Vehicle Platooning With Non-Ideal Communication Networks

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***Abstract*—The performance and effectiveness of a vehicle platoon rely on the topology of information flow and quality of communications, such as delays and dropouts. In this paper, we investigate the homogeneous and constant-time-headway-spacing-policy-based vehicle platoon longitudinal control problem, where communication impairments including the limited communication range, random packet losses, and time-varying communication delays are considered. Here, internal stability characterizes the system stability without disturbance while string stability is concerned with the error amplification when the vehicle platoon is affected by external disturbances. First, for the case when each vehicle utilizes the position, velocity, and acceleration information of multiple preceding and following vehicles, we obtain sufficient conditions to guarantee the internal stability of the vehicle platoon system based on the stability of matrix polynomials and the matrix eigenvalue perturbation theory. Then, considering random packet losses, we find that the effectiveness of platoon control relies on the frequency of packets being successfully received, or the delay till a new packet is received. We also obtain the upper bound for delays on all communication links such that the internal stability of the vehicle platoon can still be maintained when time-varying communication delays are within the bound. The L_2-string stability is also analyzed through the transfer function for both ideal communication cases, and cases with uniform and constant delays. Extensive simulink-based numerical results validate our analysis, which reveals how communication impairments affect platoon control with a realistic information flow topology.**

***Index Terms*—Delays, internal stability, longitudinal control, random packet losses, string stability, vehicle platoon.**

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I. Introduction

A VEHICLE platoon is a road train formed by a group of vehicles without physical couplings, where a short inter-vehicle distance is preserved by automation and Vehicle-to-Vehicle (V2V) communication technologies [1]. Studies have shown that a car with a velocity of $80\mathrm{km/h}$ following only one predecessor at $25\mathrm{m}$ achieves a 30% reduction in aerodynamic drag, and a 40% reduction can be attained by following two predecessors [2]. By maintaining a short inter-vehicle distance, the "follower" vehicles have smaller air drag and less fuel consumption in a vehicle platoon. This also leads to the reduction of carbon emissions in the transportation system. Moreover, vehicle platooning helps increase traffic throughput on roads, which is crucial for mitigating traffic congestions. For example, if all passenger cars form vehicle platoons, a 200% growth in the road capacity can be achieved [3]. With the urgent demand for smooth traffic flow, autonomous self-driving advancement, and fuel consumption reduction, vehicle platooning will be an important technology in the development of future intelligent transportation systems.

A vehicle platoon control system consists of longitudinal control and lateral control, where longitudinal control aims to regulate a longitudinal motion and lateral control is to make individual vehicles track the center of the lane accurately. In this paper, we focus on longitudinal control of the vehicle platoon. Different spacing policies have been proposed for the longitudinal control such as constant and variable spacing control strategies. The most popular spacing control policies are the constant distance policy and the constant time headway spacing policy. Compared to the constant distance policy, the constant time headway spacing policy improves the scalability and stability of the vehicle platoon with different information flow topologies (IFTs), e.g., one-look-ahead, two predecessors following, leader-following, leader-predecessor following, multiple-look-ahead, multiple predecessors and followers. In Adaptive Cruise Control (ACC), each vehicle in the platoon utilizes its on-board sensors (such as radar-based sensors) to measure the velocity and position of its one predecessor and maintains a small inter-vehicle distance by using these sensor measurements. Since ACC has limitations in control performance, e.g., there is an inherent limit on the headway time, Cooperative Adaptive Cruise Control (CACC) has become a more popular technique. In CACC, the inter-vehicle communication is used to maintain shorter inter-vehicle distance for the vehicle platoon and robustness against disturbance is guaranteed.

Much attention has been paid to vehicle platooning with a leader-predecessor information structure, where the leader vehicle broadcasts its local information to all following vehicles and each following vehicle combines the information with that of one immediate predecessor for local control. However, this leader-predecessor structure may not be realistic especially when the size of the platoon is large. Therefore, it is desirable to consider the case when each vehicle has a limited communication range in the platoon. Furthermore, if each vehicle knows a higher percentage of the global network information, then the vehicles may achieve faster consensus on acceleration and speed. It was also shown that the time-headway can be minimized by using multiple predecessors' information in the local controller [4], which means that the inter-vehicle distance can be minimized by using the information of more predecessors and followers. At the same time, it is vitally important that the network and the controller are jointly designed for safety applications and cooperative driving. In summary, it is of vital importance to investigate the performance of the vehicle platoon where each vehicle receives and uses the information from multiple predecessors and multiple followers.

V2V and Vehicle-to-Infrastructure (V2I) wireless communication technologies facilitate the application and development of CACC. However, the quality of wireless communication is affected by many impairments, such as channel fading, shadowing, and interference, and thus communication delays and packet losses are inevitable in vehicular communication networks [5]. As a result, it is necessary and meaningful to investigate their effects on vehicle platoon control, which provides guidelines on the design of effective controllers. Stability conditions for vehicle platoons in imperfect communication environments have received attention in recent years. In particular, for the leader-predecessor following and one-look-ahead IFTs, how communication delays and random packet losses affect the performance has been studied [6]–[8]. However, for vehicle platoons with a more general IFT such as two-predecessor following, limited communication range, and bidirectional networks, the stability, scalability, and robustness of the platoon system were mainly investigated based on the assumption that the communication networks are perfect [9]–[11]. It remains open to analyze how limited communication range-based network (where each vehicle uses the information of multiple predecessors and followers) and imperfect communication environments interact with each other and how they affect control performance of the vehicle platoon. To fill this gap, in this paper, we study the vehicle platoon where each vehicle has a limited communication range and adopts the constant time headway spacing policy in an imperfect communication environment. The main contributions of this work are summarized as follows.

1) We analyze control performance of the vehicle platoon where the vehicles have limited communication range (where each vehicle uses the information of multiple predecessors and followers) and they are in an imperfect communication environment.
2) We investigate how the vehicle platoon achieves internal stability. Specifically, by using matrix eigenvalue perturbation theory and block matrix polynomials, we obtain sufficient conditions on the information flow to guarantee internal stability.
3) We show that there exists an upper bound for the time-varying communication delays under which the internal stability is ensured. We also quantify the reliability of the vehicle platoon with random packet losses and characterize the performance of platoon control.
4) We analyze the string stability of the considered vehicle platoon model in ideal communication cases (without any delays or packet drops) and cases with uniform and constant communication delays, respectively. The sufficient conditions are provided to guarantee the L_2-string stability.

The remainder of this paper is organized as follows. Related work is summarized in Section II. The preliminaries and problem formulation are provided in Section III. We give the system design in Section IV, and analyze the system performance in Section V. Section VI verifies the main results through numerical studies. Conclusion and future work are given in Section VII.

II. Related Work

For the platoon with a leader-predecessor-following topology, many efforts have been made to the stability analysis and controller design in the case when communication delays exist. Considering the constant spacing policy, Liu *et al.* investigated the effects of time-invariant delays on the string stability of the vehicle platoon and proposed a simple method to avoid the instability caused by communication delays through clock jitter [6]. Xiao *et al.* analyzed the stability of the platoon when the communication delay on the links between the vehicle and the leader is larger than that between the vehicle and the predecessor, and they proposed PD and sliding mode-based controllers for two different spacing policies [7]. Moreover, Peters *et al.* analyzed the effects of uniform delays on the system performance for different communication strategies [12]. Fernandes *et al.* proposed a mitigation method based on the intraplatoon information management to guarantee the string stability, where only simulation results were provided [13]. Lots of works have been done regarding the platoon with other simple information structures. For CACC-based vehicle platoon under uniform time-varying delays, sufficient conditions were obtained on uncertain sampling intervals and delays to ensure the string stability in [8]. Bernardo *et al.* proposed a consensus-based control strategy for the platoon and investigated the effect of time-varying delays on the internal stability, where the leader vehicle's information is available to all other vehicles [14], [15]. Liu *et al.* provided the stability condition for the platoon with constant input delays where each vehicle measures one-look-ahead and one-look-backward vehicles' information by its local sensors [16]. For the connected cruise control system with a nonlinear local controller by using one immediate (multiple) predecessor's (predecessors') information, many efforts have been made to study the effect of the feedback delay and communication delay on the stability. For example, Jin *et al.* investigated the relationship among the driver reaction time/sensor delay, time-invariant but heterogeneous communication delays, and the stability margin of the platoon system, and then an optimal feedback law was proposed to minimize the cost function defined by headway and velocity errors [17]. They also studied whether the delayed local information can enhance the stability of the platoon system for both open-chain and ring-road configurations [18]. In [19], the influence of heterogeneous connectivity structures and information delays on the stability of the connected vehicles' systems was investigated, and a motif-based approach was designed to improve the module and scalability. Meanwhile, Qin *et al.* investigated how the stochastic delay affects the stability of the nonlinear platoon system, where a uniform packet drop rate is considered [20].

Vehicle platooning with packet losses is another hot research topic. Seiler *et al.* explored the effect of packet losses on the discrete-time vehicle following control using linear matrix inequalities, where each vehicle only uses its predecessor's information for longitudinal control [21]. Teo *et al.* investigated a discrete-time model for a platoon with leader-predecessor following structure and packet dropouts, and they proposed a mitigation method by estimating the leader vehicle's state [22]. For persistent packet losses, a state estimation based method is proposed to make the CACC gracefully switch to ACC and guarantee the string stability [23]. Guo *et al.* considered the constant spacing strategy and a leader-predecessor-following-IFT-based platoon with random packet losses and limited communication capacity, and they designed a strategy to guarantee the mean square exponential string stability [24]. Considering a heterogeneous vehicle platoon with inter-vehicle communication losses, Harfouch *et al.* proposed an extended dwell-time structure and designed an adaptive switch controller to guarantee the system stability [25]. Acciani *et al.* studied a CACC-based vehicle platoon with a one-look-ahead IFT and proposed an H-infinity-based controller to guarantee the stability in expectation [26]. Zhao *et al.* investigated the discrete-time vehicle platoon model with packet losses, where multiple predecessors' information is used for local vehicle controller and the convergence time is thoroughly analyzed [27].

