that could be charged at charging station i can be obtained. The upper bound on the expected profit of charging station i and the corresponding blocking probability $\overline{P}_{i,t}^B$ can be obtained by solving Problem **A1**:

$$\mathbf{A1}: \overline{C}_i^k = \max_{N_i^k} \hat{C}_i^k \tag{21}$$

$$M_{i,t}^k = (\lambda_{i,t}^k + \sum_{i \in O(j)} \lambda_{j,t}^k p_{ji,t})(1 - \overline{P}_{i,t}^B), \tag{22}$$

$$\hat{N}_{i,t}^k = \min\{N_i^k, \lfloor \frac{\bar{P}_{i,t}^k}{E} \rfloor\}, \tag{23}$$

$$\underline{N}_i^k \le N_i^k \le \tilde{N}_i^k. \tag{24}$$

Assumption II: Assume that all the neighboring zones of i, $\{j, j \in O(i)\}$, have been installed with sufficient chargers, and no EVs in the neighboring zones will move to charging station i for charging service. Hence, the minimal number of EVs that can be charged at charging station i can be obtained. The lower bound on the expected profit of charging station i and the corresponding blocking probability $\underline{P}_{i,t}^B$ can be obtained by solving Problem **A2**:

$$\mathbf{A2}: \underline{C}_i^k = \max_{N_i^k} \hat{C}_i^k \tag{25}$$

$$M_{i,t}^k = \lambda_{i,t}^k (1 - \underline{P}_{i,t}^B), \tag{26}$$

$$\hat{N}_{i,t}^k = \min\{N_i^k, \lfloor \frac{\bar{P}_{i,t}^k}{E} \rfloor\}, \tag{27}$$

$$\underline{N}_i^k \le N_i^k \le \tilde{N}_i^k. \tag{28}$$

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According to the results in existing works [25], [26], blocking probability $P^B_{i,t}$ is decreasing and convex with respect to the number of servers N^k_i when the other parameters are given. It is easy to prove that both Problems **A1** and **A2** are convex optimization problems. Thus, the optimal solutions to Problems **A1** and **A2** can be obtained in an efficient way [27]. Note that $\overline{C}^k_i \geq \underline{C}^k_i$ always holds for each candidate location. Based on the values of $\overline{C}^k_i$ and $\underline{C}^k_i$, we can classify all the candidate locations into three categories:

- **C1** Unprofitable location: $i \in$ **C1** if $\overline{C}^k_i < 0$;
- **C2** Possibly-profitable location: $i \in$ **C2** if $\overline{C}^k_i \geq 0$ and $\underline{C}^k_i \leq 0$;
- **C3** Profitable location: $i \in$ **C3** if $\underline{C}^k_i > 0$.

Note that, before the expansion, all the existing charging stations belong to category **C3**. Since only the candidate locations in categories **C2**&**C3** may generate profit for the charging service provider, we have the following theorem:

Theorem 1: To maximize the total profit, the charging service provider only builds new charging stations at the candidate locations in categories **C2**&**C3**.

Let $\overline{N}^k_i$ and $\underline{N}^k_i$ denote the optimal number of chargers in candidate location i by solving Problems **A1** and **A2**, respectively. We have the following theorem for the optimal number of chargers, denoted by N^{k*}_i, as:

Theorem 2: For the optimal Problem **P0**, the optimal number of chargers N^{k*}_i satisfies $\underline{N}^k_i \leq N^{k*}_i \leq \overline{N}^k_i$.

The proofs of Theorem 1 and 2 can be found in [1].

Since the charging service provider only builds charging stations at the candidate locations in categories **C2**&**C3**, for any given location in categories **C2**&**C3**, we can obtain the optimal deployment decision by solving Problem **P1** as follows:

$$\mathbf{P1}: \max_{N^k_i, i\in\{\mathbf{C2}\&\mathbf{C3}\}} \hat{C}^k \tag{29}$$

$$S.t.: M^k_{i,t} = (\lambda^k_{i,t} + \sum_{i\in O(j)} \lambda^k_{j,t} p_{ji,t}) p_{ii,t}, \quad \forall i,t, \tag{30}$$

$$\hat{N}^k_{i,t} = \min\{N^k_i, \lfloor \frac{\bar{P}^k_{i,t}}{E} \rfloor\}, \quad \forall i,t, \tag{31}$$

$$\underline{N}^k_i \leq N^k_i \leq \overline{N}^k_i, \quad \forall i. \tag{32}$$

Comparing to Problem **P0**, the variable $\{S^k_i\}$ has been removed and the range for N^k_i has been narrowed. However, since the variables are integers, the transformed problem also is an integer programming problem.

