Low-Complexity Reversible Integer-to-Integer Wavelet Transforms for Image Coding*

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Abstract

A simple exhaustive search technique is explored as a means to design low-complexity reversible integer-to-integer wavelet transforms for image coding applications. Several new transforms found with this approach are employed in an image coder in order to demonstrate their effectiveness.

1. Introduction

There has been a growing interest in reversible integer-to-integer wavelet transforms for image coding applications [1–7]. Such transforms are invertible in finite-precision arithmetic (i.e., reversible), map integers to integers, and approximate the linear wavelet transforms from which they are derived. In this paper, we employ a simple exhaustive search technique to find good low-complexity reversible integer-to-integer transforms for image coding applications. The transforms obtained are then employed in an image coding system to demonstrate their effectiveness.

2. Transforms

The fundamental building block of a wavelet transform is the uniformly maximally-decimated filter bank (UMDFB). In order to design a wavelet transform, we need only construct its corresponding UMDFB. To obtain transforms that can be made reversible and map integers to integers, we need to realize the UMDFB using the lifting framework [8,9]. When this framework is employed, a UMDFB is realized in its polyphase form with the polyphase filtering performed by a ladder network. This leads to the general structure shown in Figure 1. It is easy to see that the perfect reconstruction (PR) property is always satisfied by this type of structure. By rounding the outputs of the ladder step filters to integers, we obtain reversible integer-to-integer mappings. Since the synthesis side of the UMDFB is completely determined by the analysis side, in the discussion which follows, only the analysis side is considered. As a matter of notation, we denote the lowpass and highpass analysis filter transfer functions as $H_0(z)$ and $H_1(z)$, respectively. The number of ladder steps is denoted as N. The kth ladder step filter has transfer function $A_k(z)$ and length L_k.

Since we are concerned with the application of image coding, it is desirable to constrain the UMDFBs of interest to those with linear-phase filters. In the case of 2-channel FIR PR UMDFBs, there are two possibilities for linear-phase filters which are of practical interest:

1. Both analysis filters are of odd length and have symmetric impulse responses.
2. Both analysis filters are of even length, with the lowpass and highpass analysis filters having symmetric and antisymmetric impulse responses, respectively.

All UMDFBs of the first type can be generated (up to a scaling and delay factor) by choosing the $A_k(z)$ as

$$A_k(z) = \begin{cases} \sum_{i=0}^{L_k/2-1} \alpha_k[i](z^{-i} + z^{i+1}) & k \text{ even} \\ \sum_{i=0}^{L_k/2-1} \alpha_k[i](z^{-i-1} + z^{i}) & k \text{ odd} \end{cases} \tag{1}$$

where the L_k are even. A subset of the UMDFBs of the second type can be generated by choosing the $A_k(z)$ as

$$A_k(z) = \begin{cases} -1 & k = 0 \\ \frac{1}{2} + \sum_{i=1}^{(L_k-1)/2} \alpha_k[i](z^{-i} - z^{i}) & k = 1 \\ \sum_{i=1}^{(L_k-1)/2} \alpha_k[i](z^{-i} - z^{i}) & k \geq 2 \end{cases} \tag{2}$$

where the L_k are odd.

3. Design Method

To date, many criteria have been suggested for UMDFB design. For image coding applications, however, the following criteria have proven to be particularly useful: coding gain, analysis filter frequency selectivity, the number of vanishing moments of the analyzing and synthesizing wavelet functions, and the smoothness of the synthesizing scaling and wavelet functions. Moreover, experimental results suggest that UMDFBs which are effective for image coding generally satisfy the following conditions:

$$\begin{aligned} |H_0(e^{j0})| \neq 0 \quad \text{and} \quad |H_0(e^{j\pi})| = 0, \\ |H_1(e^{j\pi})| \neq 0 \quad \text{and} \quad |H_1(e^{j0})| = 0, \end{aligned} \tag{3a}$$