The platoon with general IFT also becomes an important topic in recent years. Richard *et al.* analyzed the stability of the vehicle platoon by the frequency domain method [28]. In their model, each vehicle has a limited communication range and broadcasts its spacing error information to neighbors. Zheng *et al.* considered the platoon with the constant spacing policy, and they investigated the scalability and robustness for general IFTs [9]. Li *et al.* proposed an eigenvalue-based method to investigate the stability and scalability of the vehicle platoon, where a general IFT and the constant spacing strategy were considered [10]. Stüdli *et al.* investigated the cyclic interconnections for vehicle platoons and provided conditions for the string stability by frequency-domain methods [29]. The stability margin of the vehicle platoon with general undirected IFTs and the constant spacing policy was analyzed in [9]. Many efforts have also been made to problems of platoon control with general IFTs [30]. Bian *et al.* studied the vehicle platoon with the constant time headway policy where multiple predecessors' information is utilized for the short inter-vehicle distance control [11].

In recent years, many efforts have been made to the dynamic analysis of vehicle platoons to avoid vehicle collisions. Bergenhem *et al.* considered the problem of coordinating emergency brake for vehicle platoons with packet losses, where a machine learning based parameters estimation method was proposed and quantitative analysis was provided through simulations [31]. Wu *et al.* proposed to apply Kalman Filter to handle the communication delays and design an adaptive acceleration for the vehicle platoon [32]. Li *et al.* considered the one-look-ahead and leader following longitudinal control for the individual platoon and cooperative lateral control for parallel platoons, where the artificial function method based controllers were proposed [33]. Al-Jhayyish *et al.* investigated the one-look-ahead and the constant time headway model, and proposed to estimate the parameter bounds for different feedback strategies to ensure the string stability of the vehicle platoon [34].

Notice that for the case when each vehicle has a limited communication range (where each vehicle uses the information of multiple predecessors and followers), how to maintain the stability of the vehicle platoon and how the imperfect communication environments affect platoon control are still open problems.

III. Preliminaries and Problem Formulation

Notation: Let $\mathbb{R}$, $\mathbb{N}$, $\mathbb{N}^+$, and $\mathbb{C}$ denote the set of real numbers, the set of natural numbers, the set of positive integers, and the set of complex numbers, respectively. Let $A = [a_{ij}] \in \mathbb{R}^{n\times m}$ be an $n \times m$ real matrix, where $n, m \in \mathbb{N}^+$, and a_{ij} is the (i, j)-th element of A. The transpose of A is denoted by $A^\top$. Let $\mathbb{R}^n$ be the space of n-dimensional real column vectors. A symmetric square matrix $A \in \mathbb{R}^{n\times n}$ is positive definite ($A \succ 0$) if for any nonzero vector $x \in \mathbb{R}^n$, we have $x^\top Ax > 0$; A is positive semidefinite ($A \succ 0$) if $x^\top Ax \geq 0$ for any $x \in \mathbb{R}^n$. We use I to denote the identity matrix, whose dimension will be made clear when it appears. The determinant of a matrix A is denoted by $\det(A)$. Let $\|\cdot\|_2$ denote the 2-norm of a vector or matrix. We use i to denote the imaginary unit and the function $\mathrm{real}(\cdot)$ takes the real part of a complex number. The square matrix A is a stable matrix (or a Hurwitz matrix) if every eigenvalue of A has a strictly negative real part. The function $\min(\cdot)$ ($\max(\cdot)$) takes the minimum (maximum) value. We use $\mathbf{e}$ to represent the base of the natural logarithm. Let $\Pr\{\cdot\}$ denote the probability that the event $\cdot$ happens.

A. Network Model

Consider a platoon consisting of $n+1 \geq 2$ homogeneous vehicles, where n is the number of following vehicles. Each vehicle has a unique ID, i.e., $i = 0, 1, 2, \ldots, n$, where $i = 0$ is the ID of the leader vehicle and the following vehicles have IDs $i \in \{1, \ldots, n\}$. The topology of the communication network is modeled by $G = (V, E)$, where V is the set of $\{1, 2, \ldots, n\}$ vehicles and $E \subset V \times V$ is the edge set. If vehicle i uses vehicle j's information, then we have $(i, j) \in E$. In this paper, we consider the case where each vehicle has a limited communication range and uses the information from r predecessors and l followers when $i \in [r, n-l]$, $r + l < n$. The parameters r and l can be set as constants before being implemented in the vehicle platoon. For $1 \leq i < r$, vehicle i communicates with i predecessors and l followers, while for $i > n - l$, vehicle i communicates with r predecessors and $n - i$ followers. Self-loops are not allowed, i.e., $(i, i) \notin E$ for all $i \in V$. The communication graph of the entire platoon including the leader vehicle is modeled by $\tilde{G} = (\tilde{V}, \tilde{E})$, where $\tilde{V} = V \cup \{0\}$ and $\tilde{E}$ consists of all communication links in the platoon. Let $A = [A_{ij}] \in \mathbb{R}^{n\times n}$ be the adjacency matrix of graph $G = (V, E)$, where $A_{ij} = 1$ if and only if $(i, j) \in E$. The in-degree matrix $\mathcal{D} = [\mathcal{D}_{ij}] \in \mathbb{R}^{n\times n}$ is a diagonal matrix with $\mathcal{D}_{ii} = \sum_{j=1}^{n} A_{ij}$. The Laplacian matrix L is defined by $L = \mathcal{D} - A$. Let $P \in \mathbb{R}^{n\times n}$ be a diagonal matrix with $P_{ii} = 1$ if vehicle i uses leader's information.

B. Vehicle Dynamic Model

Let m_i be the mass and q_i be the position of vehicle i for all $i \in \tilde{V}$. By Newton's law and the vehicle's engine dynamics [35], the dynamical model of vehicle i is

$$\begin{aligned} m_i\ddot{q}_i &= m_i\xi_i - K_i\dot{q}_i^2 - d_i, \\ \dot{\xi}_i &= -\frac{\xi_i}{\tau_i(\dot{\xi}_i)} + \frac{\mu_i}{m_i\tau_i(\dot{\xi}_i)}, \end{aligned} \tag{1}$$

where $K_i\dot{q}_i^2$ characterizes the air resistance with K_i being a constant parameter, $m_i\xi_i$ is the vehicle's engine force, and d_i is a constant representing the mechanical drag. Moreover, $\tau_i(\dot{\xi}_i)$ denotes the engine time constant, and μ_i is the throttle input to the engine. The parameters in (1) are assumed to be known a priori. By adopting the control law $\mu_i = m_iu_i + K_i\dot{q}_i^2 + d_i + 2\tau_i(\dot{\xi}_i)K_i\dot{q}_i\ddot{q}_i$, one obtains $\dddot{q}_i = -\frac{1}{\tau_i}\ddot{q}_i + \frac{1}{\tau_i}u_i$, where $\tau_i = \tau_i(\dot{\xi}_i)$ is constant when $\tau_i(\dot{\xi}_i)$ is small enough [36] and u_i is the control input. Let $v_i(t)$ and $a_i(t)$ be the absolute velocity and acceleration of vehicle i at time t, respectively. The dynamic of each vehicle i is described by,

$$\begin{aligned} \dot{q}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= a_i(t), \\ \dot{a}_i(t) &= -\frac{1}{\tau}a_i(t) + \frac{1}{\tau}u_i(k), \end{aligned} \tag{2}$$

where $\tau = \tau_i = \tau_j$ for all $i, j \in \tilde{V}$. In this paper, we mainly focus on the linear model of the system composed of connected automated vehicles, which has been extensively used in vehicle control. An important further research issue is to analyze a nonlinear model since there are nonlinear constraints for vehicles such as input saturation and bounded velocity constraints, which beckons future work.

C. Communication Losses and Delay Models

1) Communication Loss Models: We consider two different communication loss models. In the first model, we assume that the packet delivery succeeds with the same constant probability on all communication links, and in the second model, the packet delivery succeeds with probabilities depending on the relative distance between vehicles.

In the first model, the packet forwarding succeeds with the same probability on all links, i.e., $p_{ij} = p$, $(i, j) \in \tilde{E}$, $0 < p \leq 1$. For each vehicle i, if the packet from its neighbor j is transmitted successfully, then i will use the newly received information for local control. Otherwise, vehicle i will use the latest received information for the local control.

In the second model, since the wireless signal fades with the relative distance between communicating vehicles [37], [38], the success probability p_{ij}, $(i, j) \in \tilde{E}$, decreases with distance. By [39], the received signal power $\mathcal{P}_{i,j}(t)$ at the receiver of vehicle i at time t from the neighboring vehicle j is given by $\mathcal{P}_{i,j}(t) = \frac{K_0\mathcal{P}_{b,j}(t)}{|d_{i,j}(t)|^\alpha}$, where $\mathcal{P}_{b,j}(t)$ is the transmission signal power of vehicle j, K_0 and α are positive constants, and $|d_{i,j}(t)|$ is the absolute distance between vehicle i and j. Suppose the communication noise has mean power N_0, i.e., it is an additive Gaussian random variable with zero mean and variance N_0. At each time slot t, all $n + 1$ vehicles broadcast their information to their neighbors. Suppose there is no communication interference among different vehicles, then the signal-to-noise ratio (SNR) $\bar{\gamma}_{i,j}(t)$ is $\bar{\gamma}_{i,j}(t) = \frac{\mathcal{P}_{i,j}(t)}{N_0} = \frac{K_0\mathcal{P}_{b,j}(t)}{N_0|d_{i,j}(t))|^\alpha}$. If the fast fading channel model is a Rayleigh distribution, then the probability density function of the SNR is $f_i(\gamma_{i,j}(t)) = \frac{1}{\bar{\gamma}_{i,j}(t)}e^{-\frac{\gamma_{i,j}(t)}{\bar{\gamma}_{i,j}(t)}}$ for $\gamma_{i,j}(t) \geq 0$, and $f_i(\gamma_{i,j}(t)) = 0$ otherwise. Given an acceptable SNR $\tilde{\gamma}$, $\gamma_{i,j}(t)$ needs to be greater than or equal to $\tilde{\gamma}$ in order for vehicle i to decode a received packet successfully. Thus, the probability of successful packet delivery is calculated by

$$p_{ij}(t) = \Pr\{\gamma_{i,j}(t) \geq \tilde{\gamma}\} = e^{-\frac{\tilde{\gamma}}{\bar{\gamma}_{i,j}(t)}} = e^{-\frac{\tilde{\gamma}N_0|d_{i,j}(t)|^\alpha}{K_0\mathcal{P}_{b,j}(t)}}. \tag{3}$$

Notice that when the relative distance grows, the probability of successful packet delivery decreases.