To solve this problem, we relax the number of chargers N^k_i in each charging station as a continuous variable, whose range is $[\underline{N}^k_i \ \ \overline{N}^k_i]$. Then, the following theorem can be obtained by analyzing the relationship among $\hat{C}^k_i$, N^k_i, and N^k_j:

Theorem 3: The total profit $\hat{C}^k_i$ is a concave function if $\{N^k_i, \forall i \in \mathbf{C2}\&\mathbf{C3}\}$ are continuous variables.

Proof: From the expression of $\hat{C}^k_i$, it includes the total daily cost C^O_i, which is a linear function of N^k_i, and the total revenue C^I_i, which is a linear function of $M^k_{i,t}$. All the constraints are linear, and thus the transformed problem is a convex optimization problem if $M^k_{i,t}$ is a concave function of the variables $\{N^k_i, N^k_j, j \in O(i)\}$. According to the existing works [26], the blocking probability $P^B_{i,t}$ is convex in the number of servers N^k_i when the other parameters are given. Hence, $p_{ii,t}$, which equals $1 - P^B_{i,t}$, is concave with respect to N^k_i, and $p_{ji,t}$, which equals $P^B_{i,t} p^k_{ji,t}$, is convex with respect to N^k_j. To prove one function is concave with respect to several variables, Hessian matrix of the second partial derivatives of the function can be employed since a negative Hessian matrix denotes a concave function in these variables [27]. The Hessian matrix for $M^k_{i,t}$ with respect to $\{N^k_i, N^k_j, j \in O(i)\}$ is

$$\begin{bmatrix} \frac{\partial(M^k_{i,t})^2}{\partial^2 N^k_i} & \frac{\partial(M^k_{i,t})^2}{\partial N^k_i \partial N^k_j} & \cdots & \frac{\partial(M^k_{i,t})^2}{\partial N^k_i \partial N^k_m} \\ \frac{\partial(M^k_{i,t})^2}{\partial N^k_j \partial N^k_i} & \frac{\partial(M^k_{i,t})^2}{\partial^2 N^k_j} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial(M^k_{i,t})^2}{\partial N^k_m \partial N^k_i} & 0 & \cdots & \frac{\partial(M^k_{i,t})^2}{\partial^2 N^k_m} \end{bmatrix}$$

Since $\frac{\partial(M^k_{i,t})^2}{\partial^2 N^k_j} > 0$, $j \in O(i)$ and all the other elements in the matrix are smaller than 0, the Hessian matrix H is negative. Thus, the total profit $\hat{C}^k_i$ for charging station i is a concave function of the number of chargers $\{N^k_i, N^k_j, j \in O(i)\}$. ■

Thanks to its convexity, there exists only one optimal solution [27]. However, it is difficult for the existing tools to deal with the blocking probabilities due to the factorial function and the exponentiation function of variables. Furthermore, the solution is obtained when the candidate locations in categories **C2**&**C3** are given. According to the definition of the candidate locations in category **C2** , they only can generate profit when their neighboring locations have no charging stations or have but with high blocking probabilities. Actually, some of the locations in category **C2** may not generate any profit or only generate a small profit. Even for the candidate locations in category **C3**, due to the high construction cost, some of them may generate a low profit. Thus, merging some candidate locations may increase the total profit for the charging service provider by reducing the high construction cost. Generally speaking, there may exist some margins to improve the total profit for the charging service provider.