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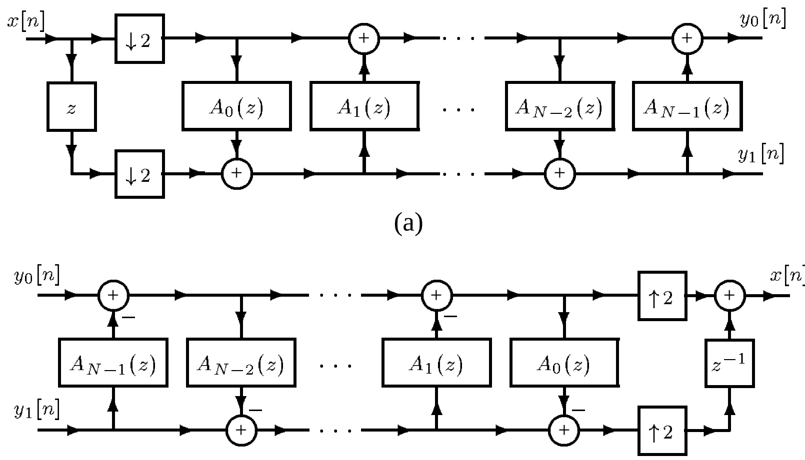

Figure 1: Lifting realization of a UMDFB. (a) Analysis side. (b) Synthesis side.

$$\tilde{\eta} \geq 2 \quad \text{or} \quad \eta \geq 2, \tag{3b}$$

$$S_{H_0} \leq 0.25 \quad \text{and} \quad S_{H_1} \leq 0.25 \tag{3c}$$

$$G_6 \geq 9 \tag{3d}$$

where

$$S_{H_0} = \int_0^{3\pi/8} \left(|H_0(e^{jw})| - |H_0(e^{j0})|\right)^2 dw + \int_{5\pi/8}^{\pi} |H_0(e^{jw})|^2 dw,$$

$$S_{H_1} = \int_0^{3\pi/8} |H_1(e^{jw})|^2 dw + \int_{5\pi/8}^{\pi} \left(|H_1(e^{jw})| - |H_1(e^{j\pi})|\right)^2 dw,$$

G_6 is the coding gain of the sixfold iterated UMDFB, and $\tilde{\eta}$ and η are the number of vanishing moments of the analyzing and synthesizing wavelet functions, respectively. The above conditions lead us to propose a simple design algorithm based on an exhaustive search as described below.

Suppose that we choose a small finite set of admissible values for the coefficients of the filters A_k. Given the type of ladder network and its associated parameters, namely the number of ladder steps N, and the filter lengths L_k, we can perform an exhaustive search of the solution space, seeking UMDFBs which satisfy the conditions given by (3). Any UMDFB satisfying these conditions can be taken as an output of the design process.

Clearly, the above design technique has one severe limitation. The size of the solution space must be kept small enough that an exhaustive search is feasible. Hence, we can only use such an approach for small design problems. As we are interested in low-complexity transforms, however, we wish to keep the number of ladder step filters and their lengths to a minimum. Provided that the number of admissible filter coefficient values is also kept small enough, the resulting design problem is computationally tractable.

4. Design Examples

Our design method was applied to several filter bank configurations. The filter coefficient values were selected to either be powers of two or dyadic rational numbers. Numerous transforms were obtained. Several of the more promising new solutions are listed in Table 1. Note that the 5/11 and 6/14 transforms have strictly power-of-two filter coefficients. A number of important parameters for the various transforms are listed in Table 2. As can be seen from the table, these transforms have both high coding gain and good frequency selectivity. In addition to the new transforms obtained, our design method also found many of the more performant transforms already described in the literature, including the $(2, 2)$, $(2 + 2, 2)$, $(4, 2)$, and $(4, 4)$ transforms of [8], the TS transform of [10], and the 6/14-CRF and 13/7-CRF transforms of [11].

5. Coding Results

To demonstrate the effectiveness of the new transforms, they were employed in the JPEG-2000 image coder (or, more precisely, version 0.0 of the JPEG-2000 verification model software) [12]. Two grayscale images from the JPEG-2000 test set [13] (e.g., `gold` and `finger`) were used as input

Table 1: Transforms

Transform	
5/11	$A_0(z) = \frac{1}{2}(-z-1)$ $A_1(z) = \frac{1}{4}(1+z^{-1})$ $A_2(z) = \frac{1}{32}(z^2 - z - 1 + z^{-1})$
6/14	$A_0(z) = -1$ $A_1(z) = \frac{1}{32}(-z+16+1)$ $A_2(z) = \frac{1}{32}(z^2 - z + z^{-1} - z^{-2})$
13/7	$A_0(z) = \frac{1}{32}(3z^2 - 19z^1 - 19 + 3z^{-1})$ $A_1(z) = \frac{1}{32}(-z + 5 + 5z^{-1} - z^{-2})$

Table 2: Transform parameters

Transform	$\tilde{\eta}$	η	G_6	S_{H_0}	S_{H_1}
5/11	2	2	9.603	0.165	0.131
6/14	3	1	9.713	0.075	0.072
13/7	2	2	9.729	0.025	0.034

data. The lossy and lossless results for the various transforms are given in Tables 3 and 4, respectively. For comparison purposes, we also include the results obtained with the well-known $(2+2,2)$ transform of [8] and TT transform of [3]. Clearly, the new transforms perform quite well for both lossy and lossless compression. In the case of lossy compression, the subjective image quality obtained with the new transforms is also good, as demonstrated by the example depicted in Figure 2.