Remark 3.1: The interference and network congestion (caused by the number of platoon members) are key sources of packet losses in vehicular networks. However, the interference and network congestion also depends on the distance between cars since interferences becomes less tolerable when the distance between the transmitter and the receiver becomes larger, while network congestion becomes weaker. Moreover, the interference and network congestion can be alleviated through communication protocol design. Thus, in our paper, the packet loss model largely depends on distance. From [40], the payload for the lower level automation is 300-400 bytes. In a vehicle platoon, only the position, velocity, and acceleration need to be transmitted, and the data volume exchanged is very small and can be fit in a single packet. Thus, the volume of data exchanged may not lead to substantial congestion losses given the capacity of vehicle-to-everything communication system such as 5 G new radio, and we ignore it for simplicity. Validating the more realistic losses models for the vehicle platoon is an interesting topic, which is left as future work.

2) Communication Delay Model: Since the outdated information will be used when the new packets are lost during the transmission, the performance analysis of the vehicle platoon under communication losses can be transformed into a problem with communication delays. In this paper, we consider heterogeneous time-varying delays on the communication links. Let

$\iota_{ij}(t)$ be the time-varying delay on link $(i, j) \in \tilde{E}$. Note that in practice, communication delays are samples depending on packet losses, which is a special expression of $\iota_{ij}(t)$. As we consider more general delay model $\iota_{ij}(t)$, the obtained analytical results are applicable for practical scenarios with time-varying delays.

D. Problem of Interests

In the literature, most work on vehicle platoon controller design and performance analysis for general IFT (including limited communication range-based IFT) relies on the assumption that the communication environment is perfect. In this paper, we answer the following questions.

- Do there exist gain parameters to guarantee the internal stability of the vehicle platoon with limited communication range in a ideal communication environment?
- How to maintain the internal stability and the string stability of the vehicle platoon when there are communication delays or random communication losses?
- When and why does the controller not work when probabilistic transmission losses exist? Can we provide other metrics to characterize the reliability of the vehicle platoon?

IV. System Design

In this section, we describe the system and protocols that we consider in this paper, which contains three parts, i.e., the constant time headway spacing policy, the distributed controller, and the communication mechanism.

A. Constant Headway Spacing Policy

The constant time headway spacing policy is used in the vehicle platoon to maintain a small relative distance between neighboring vehicles. The desirable distance $d_{i,i-1}(t)$ between vehicle i and its predecessor $i-1$ is described by

$$d_{i,i-1}(t) = d + hv_i(t), \tag{4}$$

where $h > 0$ is the constant time headway and d is a constant distance. The actual distance between vehicle i and its predecessor $i-1$ is $d_i(t) = q_{i-1}(t) - q_i(t) - \ell$, where ℓ is the length of vehicle i. Then, the spacing error between vehicle $i \geq 1$ and its predecessor $i-1$ is calculated by

$$\begin{aligned} e_i(t) &= d_i(t) - d_{i,i-1}(t), \\ &= q_{i-1}(t) - q_i(t) - \ell - (d + hv_i(t)). \end{aligned} \tag{5}$$

Since ℓ and d are the same for all vehicles in V, we neglect them in the rest of the paper. For any pair of neighboring vehicles i and j in the communication graph, the desired spacing distance is

$$d_{i,j}(t) = \begin{cases} \sum_{\eta=j+1}^{i} hv_\eta(t), & j < i, \\ -\sum_{\eta=i+1}^{j} hv_\eta(t), & j > i. \end{cases} \tag{6}$$

Note that the desired distance between each vehicle $i > 0$ and the leader vehicle is $d_{i,0}(t) = \sum_{\eta=1}^{i} hv_\eta(t)$. Using the velocity-based policy is to improve string stability. Note that when the

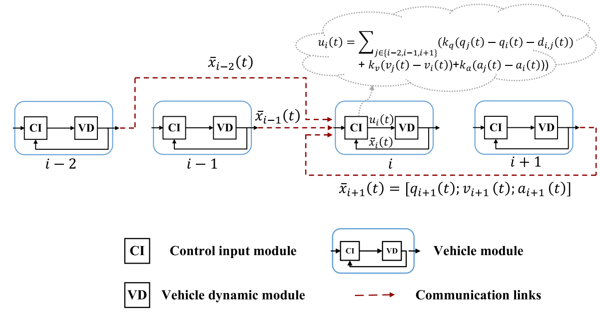

Fig. 1. Close loop diagram of vehicle i.

velocity changes, the communicable number of vehicles in a platoon is different. It should be pointed out that r and l can be set as constants. For example, r and l are set as the number of predecessors and the number of followers which are obtained for the maximum velocity, respectively. Moreover, we use different r and l for general IFTs, which can be optimized for the vehicle platoon.

B. Distributed Controller

Inspired by [9] and [11], we study the case where each vehicle uses spacing errors, velocity errors, and acceleration errors with respect to its communication neighbors to produce the control input. For each vehicle i, the control input $u_i(t)$ consists of three parts, i.e., control input $u_i^{pr}(t)$ using the information of multiple predecessors, the control input $u_i^{fl}(t)$ by using the information of multiple followers, and the control input $u_i^{le}(t)$ by using the information of the leader vehicle. Specifically, for $i \in V$, we have $u_i(t) = u_i^{pr}(t) + u_i^{fl}(t) + u_i^{le}(t)$ and

$$\begin{aligned} u_i^{pr}(t) + u_i^{fl}(t) &= \sum_{j=1}^{n} A_{ij}(k_q(q_j(t) - q_i(t) - d_{i,j}(t)) \\ &\quad + k_v(v_j(t) - v_i(t)) + k_a(a_j(t) - a_i(t))), \\ u_i^{le}(t) &= P_{ii}(k_q(q_0(t) - q_i(t) - d_{i,0}) \\ &\quad + k_v(v_0(t) - v_i(t)) + k_a(a_0(t) - a_i(t))), \end{aligned} \tag{7}$$

where k_q, k_v, and k_d are the corresponding gain parameters.

An example is given to show the closed control loop of the controller (7). For vehicle i, with $r = 2$ and $l = 1$, we have the feedback control loop of vehicle i shown in Fig. 1, which includes both information flow and control structure of vehicle i. The control input is the combination of local state information $\bar{x}_i(t) = [q_i(t)\ v_i(t)\ a_i(t)]^\top$, and neighbors' transmitted information $\bar{x}_{i-2}(t) = [q_{i-2}(t)\ v_{i-2}(t)\ a_{i-2}(t)]^\top$, $\bar{x}_{i-1}(t) = [q_{i-1}(t)\ v_{i-1}(t)\ a_{i-1}(t)]^\top$, and $\bar{x}_{i+1}(t) = [q_{i+1}(t); v_{i+1}(t); a_{i+1}(t)]$, which is shown in equation (7).

C. Communication Mechanism

Suppose the communication interval among vehicles is T_s and a zero-order hold is used by each vehicle i after it receives the sampled data from neighbors. Then, during the interval $[k\mathrm{T}_s, (k+1)\mathrm{T}_s)$ for all $k \in \mathbb{N}$, the information received by vehicle i from vehicle j is described by

$$\begin{aligned}&\hat{q}_j(k\mathrm{T}_s+\kappa)\\&=\vartheta_{ji}(k\mathrm{T}_s)\hat{q}_j((k-1)\mathrm{T}_s)+(1-\vartheta_{ji}(k\mathrm{T}_s))q_j(k\mathrm{T}_s),\\&\hat{v}_j(k\mathrm{T}_s+\kappa)\\&=\vartheta_{ji}(k\mathrm{T}_s)\hat{v}_j((k-1)\mathrm{T}_s)+(1-\vartheta_{ji}(k\mathrm{T}_s))v_j(k\mathrm{T}_s),\\&\hat{a}_j(k\mathrm{T}_s+\kappa)\\&=\vartheta_{ji}(k\mathrm{T}_s)\hat{a}_j((k-1)\mathrm{T}_s)+(1-\vartheta_{ji}(k\mathrm{T}_s))a_j(k\mathrm{T}_s),\end{aligned}\quad(8)$$

where $0\le\kappa<\mathrm{T}_s$, $\hat{q}_j(k\mathrm{T}_s+\kappa)$, $\hat{v}_j(k\mathrm{T}_s+\kappa)$, and $\hat{a}_j(k\mathrm{T}_s+\kappa)$ are the estimates of the position, velocity, and acceleration of vehicle j at time $k\mathrm{T}_s+\kappa$ through the zero-order hold, respectively, and $\vartheta_{ji}(k\mathrm{T}_s)$ is a binary random variable with the distribution given by $P\{\vartheta_{ji}(k\mathrm{T}_s)=1\}=1-p_{ji}(k\mathrm{T}_s)$ and $P\{\vartheta_{ji}(k\mathrm{T}_s)=0\}=p_{ji}(k\mathrm{T}_s)$.

V. Internal and String Stability Analysis

In this section, we first provide the sufficient conditions on gain parameters and the IFT so as to ensure the internal stability of the vehicle platoon. Then, we present stability analysis from the perspective of the reliability for the platoon with random packet losses. We also analyze how time-varying communication delays influence the internal stability of the platoon.