Thus, we propose a heuristic algorithm, named Removing and Merging Candidate Locations with Expansion (RMCL-E) based on RMPL in our previous work [1], to expand the existing charging network, which can be summarized as Algorithm 1. In the RMCL-E algorithm, we exclude the unprofitable locations in categories **C1**&**C2** and merge the candidate locations in categories **C2**&**C3** if beneficial. Generally, the algorithm can be divided into three steps: **I)** Removing Candidate Locations in **C1**, **II)** Removing Candidate Locations in **C2**, and **III)** Merging Candidate Locations in **C2**&**C3**. Taking the profit of each candidate location as the reference, some unprofitable candidate locations are removed in steps **I)** and **II)** since they cannot generate any profit to the charging network. Then, we attempt to remove each candidate location to reduce the high construction cost, which can increase the total profit of the charging network until removing any candidate location will decrease the total profit. Such that, the total profit of the charging network can be maximized.

Note that, by updating $\lambda^{k'}_{i,t}$ and $\lambda^{k''}_{i,t}$ and the corresponding $\underline{P}^B_{i,t}$ in each step of Algorithm 1 respectively, the lower bound

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of blocking probability $\underline{P}_{i,t}^B$ reflects the change of the blocking probability $P_{i,t}^B$ with different charging station deployment strategies. When the deployment of charging stations is determined, the final lower bound of blocking probability $\underline{P}_{i,t}^B$ in the proposed RMCL-E gives the blocking probability $P_{i,t}^B$.

Algorithm 1 RMCL-E

Initialization: Input $\lambda_{i,t}^k$, $\alpha_{ij,t}^k$, $\bar{P}_{i,t}^k$, E, $\bar{Q}_{i,t}$, $C_{1,i}$, $C_{2,i}$, C_3, $\check{N}_i^k$, $\underline{N}_i^k$, $\overline{N}_i^k$, and $\tilde{N}_i^k$, for each location i at time stage k;
I. Removing Candidate Locations in C1
  Set $\underline{N}_i^k = 0$ and $\underline{P}_{i,t}^B = 1$ if $i \in$ **C1**;
  For each candidate location i, $i \in$ **C2**&**C3**
    1) Set $\lambda_{i,t}^{k'} = \lambda_{i,t}^k + \sum_{j\in \mathbf{C1}} \lambda_{j,t}^k \alpha_{ji,t}^k$;
    2) Update $\underline{N}_i^k$, $\underline{P}_{i,t}^B$ and $\underline{C}_i^k$ by solving Problem **A2**;
  End for and go to **II**.
II. Removing Candidate Locations in C2
  1) Set $\lambda_{i,t}^{k''} = \lambda_{i,t}^{k'} + \sum_{j\in \mathbf{C2\&C3}} \lambda_{j,t}^k \underline{P}_{j,t}^B \alpha_{ji,t}^k$;
  2) Update $\underline{N}_i^k$, $\underline{P}_{i,t}^B$, and $\underline{C}_i^k$ by solving Problem **A2**;
  3) Sort $\{\underline{C}_i^k, \forall i \in \mathbf{C2\&C3}\}$ from the smallest to largest;
  4) If $\underline{C}_i^k < 0 \& \check{N}_i^k = 0$, remove candidate location i from **C2** and set $\overline{N}_i^k = 0$ and $\underline{P}_{i,t}^B = 1$; Otherwise, **end** and go to **III**.
III. Merging Candidate Locations in C2&C3
  For each candidate location i, $\check{N}_i^k = 0$ and $i \in$ **C2**&**C3**
    1) Remove location i from **C2**&**C3** and set $\underline{P}_{i,t}^B = 1$;
    2) Update $\{\underline{C}_j^k, \forall j \in \mathbf{C2\&C3}\}$ according to step **II** 1)–2);
    3) If the total profit increases, remove candidate location i and set $\underline{N}_i^k = 0$ and $\underline{P}_{i,t}^B = 1$; Otherwise, return i to **C2**&**C3** and recover $\underline{N}_i^k$ and $\underline{P}_{i,t}^B$;
  End for.
Return $\{i, N_i^k = \underline{N}_i^k, i \in \mathbf{C2\&C3}\}$.

IV. Case Study and Numerical Simulations

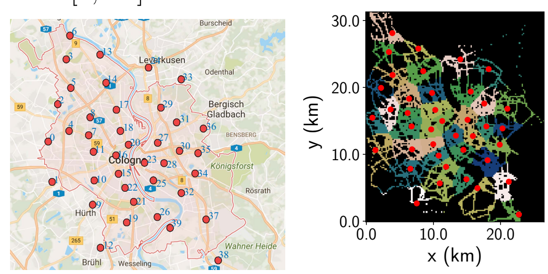
A. Case Study

In this paper, we use the traffic data in Cologne in Germany as the case study [28]. The trace of the car traffic covers a region of 400 square kilometers for a period of 24 hours in a typical work day, and comprises more than 700,000 individual car trips. From the data, we can obtain not only the real-time traffic density information, but also the statistics of the travel direction, the traffic volume in a certain area, etc. The traffic densities are time-varying and the numbers of vehicles at different locations and different time stages are different. We set the EV penetration ratios for different time stages change from 10% to 25% by 1%. We assume that the EV charging requirements during different time slots are different and the percentages of EVs requiring charging services are 5% during [6am, 10am), 8% during [10am, 2pm), and 10% during [2pm, 9pm], respectively. The charging requirement of one EV follows a Gaussian distribution (R, δ^2), where $R = 40kWh$ and $\delta = 4kWh$. Given the EV penetration ratio and their charging requirements ratios, the total EV charging requirement changes from 1x to 2.5x of the EV charging requirements at time stage 1 by 10%. The charging rate for each charger is $E = 120kW$ and the revenue for charging one kWh energy to EVs is $C_3 = \$0.125/kWh$.

The candidate locations and their service areas are shown in Fig. 1. Here, the service areas are calculated using the Voronoi diagram. Based on the construction cost estimation in [29], we assume that the range of the average daily cost for building one charging station is [\$150–\$300] depending on the locations and that for building and maintaining one charger at the candidate locations is [\$30–\$35]. Different locations may have different space limits and charging load limits from the power grid. The costs and the limits can be found in Table I. Considering the EV owners' behaviors, we model the transfer probabilities based on the statistics of the traffic between two candidate locations. The time-varying transfer probabilities among two neighboring charging stations at 7am and 5pm are shown in Fig. 2. The maximal queue length at each charging station is $\bar{Q}_{i,t} = 10$ and the probability that one EV leaving the charging system directly $\alpha_{il,t}^k$ is randomly selected in $[0, 0.3]$.

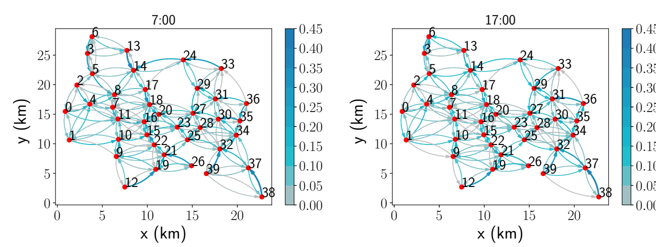
TABLE I
The Construction Cost and Limits of Candidate Locations

i	0–14, 33–39	15–18	19, 26–28	20–25	29–32
$C_{1,i}$	\$150	\$260	\$220	\$300	\$180
$C_{2,i}$	\$35/pile	\$30/pile	\$30/pile	\$40/pile	\$30/pile
$\tilde{N}_i$	50	30	30	30	50
$\bar{P}_{i,t}^k$	5000kW	4000kW	3500kW	3000kW	5000kW



(a) Candidate locations (b) Service areas

Fig. 1. The candidate locations and their service areas in the city.



(a) Transfer at 7am (b) Transfer at 5pm

Fig. 2. The transfer probabilities at 7am and 5pm during one day.