A. Internal Stability of Platoon With Ideal Communication

In this subsection, we first establish the internal stability of the platoon where vehicles have limited communication range. Then, by using the eigenvalue perturbation theory, we obtain a sufficient condition for the internal stability. Similar to [11], let $\tilde{q}_i(t)$, $\tilde{v}_i(t)$, and $\tilde{a}_i(t)$ be the relative distance, relative velocity, and relative acceleration between vehicle i and leader vehicle, i.e.,

$$\begin{aligned}\tilde{q}_i(t)&=q_i(t)-(q_0(t)+d_{i,0}(t)),\\\tilde{v}_i(t)&=v_i(t)-v_0(t),\\\tilde{a}_i(t)&=a_i(t)-a_0(t).\end{aligned}$$

Then, the control input can be described by

$$\begin{aligned}u_i^{pr}(t)+u_i^{fl}(t)=&\sum_{j=1}^{n}A_{ij}(k_q(\tilde{q}_j(t)-\tilde{q}_i(t))\\&+k_v(\tilde{v}_j(t)-\tilde{v}_i(t))+k_a(\tilde{a}_j(t)-\tilde{a}_i(t))),\\u_i^{le}(t)=&P_{ii}(-k_q\tilde{q}_i(t)-k_v\tilde{v}_i(t)-k_a\tilde{a}_i(t)),\end{aligned}\quad(9)$$

where the desirable distance has been eliminated. Suppose that the leader vehicle maintains a constant speed, i.e., $q_0=v_0t$ and $a_0=0$, then one obtains

$$\begin{aligned}\dot{\tilde{q}}_i(t)&=\tilde{v}_i(t)-\sum_{\eta=1}^{i}h\tilde{a}_\eta(t),\\\dot{\tilde{v}}_i(t)&=\tilde{a}_i(t),\\\dot{\tilde{a}}_i(t)&=-\frac{1}{\tau}\tilde{a}_i(t)+\frac{1}{\tau}u_i(t).\end{aligned}\quad(10)$$

Let $\tilde{q}(t)=[\tilde{q}_1(t)\ \tilde{q}_2(t)\ \cdots\ \tilde{q}_n(t)]^\top$, $\tilde{v}(t)=[\tilde{v}_1(t)\ \tilde{v}_2(t)\cdots\tilde{v}_n(t)]^\top$, $\tilde{a}(t)=[\tilde{a}_1(t)\ \tilde{a}_2(t)\cdots\tilde{a}_n(t)]^\top$, $u(t)=[u_1(t)\ u_2(t)\cdots u_n(t)]^\top$. By (9) and (10), the closed-loop dynamics of the platoon can be written as

$$\dot{x}(t)=\tilde{A}x(t),\quad(11)$$

where $x(t)=[\tilde{q}(t)^\top\ \tilde{v}(t)^\top\ \tilde{a}(t)^\top]^\top$. The system matrix $\tilde{A}$ in (11) has the following structure

$$\tilde{A}=\begin{bmatrix}0_n&I_n&-H\\0_n&0_n&I_n\\-\frac{k_q}{\tau}\tilde{L}&-\frac{k_v}{\tau}\tilde{L}&\frac{1}{\tau}(-I_n-k_a\tilde{L})\end{bmatrix},$$

where $\tilde{L}=L+P$, 0_n is an $n\times n$ zero matrix and

$$H=\begin{bmatrix}h&0&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\h&h&\cdots&h\end{bmatrix}.$$

The system (11) is globally asymptotically stable if and only if all the eigenvalues of $\tilde{A}$ have negative real parts. By analyzing the characteristic polynomials of $\tilde{A}$, we obtain the lemma as follows.

Lemma 5.1: Let C be the block companion matrix, i.e.,

$$C=\begin{bmatrix}0_n&I_n&0_n\\0_n&0_n&I_n\\-\frac{k_q}{\tau}\tilde{L}&\frac{k_q}{\tau}\tilde{L}H-\frac{k_v}{\tau}\tilde{L}&\frac{1}{\tau}(-I_n-k_a\tilde{L})\end{bmatrix}.$$

Then, $\tilde{A}$ and C have the same set of eigenvalues.

Remark 5.2: The relationship between the eigenvalues of C and $\tilde{A}$ can be found in [41], which provides a way to select system parameters and the IFT for better internal stability of the vehicle platoon. Then, we can further investigate the interaction between the IFT contained in $\tilde{L}$ and the internal stability of the vehicle platoon described by $\tilde{A}$.

Using the matrix eigenvalue perturbation theory, we analyze the internal stability of the platoon with limited communication range. We write C in a matrix perturbation form, i.e., $C=C_0+\Delta$, where C_0 and Δ satisfy

$$C_0=\begin{bmatrix}0_n&I_n&0_n\\0_n&0_n&I_n\\-\frac{k_q}{\tau}\tilde{L}&-\frac{k_v}{\tau}\tilde{L}&\frac{1}{\tau}(-I_n-k_a\tilde{L})\end{bmatrix},$$

$$\Delta=\begin{bmatrix}0_n&0_n&0_n\\0_n&0_n&0_n\\0_n&\frac{k_q}{\tau}\tilde{L}H&0_n\end{bmatrix}.$$

Let $\sigma_{C_0}=\{\lambda_1,\lambda_2,\ldots,\lambda_{3n}\}$ and $\sigma_C=\{\tilde{\lambda}_1,\tilde{\lambda}_2,\ldots,\tilde{\lambda}_{3n}\}$ be the set of eigenvalues of C_0 and C, where $|\lambda_{3n}|\ge\cdots\ge|\lambda_2|\ge|\lambda_1|$ and $|\tilde{\lambda}_{3n}|\ge\cdots\ge|\tilde{\lambda}_2|\ge|\tilde{\lambda}_1|$. Since it is difficult to obtain the sufficient condition for the stability of a general block companion matrix, we only provide the sufficient condition for a special case where the Laplacian matrix is symmetric, i.e., $r=l$. For $r=l$, we can obtain the Jordan normal form of matrix $\tilde{L}$, i.e., $\tilde{L}=D\Sigma D^{-1}$, where Σ is a diagonal matrix.

Lemma 5.3: If $r = l$ and the following condition holds,

$$\frac{k_v}{\tau}\Sigma - k_q(k_a\Sigma + I_n)^{-1} \succ 0, \tag{12}$$

then all eigenvalues of C_0 have negative real parts, i.e.,

$$\text{real}(\lambda_i) < 0, \quad \forall \lambda_i \in \sigma_{C_0}. \tag{13}$$

Proof: When $r = l$, by performing the matrix Routh array test [42], one obtains

$$\begin{aligned}
&\mathcal{C}_{11} = I_n && \mathcal{C}_{12} = \frac{k_v}{\tau}\tilde{L} && \mathcal{C}_{13} = 0_n\\
&\mathcal{C}_{21} = \frac{1}{\tau}(I_n + k_a\tilde{L}) && \mathcal{C}_{22} = \frac{k_q}{\tau}\tilde{L} && \mathcal{C}_{23} = 0_n\\
&\mathcal{C}_{31} = \frac{k_v}{\tau}\tilde{L} - (\frac{1}{\tau}(I_n + k_a\tilde{L}))^{-1}\frac{k_q}{\tau}\tilde{L} && \mathcal{C}_{32} = 0_n && \mathcal{C}_{33} = 0_n\\
&\mathcal{C}_{41} = \frac{k_q}{\tau}\tilde{L} && \mathcal{C}_{42} = 0_n && \mathcal{C}_{43} = 0_n
\end{aligned}$$

Then, the matrix quotients can be expressed as

$$\begin{aligned}
\tilde{H}_1 &= \mathcal{C}_{11}\mathcal{C}_{21}^{-1}\\
&= \{\frac{1}{\tau}(I_n + k_a\tilde{L})\}^{-1} = D\{\tau(I_n + k_a\Sigma)\}^{-1}D^{-1},\\
\tilde{H}_2 &= \mathcal{C}_{21}\mathcal{C}_{31}^{-1}\\
&= \frac{1}{\tau}(I_n + k_a\tilde{L})\{\frac{k_v}{\tau}\tilde{L} - \{\frac{1}{\tau}(I_n + k_a\tilde{L})\}^{-1}\frac{k_q}{\tau}\tilde{L}\}^{-1}\\
&= \frac{1}{\tau}D(I_n + k_a\Sigma)\{\frac{k_v}{\tau}\Sigma - k_q(k_a\Sigma + I_n)^{-1}\}^{-1}D^{-1},\\
\tilde{H}_3 &= \mathcal{C}_{31}\mathcal{C}_{41}^{-1}\\
&= \frac{k_v}{\tau}\tilde{L} - \{\frac{1}{\tau}(I_n + k_a\tilde{L})\}^{-1}\frac{k_q}{\tau}\tilde{L}\{\frac{k_q}{\tau}\tilde{L}\}^{-1}\\
&= \frac{k_q}{\tau}D(\frac{k_v}{\tau}\Sigma - k_q(k_a\Sigma + I_n)^{-1})\Sigma D^{-1}.
\end{aligned}$$

If (12) holds, then we have that $\tilde{H}_1 \succ 0$, $\tilde{H}_2 \succ 0$, and $\tilde{H}_3 \succ 0$. By the sufficient condition in [42], we have (13). ■

The spectral variation of C with respect to C_0 and that of C_0 with respect to C are defined as follows

$$\text{sv}_C(C_0) = \max_i \min_j |\tilde{\lambda}_i - \lambda_j|,$$

$$\text{sv}_{C_0}(C) = \max_i \min_j |\lambda_i - \tilde{\lambda}_j|.$$

Then, the Hausdorff distance between the eigenvalues of C_0 and C can be written as $\text{hd}(C_0, C) = \max(\text{sv}_C(C_0), \text{sv}_{C_0}(C))$.

Theorem 5.4: If all the eigenvalues of C_0 have strictly negative real parts, then there must exist Δ such that all the eigenvalues of $\tilde{A}$ have strictly negative real parts, i.e., the vehicle platoon (11) is asymptotically stable.

Proof: We can write $\Delta = h\Gamma$, where Γ is independent of h. As the variation of eigenvalues of C_0 corresponding to the perturbation Δ is continuous, if all the eigenvalues of C_0 have strictly negative real parts, then there must exist a small h such that all eigenvalues of C have strictly negative real parts. Since C and $\tilde{A}$ have the same eigenvalues, we conclude the theorem. ■

Theorem 5.5: If all the eigenvalues of C_0 have strictly negative real parts and the following condition holds,

$$\text{hd}(C_0, C) < \min_{\lambda_i \in \sigma_{C_0}} (|\text{real}(\lambda_i)|),$$

then the vehicle platoon (11) is asymptotically stable.

Proof: We prove that $\text{real}(\tilde{\lambda}_i) < 0$ for all $\tilde{\lambda}_i \in \sigma_C$. By definition, we have $\max_i \min_j |\text{real}(\tilde{\lambda}_i) - \text{real}(\lambda_j)| \leq \text{hd}(C_0, C)$. If $\text{hd}(C_0, C) < \min(|\text{real}(\lambda_i)|)$, then for all $\tilde{\lambda}_i \in \sigma_C$, we have $\min_j |\text{real}(\tilde{\lambda}_i) - \text{real}(\lambda_j)| < \min_{\lambda_i \in \sigma_{C_0}}(|\text{real}(\lambda_i)|)$. Note that $\text{real}(\lambda_j) < 0$ for all $\lambda_j \in \sigma_{C_0}$, therefore, we must have $\text{real}(\tilde{\lambda}_i) < 0$ for all $\tilde{\lambda}_i \in \sigma_C$. ■

By [43], we have the bound on the Hausdorff distance.