The decision for each step in the RMPL-E algorithm at the first time stage with 1x EV charging requirement can be found in Figs. 3(a)–3(c). After step I shown in Fig. 3(a), the unprofitable locations in category **C1** have been removed, e.g., locations No. 6 and 38. It can be found that these locations are located in the outlying areas with low EV charging requirements in their service areas and few EV transfers. After step **II** shown in Fig. 3(b), the unprofitable locations in category **C2**, e.g., locations No. 13 and 26, have been removed due to low EV charging requirements in their service areas and low blocking probabilities of their neighboring locations. After step **III** shown in Fig. 3(c), the candidate locations in categories **C2**&**C3**, e.g., locations No. 10, 20, 21, and 25, have been removed. This is because removing these locations can reduce the high construction cost and more profit can be generated

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TABLE II
THE FINAL DECISION OF THE CHARGING SERVICE PROVIDER

Location No.	**4**	**5**	**6**	**7**	**8**	**10**	**11**	**14**	**15**
N_i with 1x	9	0	0	0	11	0	15	11	26
N_i with 1.5x	13	0	0	0	16	18	21	17	30
N_i with 2x	18	0	0	0	22	26	30	23	30
N_i with 2.5x	20	7	4	12	27	35	36	28	30

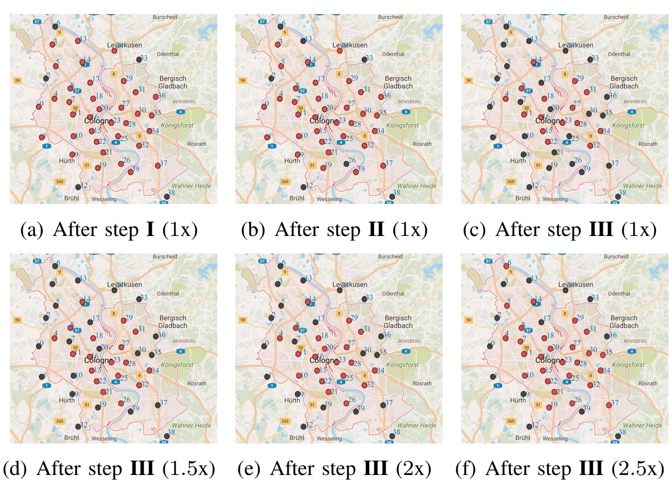
Location No.	**16**	**18**	**19**	**20**	**21**	**22**	**23**	**25**	**26**
N_i with 1x	30	24	0	0	0	11	23	0	0
N_i with 1.5x	30	30	0	0	17	15	25	16	5
N_i with 2x	30	30	0	25	23	19	25	20	5
$N_i$with 2.5x	30	30	9	25	25	25	25	25	6

Location No.	**27**	**28**	**29**	**30**	**31**	**32**	**34**	**35**	**37**
N_i with 1x	25	9	8	0	10	19	9	0	0
N_i with 1.5x	29	12	12	0	15	26	13	0	0
N_i with 2x	29	16	16	0	21	33	17	0	0
N_i with 2.5x	29	20	21	21	25	34	17	6	19

Note that $N_i \neq 0$ implies $S_i = 1$ and $N_i = 0$ implies $S_i = 0$.

by their neighboring charging stations. It can be found that all the charging stations are installed at the candidate locations with high EV charging requirements.

With the increase of EV charging requirements, the dynamic deployment results of the charging network are shown in Fig. 3 and Table II. It can be found that both the number of charging stations and that of chargers increase. When EV charging requirements increase from 1x to 2.5x, most of the existing charging stations have been expanded, and some of them reach to the upper bounds on available chargers, i.e., $\tilde{N}_i$ or $\lfloor \frac{\bar{P}^k_{i,t}}{E} \rfloor$. Also, the minimal number of chargers in each location has a lower bound. When the profit loss for one location is larger than the construction cost, one new charging station will be built. From Fig. 3, it can be found that candidate locations, which are close to the downtown, have been installed with more chargers. When the EV charging requirements are doubled, all the candidate locations in the downtown area have been installed with chargers. This is because EV charging requirements in downtown areas are much higher than other areas and can generate profit much easier.



(a) After step **I** (1x) (b) After step **II** (1x) (c) After step **III** (1x)

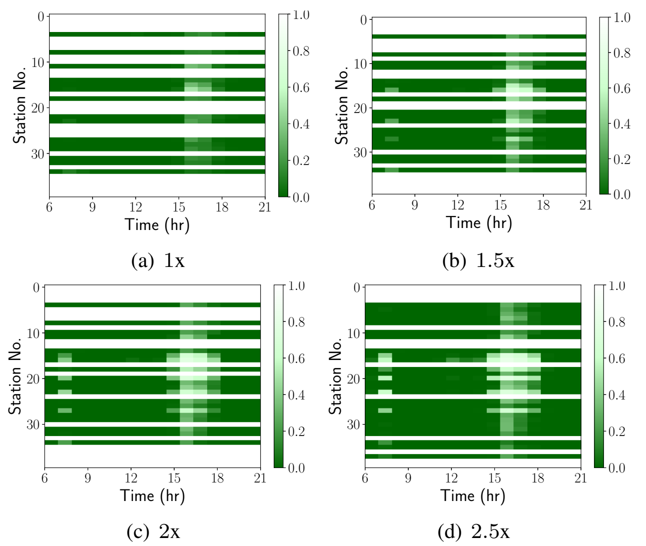
(d) After step **III** (1.5x) (e) After step **III** (2x) (f) After step **III** (2.5x)

Fig. 3. The dynamic decision of the RMCL-E algorithm (black dots denote the unselected location and red ones denote the selected location).

The expected blocking probabilities with different EV charging requirements under the corresponding charging network can be found in Fig. 4. Comparing the blocking probabilities with different EV charging requirements, the trend of blocking probabilities is the same, i.e., higher blocking probabilities during the rush hours and lower blocking probabilities during the non-rush hours. Also, with higher EV charging requirements, the charging network has more congested hours (high blocking probabilities). It can be found that an appropriate congestion level can improve the utilization of chargers and the return on investment, and increase the total profit.

TABLE III
DYNAMIC DEPLOYMENT OF THE CHARGING NETWORK WITH DIFFERENT EV CHARGING REQUIREMENTS

Increment	**Stations**	**Chargers**	**Increment**	**Stations**	**Chargers**
1x	15	240	1.1x	15	255
1.2x	15	274	1.3x	17	305
1.4x	17	318	1.5x	19	360
1.6x	19	370	1.7x	20	408
1.8x	20	426	1.9x	20	442
2x	20	458	2.1x	21	489
2.2x	24	519	2.3x	24	534
2.4x	26	573	2.5x	27	591



(a) 1x (b) 1.5x

(c) 2x (d) 2.5x

Fig. 4. The expected blocking probabilities with different EV charging requirements under the corresponding charging networks (the blocking probabilities at unselected locations are 100%).

In order to explore the variation of charging networks, we increase the EV charging requirements by 10% from 1x to 2.5x and the deployment results are shown in Table II. The number of charging stations increases slowly while the number of chargers in the charging network increases in proportion to the growth of EV charging requirements. This is because the charging capacity of the charging network mainly depends on the number of chargers. Furthermore, the high construction cost of charging stations makes the new charging station difficult to generate profit. Thus, the charging service provider can make a trade-off between the profit loss and the construction cost and only build one new charging station when the profit loss is larger than the construction cost. That also is the fundamental basis for us to remove some candidate locations in categories **C2** and **C3** in **Algorithm 1-III**.

The number of served and blocked EVs can be found in Fig. 5(a) while the corresponding total profit and construction cost can be found in Fig. 5(b), respectively. It can be seen that, with the increase of EV charging requirements, the number of served EVs increases quickly while that of blocked EVs keeps

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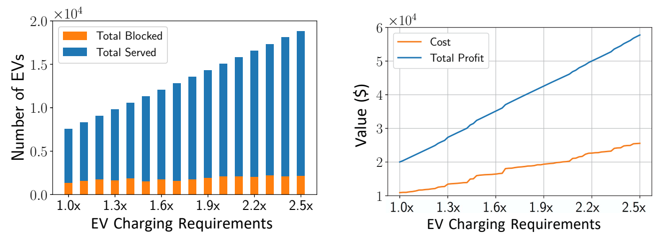
at a low level. This is because the proposed RMCL-E always makes a good trade-off between blocked EVs (profit loss) and the construction cost. When the number of blocked EVs reaches a certain level at some given locations, new charging stations will be built. From Fig. 5(b), it can be found that both the total profit and the total construction cost increase near linearly with the increase of EV charging requirements. The increasing rate of the total profit is much higher than that of the total construction cost, which means that the investment yields a higher profit. More simulation results, including the effects of the construction cost and real-time EV transfer probabilities, can be found in [1].