Lemma 5.6: For C_0 and C, we have

$$\text{hd}(C_0, C) \leq (\|C_0\|_2 + \|C\|_2)^{(1-\frac{1}{3n})}\|P\|_2^{\frac{1}{3n}}. \tag{14}$$

Remark 5.7: If (13) holds and

$$|\frac{k_q}{\tau}|^{\frac{1}{3n}}(\|C_0\|_2 + \|C\|_2)^{(1-\frac{1}{3n})}\|\tilde{L}H\|_2^{\frac{1}{3n}} < \min(|\text{real}(\lambda_i)|), \tag{15}$$

then all eigenvalues of C are negative. By Lemma 5.6, the max-min relative distance among σ_C and σ_{C_0} is less than $\min(|\text{real}(\lambda_i)|)$. Therefore, we have $\text{real}(\tilde{\lambda}_i) < 0$ for all $i \in \{1, 2, \ldots, 3n\}$, and all eigenvalues of C have strictly negative real parts. Condition (15) shows the relationship among the system stability, IFTs, and the system parameters.

B. Sampled Communication With Random Packet Drops

In this subsection, we investigate the influence of non-ideal communication environments on the stability of platoon and define reliability for the platoon control. Before providing the analysis, we give a basic assumption to guarantee the stability of the vehicle platoon with (8) as follows.

Assumption 5.8: For the vehicle platoon described by (2), (4), (5) and (6) with (8), there exists a sufficiently small T_s which guarantees the system stability.

Let the sampling frequency be $f_s = 1/\mathrm{T}_s$. By the Nyquist-Shannon sampling theorem, the original communication signal $q_j(t)$, $v_j(t)$, and $a_j(t)$ should have frequency less than $f_s/2$ and the original period should be large than or equal to $2\mathrm{T}_s$. Suppose that the minimum period of the original signal is T, which is approximated to $M\mathrm{T}_s$. We divide the communication interval $[k\mathrm{T}, (k+1)\mathrm{T})$ into $[k\mathrm{T}, k\mathrm{T} + \mathrm{T}_s)$, $[k\mathrm{T} + \mathrm{T}_s, k\mathrm{T} + 2\mathrm{T}_s)$, $\cdots$, $[k\mathrm{T} + (M-1)\mathrm{T}_s, k\mathrm{T} + M\mathrm{T}_s)$, then at least two samples are needed during this period to guarantee the recovery of the original signal, which is defined as an event Θ. Then, we have the following

$$\begin{aligned}
\Pr\{\Theta\} = 1 &- \prod_{\mu=1}^{M}(1 - p_{ji}(k\mathrm{T} + \mu\mathrm{T}_s))\\
&- \sum_{\nu=1}^{M} p_{ji}(k\mathrm{T} + \nu\mathrm{T}_s) \prod_{\mu=1,\mu\neq\nu}^{M}(1 - p_{ji}(k\mathrm{T} + \mu\mathrm{T}_s)).
\end{aligned}$$

Note that if T_s is sufficiently small, then M is large and the probability will be large even if the successful packet delivery probability is small. When T_s increases, the probability decreases. Therefore, for each vehicle j, we define the reliability by $\Pr\{\int_{k\mathrm{T}}^{(k+1)\mathrm{T}}(k_q(q_j(t)-\hat{q}_j(t)+k_v(v_j(t)-\hat{v}_j(t)+k_a(a_j(t)-\hat{a}_j(t)))^2\mathrm{d}t\leq\epsilon\}$, which depends on the sampling period, the input signal, and the packet loss probability.

C. Internal Stability Under Time-Varying Delays

Since the wireless medium is shared among vehicles and there are wireless network dynamics, the channel access time of each message is random, which results in random delays. Moreover, some packets may be corrupted and non-decodable at the receiver and the receiver needs to use the following non-corrupted packets for control purposes. Therefore, communication links may suffer from time-varying and non-uniform communication delays. Inspired by [44], we show that there exists an upper bound for the time-varying delays such that the internal stability can always be guaranteed. This upper bound provides important guidelines for the communication network design and QoS provisioning.

Theorem 5.9: The vehicle platoon (11) is asymptotically stable if $\tilde{A}$ is Hurwitz and there exists an upper bound $\bar{\iota}$ such that when time-varying delays $\iota_{ij}(t)$ on all communication links satisfy $0\leq\iota_{ij}(t)<\bar{\iota}$, $\bar{\iota}$ satisfying

$$\bar{\iota}<\frac{\lambda_{\min}(Q)}{\lambda_{\max}(F)},\tag{16}$$

where for any $Q\succ 0$ and $Q=Q^\top$, there exists an $O\succ 0$ such that $O\tilde{A}+\tilde{A}^\top O=-Q$, and

$$\begin{aligned}F=&\sum_{j=1}^{m}(O\tilde{A}_{d_j}\tilde{A}_0O^{-1}\{\tilde{A}_{d_j}\tilde{A}_0\}^\top O+cO)\\&+\sum_{j=1}^{m}\sum_{i=1}^{m}(O\tilde{A}_{d_j}\tilde{A}_{d_i}O^{-1}\{\tilde{A}_{d_j}\tilde{A}_{d_i}\}^\top O+c^2O),\end{aligned}$$

Proof: Let m be the number of total communication links. Then, one infers

$$\dot{x}(t)=\tilde{A}_0x(t)+\sum_{i=1}^{m}\tilde{A}_{d_i}x(t-\iota_{d_i}(t)),\tag{17}$$

where $\tilde{A}_{d_i}$ is the matrix corresponding to the ith communication link. By the Leibniz-Newton formula, we have

$$\begin{aligned}x(t-\iota_{d_i}(t))&=x(t)-\int_{-\iota_{d_i}(t)}^{0}\dot{x}(t+\varsigma)\mathrm{d}\varsigma\\&=x(t)-\int_{-\iota_{d_i}(t)}^{0}(\tilde{A}_0x(t+\varsigma)\\&\quad+\sum_{i=1}^{m}\tilde{A}_{d_i}x(t+\varsigma-\iota_{d_i}(t+\varsigma)))\mathrm{d}\varsigma.\end{aligned}$$

Hence, there holds

$$\begin{aligned}\dot{x}(t)&=\tilde{A}_0x(t)+\sum_{j=1}^{m}\tilde{A}_{d_j}x(t-\iota_{d_j}(t))\\&=\tilde{A}_0x(t)+\sum_{j=1}^{m}\tilde{A}_{d_j}x(t)-\sum_{j=1}^{m}\tilde{A}_{d_j}\tilde{A}_0\int_{-\iota_{d_j}(t)}^{0}x(t+\varsigma)\mathrm{d}\varsigma\\&\quad-\sum_{j=1}^{m}\sum_{i=1}^{m}\tilde{A}_{d_j}\tilde{A}_{d_i}\int_{-\iota_{d_j}(t)}^{0}x(t+\varsigma-\iota_{d_i}(t+\varsigma))\mathrm{d}\varsigma.\end{aligned}$$

As $\tilde{A}$ is Hurwitz, for any $Q\succ 0$ and $Q=Q^\top$, there exists an $O\succ 0$ such that $O\tilde{A}+\tilde{A}^\top O=-Q$. Let $V(x(t))=x(t)^\top Ox(t)$ be the candidate Lyapunov function. We obtain

$$\begin{aligned}\dot{V}(x(t))&=x(t)^\top(O\tilde{A}+\tilde{A}^\top O)x(t)\\&\quad-\sum_{j=1}^{m}2x(t)^\top O\tilde{A}_{d_j}\tilde{A}_0\int_{-\iota_{d_j}(t)}^{0}x(t+\varsigma)\mathrm{d}\varsigma\\&\quad-\sum_{j=1}^{m}\sum_{i=1}^{m}2x(t)^\top O\tilde{A}_{d_j}\tilde{A}_{d_i}\int_{-\iota_{d_j}(t)}^{0}x(t+\varsigma-\iota_{d_i}(t+\varsigma))\mathrm{d}\varsigma.\end{aligned}$$

Let $\tilde{a}^\top=-x(t)^\top O\tilde{A}_{d_j}\tilde{A}_0$ and $\tilde{c}=x(t+\varsigma)$. Since $2\tilde{a}^\top\tilde{c}\leq\tilde{a}^\top\Upsilon\tilde{a}+\tilde{c}^\top\Upsilon^{-1}c$ for any $\Upsilon\succ 0$, there holds

$$\begin{aligned}&-\sum_{j=1}^{m}2x(t)^\top O\tilde{A}_{d_j}\tilde{A}_0\int_{-\iota_{d_j}(t)}^{0}x(t+\varsigma)\mathrm{d}\varsigma\\&\leq\sum_{j=1}^{m}(\iota_{d_j}(t)x(t)^\top O\tilde{A}_{d_j}\tilde{A}_0O^{-1}\{\tilde{A}_{d_j}\tilde{A}_0\}^\top Ox(t)\\&\quad+\int_{-\iota_{d_j}(t)}^{0}x(t+\varsigma)^\top Ox(t+\varsigma)\mathrm{d}\varsigma).\end{aligned}$$

Similarly,

$$\begin{aligned}&-\sum_{j=1}^{m}\sum_{i=1}^{m}2x(t)^\top O\tilde{A}_{d_j}\tilde{A}_{d_i}\int_{-\iota_{d_j}(t)}^{0}x(t+\varsigma-\iota_{d_i}(t+\varsigma))\mathrm{d}\varsigma\\&\leq\sum_{j=1}^{m}\sum_{i=1}^{m}(\iota_{d_j}(t)x(t)^\top O\tilde{A}_{d_j}\tilde{A}_{d_i}O^{-1}\{\tilde{A}_{d_j}\tilde{A}_{d_i}\}^\top Ox(t)\\&\quad+\int_{-\iota_{d_j}(t)}^{0}x(t+\varsigma-\iota_{d_i}(t+\varsigma))^\top Ox(t+\varsigma-\iota_{d_i}(t+\varsigma))\mathrm{d}\varsigma).\end{aligned}$$

Therefore, one obtains

$$\begin{aligned}\dot{V}(x(t))&\leq x(t)^\top(O\tilde{A}+\tilde{A}^\top O)x(t)\\&\quad+\sum_{j=1}^{m}[\iota_{d_j}(t)x(t)^\top O\tilde{A}_{d_j}\tilde{A}_0O^{-1}\{\tilde{A}_{d_j}\tilde{A}_0\}^\top Ox(t)\\&\quad+\int_{-\iota_{d_j}(t)}^{0}x(t+\varsigma)^\top Ox(t+\varsigma)\mathrm{d}\varsigma]\\&\quad+\sum_{j=1}^{m}\sum_{i=1}^{m}(\iota_{d_j}(t)x(t)^\top O\tilde{A}_{d_j}\tilde{A}_{d_i}O^{-1}\{\tilde{A}_{d_j}\tilde{A}_{d_i}\}^\top Ox(t)\end{aligned}$$

$$+\int_{-\iota_{d_j}(t)}^{0} x(t+\varsigma-\iota_{d_i}(t+\varsigma))^\top O x(t+\varsigma-\iota_{d_i}(t+\varsigma))\mathrm{d}\varsigma).$$

We pick a continuous non-decreasing function $\varphi_1(y) = cy$, where $c > 1$ is a constant, and a continuous, non-negative, non-decreasing function $\varphi_2(y) = (\lambda_{\min}(Q) - \bar{\iota}\lambda_{\max}(F))y^2$, where $\lambda_{\min}(Q) - \bar{\iota}\lambda_{\max}(F) > 0$, $\lambda_{\min}(Q)$ is the minimum eigenvalue of Q, and $\lambda_{\max}(F)$ is the maximum eigenvalue of the following matrix

$$F = \sum_{j=1}^{m}(O\tilde{A}_{d_j}\tilde{A}_0 O^{-1}\{\tilde{A}_{d_j}\tilde{A}_0\}^\top O + cO) + \sum_{j=1}^{m}\sum_{i=1}^{m}(O\tilde{A}_{d_j}\tilde{A}_{d_i}O^{-1}\{\tilde{A}_{d_j}\tilde{A}_{d_i}\}^\top O + c^2 O).$$

When $V(x(t+\varsigma)) < \varphi_1(V(x(t)) = cV(x(t))$ for $-\bar{\iota} \le \varsigma \le 0$ and $V(x(t+\varsigma-\iota_{d_i}(t+\varsigma))) < \varphi_1(V(x(t+\varsigma))) = cV(x(t+\varsigma))$ for $-\bar{\iota} \le \iota_{d_i}(t+\varsigma) \le 0$, we have for $-\bar{\iota} \le \varsigma \le 0$, $V(x(t+\varsigma-\iota_{d_i}(t+\varsigma))) < c^2 V(x(t))$, for all $-\bar{\iota} \le \iota_{d_i}(t+\varsigma) \le 0$. Thus, we have

$$\dot{V}(x(t)) \le -(\lambda_{\min}(Q) - \bar{\iota}\lambda_{\max}(F))\|x(t)\|^2 = -\varphi_2(\|x(t)\|),$$

which completes the proof. ■

Remark 5.10: Notice that the upper bound of communication delays depends on IFTs of the vehicle platoon as well as the weights on the communicated information which are elements of matrix $\tilde{A}_{d_i}$ that correspond to the ith communication link, and local states. Although we can obtain analytical expression on the communication delay bound, it is difficult to present it in an explicit way in terms of the parameter setting ranges, as the expression is coupled with many variables. It is interesting to design effective numerical methods to obtain the maximum delay $\bar{\iota}$, which is left as one of our future work.

D. String Stability Analysis

In this part, we investigate whether and how the disturbance will propagate in the above-given vehicle platoon protocol. We first consider the case with perfect communications and then extend the results to the case where time delays are constant and the same on all communication links (uniform).

1) Ideal Communication Cases: By (5), we have

$$e_i(t) = q_{i-1}(t) - q_i(t) - hv_i(t),$$
$$\dot{e}_i(t) = v_{i-1}(t) - v_i(t) - ha_i(t),$$
$$\ddot{e}_i(t) = a_{i-1}(t) - a_i(t) - h\dot{a}_i(t).$$

Then, for $r < i < n-l$, by differentiating input, we get

$$\dot{u}_i^{pr}(t) = \sum_{\eta=1}^{r} A_{i(i-\eta)}(k_q(v_{i-\eta}(t) - v_i(t) - \sum_{j=i-\eta+1}^{i} ha_j(t)) + k_v(a_{i-\eta}(t) - a_i(t)) + k_a(\dot{a}_{i-\eta}(t) - \dot{a}_i(t))),$$

and

$$u_i^{pr}(t) = \sum_{\eta=1}^{r} A_{i(i-\eta)}(k_q(q_{i-\eta}(t) - q_i(t) - \sum_{j=i-\eta+1}^{i} hv_j(t)) + k_v(v_{i-\eta}(t) - v_i(t)) + k_a(a_{i-\eta}(t) - a_i(t))).$$

For vehicle $i-1$, there holds

$$u_{i-1}^{pr}(t) = \sum_{\eta=1}^{r} A_{(i-1)(i-1-\eta)}(k_q(q_{i-1-\eta}(t) - q_{i-1}(t) - \sum_{j=i-\eta}^{i-1} hv_j(t)) + k_v(v_{i-1-\eta}(t) - v_{i-1}(t)) + k_a(a_{i-1-\eta}(t) - a_{i-1}(t))).$$

Then, from $u_{i-1}^{pr}(t) + u_{i-1}^{fl}(t) - (u_i^{pr}(t) + u_i^{fl}(t)) - h(\dot{u}_i^{pr}(t) + \dot{u}_i^{fl}(t))$ and $u_i(t) = \tau\dot{a}_i(t) + a_i(t)$, we can obtain the equation characterizing the relationship between the distance errors as follows

$$\begin{aligned}
&\tau\dddot{e}_i(t) + \ddot{e}_i(t) \\
&= \sum_{\eta=1}^{r} A_{i(i-\eta)}(k_q(e_{i-\eta}(t) - e_i(t) - \sum_{j=i-\eta+1}^{i} h\dot{e}_j(t)) \\
&\quad + k_v(\dot{e}_{i-\eta}(t) - \dot{e}_i(t)) + k_a(\ddot{e}_{i-\eta}(t) - \ddot{e}_i(t))) \\
&\quad + \sum_{\eta=1}^{l} A_{i(i+\eta)}(k_q(e_{i+\eta}(t) - e_i(t) + \sum_{j=i+1}^{i+l} h\dot{e}_j(t)) \\
&\quad + k_v(\dot{e}_{i+\eta}(t) - \dot{e}_i(t)) + k_a(\ddot{e}_{i+\eta}(t) - \ddot{e}_i(t))).
\end{aligned} \tag{18}$$

Taking Laplacian transform on both sides of (18)

$$\begin{aligned}
&\tau s^3 e_i(s) + s^2 e_i(s) \\
&= \sum_{j=i-r, j\ne i}^{i+l}(k_q e_j(s) + k_v s e_j(s) + k_a s^2 e_j(s)) \\
&\quad - (r+l)(k_q e_i(s) + k_v s e_i(s) + k_a s^2 e_i(s)) \\
&\quad - \sum_{j=i-r}^{i-1}\sum_{\eta=j+1}^{i} k_q h s e_\eta(s) + \sum_{j=i+1}^{i+l}\sum_{\eta=i+1}^{j} k_q h s e_\eta(s)
\end{aligned} \tag{19}$$

As a result, we have

$$e_i(s) = \sum_{j=i-r, j\ne i}^{i+l} \mathbf{H}_{ij}(s) e_j(s) \tag{20}$$

where $\mathbf{H}_{ij}$ can be written explicitly as

$$\mathbf{H}_{ij}(s) = \begin{cases} \frac{k_a s^2 + (k_v - k_q h(r-(i-j)))s + k_q}{\tau s^3 + (\phi k_a + 1)s^2 + (\phi k_v + r k_q h)s + \phi k_q}, & i-r \le j \le i-1 \\ \frac{k_a s^2 + (k_v + k_q h(l-(j-i-1)))s + k_q}{\tau s^3 + (\phi k_a + 1)s^2 + (\phi k_v + r k_q h)s + \phi k_q}, & i+1 \le j \le i+l \\ 0, & \text{otherwise} \end{cases} \tag{21}$$

with $\phi = r + l$. For each node $r \leq i \leq n - l$, $\mathbf{H}_{ij}(s)$ only depends on the relative value between j and i.

Definition 5.11: If

$$\|e_i(t)\|_2^2 \leq \frac{1}{r+l} \sum_{j=i-r, j\neq i}^{j+l} \|e_j(t)\|_2^2 \tag{22}$$

where $\|e_i(t)\|_2^2 = \int_{-\infty}^{+\infty} |e_i(t)|^2 \mathrm{d}t$, then the L_2 string stability is achieved.

Next, we consider the following sufficient condition to guarantee the L_2-string stability.

Lemma 5.12: If there holds

$$\|\mathbf{H}_{ij}(\mathrm{i}\omega)\|_\infty \leq \frac{1}{r+l}, \forall j - r \leq j \leq j + l, j \neq i, \tag{23}$$

then the L_2-string stability (22) is achieved.

Theorem 5.13: If one of the following conditions hold for all j

$$\begin{aligned} &1): \ \alpha_2 \geq 0 \ \&; \ \alpha_3 \geq 0, \\ &2): \ \alpha_2 < 0 \ \&; \ \alpha_3 \geq 0 \ \&; \ 4\alpha_3\alpha_1 - \alpha_2^2 \leq 0 \end{aligned} \tag{24}$$

where

$$\begin{aligned} \alpha_1 &= \tau^2 \\ \alpha_2 &= (\phi k_a + 1)^2 - 2\tau(\phi k_v + r k_q h) - \phi^2 k_a^2 \\ \alpha_3 &= (\phi k_v + r k_q h)^2 - 2\phi k_q + \phi^2 g(j) \end{aligned} \tag{25}$$

$$g(j) = \begin{cases} (k_v - k_q h(r-j))^2, & 1 \leq j \leq r \\ (k_v + k_q h(l-j))^2, & 0 \leq j \leq l-1 \end{cases} \tag{26}$$

then the L_2-string stability (22) is achieved.

2) Cases With Constant and Uniform Delays: We hereby analyze the effect of communication delays on the string stability. We consider the scenario where the communication links connecting neighboring agents have constant and uniform delays over time. This is a reasonable assumption. For instance, when delays in communication networks are time-varying but bounded, by referring to [45], they can be made constant by buffering data up to the known worst-case maximum delays with time stamping. By data buffering, the case with time-varying delays is transformed into that with constant time delays, which makes the analysis more tractable.

Suppose the communication time delay is ι on all communication links, we obtain the following Laplacian transform from (20) and (21),

$$e_i(s) = \sum_{j=i-r, j\neq i}^{i+l} \mathbf{H}_{ij}(s)\mathbf{e}^{-\iota s} e_j(s). \tag{27}$$

Then, we can obtain the sufficient conditions on the L_2-string stability for the platoon.