(a) Served and blocked EVs (b) Total profit and construction cost

Fig. 5. The served and blocked EVs & total profit and cost.

V. Conclusion

In this paper, we addressed the dynamic charging station deployment problem for charging service providers, who intend to build or expand their charging networks with the increase of EV charging requirements. Taking the limits of candidate locations and the power grid into consideration, as well as the increasing EV charging requirements, we formulated a profit maximization problem at each time stage to maximize the total profit of charging service providers by building or expanding their charging networks. However, the problem is a mixed-integer problem and difficult to solve directly. By analyzing the profitability of each candidate location at each time stage, all the candidate locations can be classified into three categories, and part of them can be excluded to simplify the primal optimization problem since they cannot generate any profit. Also, we have proved that the simplified problem is a convex optimization problem given any candidate locations. Then, we designed a Removing and Merging Candidate Locations with Expansion (RMCL-E) algorithm to remove the unprofitable candidate locations and merge some low profitable candidate locations to reduce the total construction cost, such that the total profit for the charging service providers can be increased. A case study using the real vehicle traces with different EV charging requirements has been conducted to verify the performance of RMCL-E. The simulation results showed that RMCL-E can build or expand the charging network in an efficient way to increase the total profit. In our future work, we intend to design an algorithm to determine the optimal upper bound on the charging load for each candidate location, such that both the charging network and the power grid can be benefited.

Appendix A

Let μ denote the mean service rate of one charger, c denote the total number of the available chargers in charging station i during time slot t, λ denote the arrival rate of EVs that need charging services at charging station i, and ρ denote the utilization factor of charging station i during time slot t, respectively. Thus, we have $\mu = E/R_i$, $c = \hat{N}_{i,t}^k$, $\lambda = \lambda_{i,t}^k + \sum_{j\in O(i)} \lambda_{j,t}^k p_{ji,t}$, $\rho = \lambda/(c\mu)$, and $N = \hat{Q}_{i,t}$ for charging station i. According to the M/M/c/N model, the steady state distribution of this queue is given by

$$P_n = \begin{cases} \frac{1}{n!}(\frac{\lambda}{\mu})^n P_0, & 1 \leq n \leq c, \\ \frac{1}{c^{n-c}c!}(\frac{\lambda}{\mu})^n P_0, & c \leq n \leq N, \end{cases} \tag{33}$$

where

$$P_0 = \begin{cases} \left[\sum_{n=0}^{c-1} \frac{1}{n!}(\frac{\lambda}{\mu})^n + \frac{1}{c!}(\frac{\lambda}{\mu})^c \frac{1-\rho^{N-c+1}}{1-\rho}\right]^{-1}, & \text{if } \rho \neq 1, \\ \left[\sum_{n=0}^{c-1} \frac{1}{n!}(\frac{\lambda}{\mu})^n + \frac{1}{c!}(\frac{\lambda}{\mu})^c (N-c+1)\right]^{-1}, & \text{if } \rho = 1. \end{cases}$$

Note that $Q_{i,t} = \max(0, n-c)$, so the distribution of $Q_{i,t}$ can be obtained from (33). The blocking probability $P_{i,t}^B$ is

$$P_{i,t}^B = \frac{1}{c^{N-c}c!}(\frac{\lambda}{\mu})^N P_0. \tag{34}$$

It means that the probability of EVs in Φ_i to leave charging station i during time slot t is $P_{i,t}^B$.

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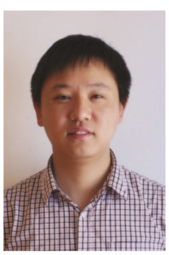

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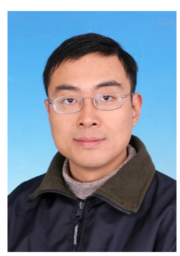

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