Theorem 5.14: The uniform and constant communication delays have no effect on the string stability of the platoon.

Proof: For $i - r \leq j \leq i - 1$, we have

$$\begin{aligned} \|\mathbf{H}_{ij}(\mathrm{i}\omega)\mathbf{e}^{-\iota \mathrm{i}\omega}\|_\infty^2 &\leq \|\mathbf{H}_{ij}(\mathrm{i}\omega)\|_\infty^2 \|\mathbf{e}^{-\iota \mathrm{i}\omega}\|_\infty^2 \\ &\leq \|\mathbf{H}_{ij}(\mathrm{i}\omega)\|_\infty^2. \end{aligned} \tag{28}$$

For $i + 1 \leq j \leq i + l$, there holds

$$\begin{aligned} \|\mathbf{H}_{ij}(\mathrm{i}\omega)\mathbf{e}^{-\iota \mathrm{i}\omega}\|_\infty^2 &\leq \|\mathbf{H}_{ij}(\mathrm{i}\omega)\|_\infty^2 \|\mathbf{e}^{-\iota \mathrm{i}\omega}\|_\infty^2 \\ &\leq \|\mathbf{H}_{ij}(\mathrm{i}\omega)\|_\infty^2. \end{aligned} \tag{29}$$

As a result, for all $i - r \leq j \leq i - 1$ and $i + 1 \leq j \leq i + l$ when $\|\mathbf{H}_{ij}(\mathrm{i}\omega)\|_\infty^2 \leq \frac{1}{(r+l)^2}$, there must hold

$$\begin{aligned} \|\mathbf{H}_{ij}(\mathrm{i}\omega)\mathbf{e}^{-\iota \mathrm{i}\omega}\|_\infty^2 &\leq \|\mathbf{H}_{ij}(\mathrm{i}\omega)\|_\infty^2 \|\mathbf{e}^{-\iota \mathrm{i}\omega}\|_\infty^2 \\ &\leq \|\mathbf{H}_{ij}(\mathrm{i}\omega)\|_\infty^2 \leq \frac{1}{(r+l)^2} \end{aligned} \tag{30}$$

Thus, under the uniform communication delays, the string stability will not be affected. ■

Remark 5.15: Although by Theorem 5.14, the uniform delay will not affect the string stability of the platoon system, we still have an upper bound on the delay to ensure internal stability which is shown in Theorem 5.9. When the delay increases, the stability performance may degrade. It also should be pointed out that the result only holds when communication delays on all links are the same, which is rather a simplified assumption. Yet, the resulting theoretical conclusion of the effects of delays is of explicit form, and will further facilitate future research on more general cases. Moreover, we only focus on the case when all information are collected through communication links. Meanwhile, when the measurement delays are the same as the uniform communication delays, our results still provide guarantees for string stability of the vehicle platoon. For the case when a part of information is collected by sensors and measurement delays are different, it is hard to analyze the effect of delays on string stability, which is left as future work.

Remark 5.16: Note that we concentrate on the analysis of internal stability and string stability of the platoon with time-invariant and uniform delays. It is interesting and realistic to study how the probabilistic delay affects the stability of the platoon system when we consider the bounded wireless communication range. For the linear system model with a constant packet drop rate on all communication links, we can define the second-moment stability to characterize the system stability based on [20]. For the linear system model with the time-independent packet drop rate, the system stability analysis can be approached through Markov jump linear system theory. Furthermore, our analysis for the string stability cannot be applied to the case with time-varying or non-uniform delays directly since it is hard to construct the spacing error $e_i(t)$ through the control input with delayed information explicitly. There are two possible ways to solve this problem, one is to define the string stability through the acceleration variable, and then we can analyze the string stability according to [18]. Another way is to apply the L_p-string stability definition for spacing errors by constructing the spacing error variable through the H_∞ control theory [27].

VI. SIMULATION RESULTS

In this section, we validate the theoretical results and investigate the platoon system performance through extensive simulation studies. We consider a single vehicle platoon with $1 + n$ vehicles. In each time slot, each vehicle in the platoon

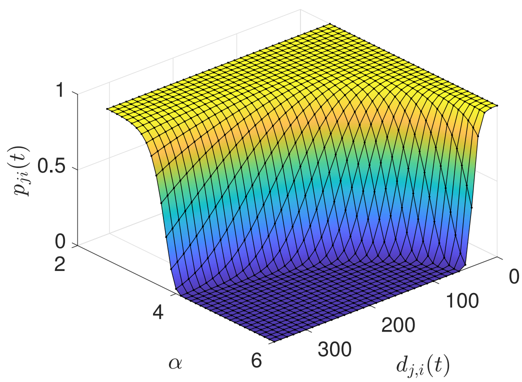

Fig. 2. Packet delivery successful probability.

follows the linear model (2) and uses the control law (7) and the state information from up to r preceding cars and l following cars to adjust its acceleration. Each vehicle also broadcasts its own state information including position, velocity, and acceleration to its neighbors (l cars in front of it and r cars behind it). The communication suffers from random packet losses that depend on the relative distance between vehicles, and the packet loss probability is set inspired by (3), where the parameters in (3) are as follows: $P_{b,i}(t) = 200\text{mw}$, $N_0 = 10^{-17.4} \times 1.8 \times 10^5$, $\alpha \in [2.6, 6]$, $K_0 = 10^{-4.38}$, $d_{j,i}(t) = |d_{j,i}(t)| \in [0350]$. Under these parameters, the successful packet delivery probability p_{ji} for all $(j,i) \in \tilde{E}$ as a function of the relative distance $d_{j,i}(t)$ and the parameter α is shown in Fig. 2. As the relative distance between communicating neighboring vehicles does not change much, we simply set p_{ji} for all $(j,i) \in \tilde{E}$ to be time invariant in our simulations.

The parameters in the vehicle model are as follows: $k_a = 9$, $k_v = 10$, $k_q = 5$, and $\tau = 0.08$. We consider three vehicle platoons of different sizes.

VP1: $1 + n = 5$, $r = 2$ and $l = 2$. The initial conditions are set to be: $q(0) = [500\ 400\ 300\ 200\ 100]^\top$, $v(0) = [13\ 12\ 12\ 11\ 10]^\top$, and $a_i(0) = 0$ for all $i \in V$.

VP2: $n + 1 = 13$, $r = 4$ and $l = 3$. The standstill distance is set to be $15\ m$. The maximum allowable acceleration is 2.3m/s^2[44]. The initial conditions are $q(0) = [240\ 220\ 200\ 180\ 160\ 140\ 120\ 100\ 80\ 60\ 40\ 20\ 0]^\top$, $v_i(0) = 10\text{m/s}$ for all $i \in V$, and $a_i(0) = 0$ for all $i \in V$.

VP3: $n + 1 = 25$, $r = 4$, and $l = 3$. The standstill distance is 15m. The maximum acceleration is 2.3m/s^2. The initial relative distance between all physical neighboring vehicles is set to be 20m, and $a_i(0) = 0$ for all $i \in V$.

Due to the space limitation, we omit the simulink models of VP2 and VP3. We consider the following three different scenarios: **1)** The vehicle platoon moves at a constant speed with unbounded acceleration and continuous communication, and the communication environment is assumed to be ideal; **2)** The communication happens periodically and suffers from packet losses. Each vehicle has a bounded acceleration. We investigate the braking process of the vehicle platoon with different packet losses settings; **3)** For the models where vehicles have bounded accelerations, we investigate the platoon braking process and the process of moving at a constant speed where there exist time-varying communication delays.

A. *Internal Stability for Perfect Communication Networks*

When VP1 moves at a constant speed, the set of eigenvalues of matrix C, that of matrix C_0 with $h = 0.1$ and $h = 1$, respectively, are shown in Fig. 3(a). When all eigenvalues of C_0 have negative real parts, the eigenvalues of C have the same property, which indicates that there exist parameters to guarantee the internal stability of the platoon. Thus, Theorem 5.4 is validated. Moreover, variations of all eigenvalues become larger when h grows, which verifies the results of Theorem 5.5. Meanwhile, Fig. 3(b) and 3(c) indicates that when $h = 1$, the internal stability is achieved, which demonstrates our theoretical results. In addition, under the perturbation of Δ, variations of eigenvalues of C_0 that are far away from the imaginary axis are negligible compared with those close to the imaginary axis. Although a smaller headway time h improves the internal stability of the vehicle platoon, it may cause the propagation of small perturbations within the string of the vehicles. In the following, we further investigate the platoon performance with realistic communication impairments.

B. *Effects of Random Packet Drops*

We investigate how the random packet drops affect the performance of the braking process of the vehicle platoon. Let the sampling period/communication rate be 10 ms. To show a clear picture of the impact of packet losses, we set the transmission success probability to be the same for all pairs of communicating vehicles, i.e., $p_{ij} = 0.8$. It is shown in Fig. 4 that all cars stop quickly and the distance between neighboring vehicles converges to the standstill value. We also observe that the random packet drops introduce disturbances with a high-frequency and small magnitude to the transmission signal. When we increase the sampling period to 100 ms and keep the probabilities of successful packet transmission on all communication links the same, we find from Fig. 5(a) that the stability is still guaranteed. However, the available spacing errors based on communication information experience high-frequency fluctuations. When we reduce the sampling period to 50 ms, and with the same successful packet transmission probability, we observe from Fig. 5(a) that the upper and lower bounds of the available space error become smaller.

We hereby consider the case when the packet drop probability increases with the relative distance. Let the successful packet transmission probability between two vehicles be $0.9 - 0.1\ k$ where $k \in \{0, 1, 2, 3\}$ is the number of vehicles between them. From Fig. 5(c), we observe that all velocities of vehicles converge to zero relatively quickly during the braking process of the platoon. For VP3, we observe in Fig. 5(d) that when the packet drop probability grows to $0.9 - 0.1\ k$ where $k \in \{0, 1, 2, 3\}$, although the response curve suffers from more fluctuations during the deceleration process, all vehicles can stop with constant small relative distance. It is noted that the problem of the platoon with random packet losses can be viewed as unbounded time-varying delays. When the communication delays can be bounded by an upper bound in a large probability, the reliability

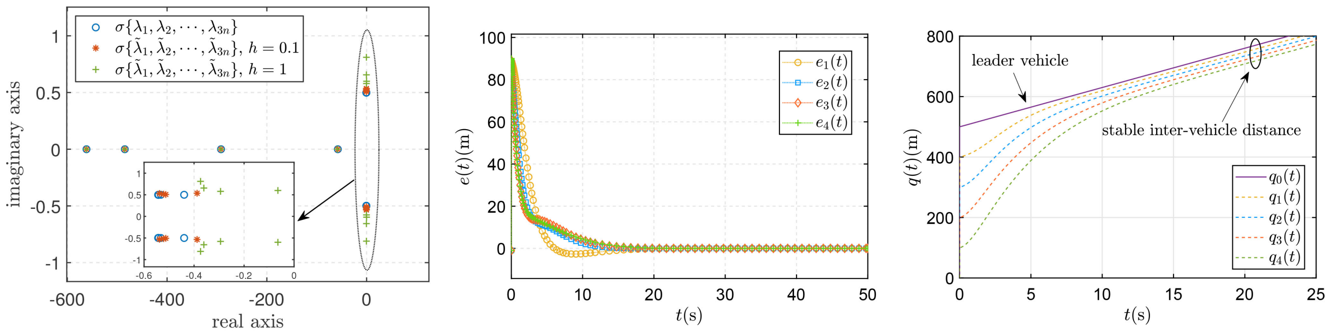

Fig. 3. Positions stability $h = 1$ of VP1 with perfect communication networks. (a) Eigenvalues distribution under perturbations. (b) Spacing error stability. (c) Position stability.

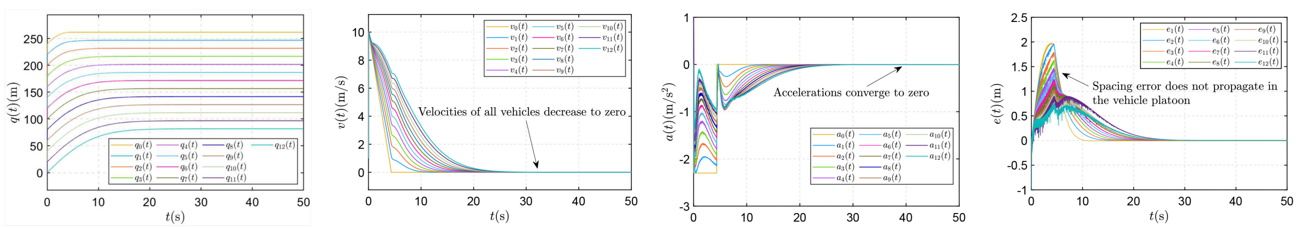

Fig. 4. Performance for VP2 with $h = 0.5$, $T_s = 10\,\text{ms}$, $p_{ij} = 0.8$. (a) Positions of all vehicles. (b) Velocities of all vehicles. (c) Accelerations of all vehicles. (d) Available space errors.

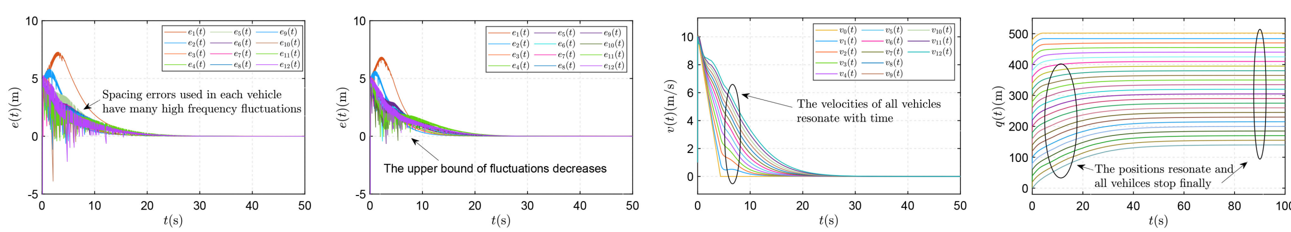

Fig. 5. Different packet delivery successful probability settings. (a) $p_{ij} = 0.8$, $T_s = 100\,\text{ms}$. (b) $p_{ij} = 0.8$, $T_s = 50\,\text{ms}$. (c) $p_{ij} \in \{0.9, 0.8, 0.7, 0.6\}$. (d) $p_{ij} \in \{0.9, 0.8, 0.7, 0.6\}$.

of the platoon is still high and thus Theorem 5.9 can still be used to guarantee the performance of the platoon with random packet losses.

C. Effect of Communication Delays

In this subsection, we investigate how bounded time-varying delays affect the control performance of VP2 and VP3, where two cases for the leader vehicle are considered, i.e., the braking process and moving at a constant speed.

1) Braking Process: First, the delays on the communication links are uniformly selected from the interval [0200] ms, and the sampling time is set to 50 ms. We consider the scenario where the vehicle platoon moves forward stably and suddenly the leader vehicle starts to brake. From Fig. 6(a) and Fig. 6(b), we observe that all vehicles stop in 30 seconds and all relative distances converge to the standstill distance and no collisions happen. The reason is that when the delay is small, the available information is relatively reliable and thus the stability can be ensured. Then, we increase the sampling time to 100 ms and the communication delay upper/lower bounds to be 10 s/1 s. From Fig. 6(c), we observe that the second vehicle stops very close to the leader vehicle, which is undesirable. Moreover, vehicles in the front and rear of the platoon cannot keep the desired relative distance with their predecessors, and vehicles in the middle have constant but larger than the standstill distance with their predecessors. Under the same time-varying delays setting, we also investigate the performance of the vehicle platoon of the same size. The difference is that now each vehicle only uses 4 predecessors' information for longitudinal control. From Fig. 6(d), we find that with only predecessors' information, when the delays vary from 1 s to 10 s, the crash happens with local longitudinal control and

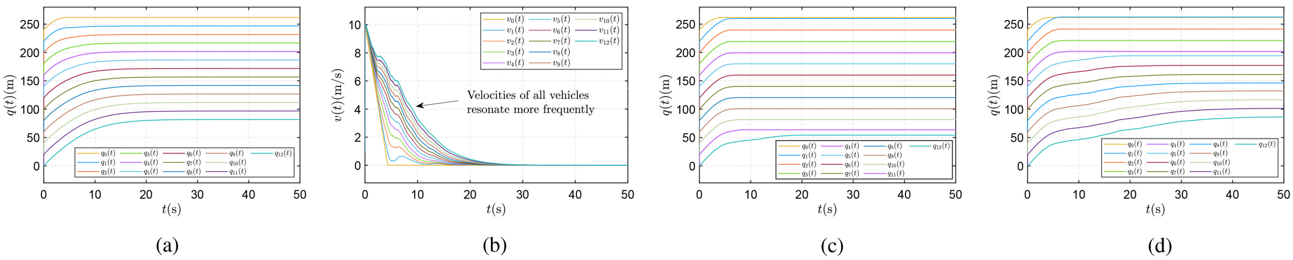

Fig. 6. Platoon stability with $\bar{\iota} = 200$ ms & Comparison of the effect of delays $\iota(t) \in [1, 10]$ s on different IFTs. (a) $q(t)$ for $\bar{\iota} = 200$ ms. (b) $v(t)$ for $\bar{\iota} = 200$ ms. (c) $r = 4$, $l = 3$. (d) $r = 4$, $l = 0$.

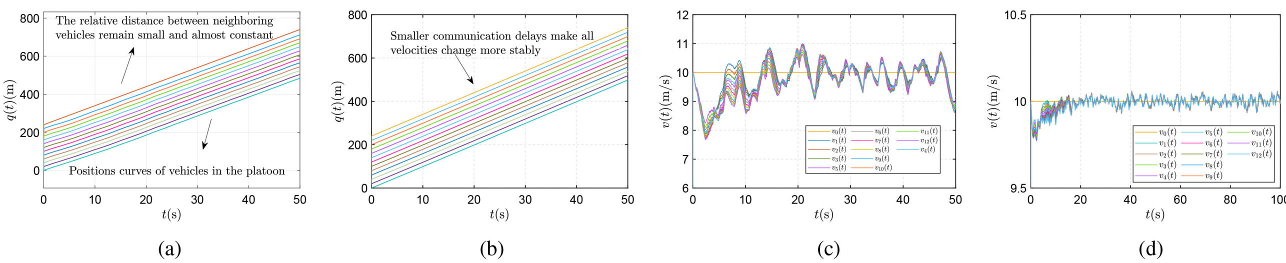

Fig. 7. Effect of different delays on the platoon moving with a constant speed. (a) $q(t)$ for $\bar{\iota} = 0.2$ s. (b) $q(t)$ for $\bar{\iota} = 0.02$ s. (c) $v(t)$ for $\bar{\iota} = 0.2$ s. (d) $v(t)$ for $\bar{\iota} = 0.02$ s.

longer time are required for all vehicles to stop. By using both predecessors and followers' information, the vehicle platoon has a quicker response and are more reliable when there are communication delays.

2) Leader Vehicle With a Constant Speed: We hereby investigate the effect of time-varying delays on the vehicle platoon where the leader vehicle moves with a constant speed. We set the sampling time to 10 ms and the maximum communication delay to 20 ms, and the sampling time to 50 ms and the maximum communication delay to 200 ms. Fig. 7 shows that even small communication delays cause large disturbances in the platoon. When there are upper and lower bounds for the acceleration, the platoon control will be more sensitive to time-varying communication delays.

VII. Conclusion

In this paper, we explored the performance of the vehicle platoon with limited communication range (where each vehicle uses the information of multiple predecessors and followers), time-varying communication delays, and random lossy links. We found that when the leader vehicle moves at a constant speed, the internal stability of the platoon can be viewed as the stability of a block companion matrix, which is difficult to solve. Through matrix eigenvalue perturbation theory, a sufficient condition was established on the system parameter gains and IFTs to ensure the internal stability of the vehicle platoon, which shows the effect of the limited communication range based IFTs on the control performance of the platoon. Then, we found that when the variation of the disturbance is smooth, the platoon system still achieves high reliability even with high packet delivery loss probability. Moreover, we showed that there always exists a sufficiently small upper bound for the time-varying communication delays such that the internal stability can always be achieved. The L_2-string stability was also investigated for both ideal and uniform constant delay cases. For future work, we will design the controller with safety constraints and analyze its dynamic analysis. Moreover, how to using more realistic communication simulation tools to validate the performance of the vehicle platoon with limited communication range as well as the non-ideal communication is another important direction. Handling the model with onboard sensors measurements and is also an important research issue that beckons further investigation.

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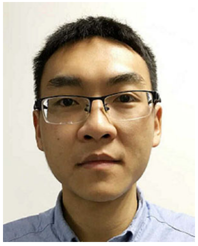

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