6. Conclusions

A simple exhaustive search technique was explored as a means to design low-complexity reversible integer-to-integer wavelet transforms for image coding. Several good transforms were obtained. Two of these transforms have strictly power-of-two filter coefficients which is attractive for shift-and-add implementations. As lossy and lossless compression results demonstrate, these transforms are all particularly effective, often outperforming the well-known $(2+2,2)$ and TT transforms.

Table 3: Lossy compression results for the (a) `finger` and (b) `gold` images

(a)

Bit Rate (bpp)	PSNR (dB) 5/11	6/14	13/7	(2+2,2)	TT
0.0625	19.93	20.44	20.31	19.90	20.36
0.125	21.52	21.93	21.90	21.55	21.88
0.250	24.15	24.41	24.64	24.27	24.37
0.500	27.66	27.56	28.27	27.91	27.57
1.000	31.51	31.08	32.06	31.77	31.13
2.000	37.52	36.92	37.91	37.77	36.99

(b)

Bit Rate (bpp)	PSNR (dB) 5/11	6/14	13/7	(2+2,2)	TT
0.0625	27.21	27.36	27.33	27.18	27.27
0.125	28.99	29.13	29.12	28.92	29.00
0.250	31.21	31.25	31.26	31.13	31.13
0.500	33.85	33.82	33.85	33.75	33.67
1.000	37.29	37.14	37.16	37.15	36.95
2.000	42.34	42.15	41.85	42.10	42.04

Table 4: Lossless compression results

Image	Bit Rate (bpp) 5/11	6/14	13/7	(2+2,2)	TT
`finger`	5.380	5.457	5.305	5.323	5.415
`gold`	4.467	4.488	4.483	4.472	4.498

References

[1] A. R. Calderbank, I. Daubechies, W. Sweldens, and B.-L. Yeo, "Lossless image compression using integer to integer wavelet transforms," in *Proc. of IEEE International Conference on Image Processing*, Santa Barbara, CA, USA, Oct. 1997, vol. 1, pp. 596–599.

[2] A. Said and W. A. Pearlman, "An image multiresolution representation for lossless and lossy compression," *IEEE Trans. on Image Processing*, vol. 5, no. 9, pp. 1303–1310, Sept. 1996.

[3] M. J. Gormish, E. L. Schwartz, A. F. Keith, M. P. Boliek, and A. Zandi, "Lossless and nearly lossless compression of high-quality images," in *Proc. of SPIE*, San Jose, CA, USA, Mar. 1997, vol. 3025, pp. 62–70.

[4] F. Sheng, A. Bilgin, P. J. Sementilli, and M. W. Marcellin, "Lossy and lossless image compression using reversible integer wavelet transforms," in *Proc. of IEEE International Conference on Image Processing*, Chicago, IL, USA, Oct. 1998, vol. 3, pp. 876–880.

[5] M. D. Adams, "Reversible wavelet transforms and their application to embedded image compression," M.A.Sc. thesis, Department of Electrical and Computer Engineering, University of Victoria, Victoria, BC, Canada, Jan. 1998, Available from `http://www.ece.ubc.ca/~mdadams`.

[6] M. D. Adams and F. Kossentini, "Performance evaluation of several reversible integer-to-integer wavelet transforms in the JPEG-2000 baseline system (VM release 0.0)," ISO/IEC JTC 1/SC 29/WG 1 N 866, June 1998, Available from `http://www.ece.ubc.ca/~mdadams`.

[7] M. D. Adams and F. Kossentini, "Evaluation of reversible integer-to-integer wavelet transforms for image compression," in *Proc. of IEEE International Conference on Image Processing*, Oct. 1999.

[8] A. R. Calderbank, I. Daubechies, W. Sweldens, and B.-L. Yeo, "Wavelet transforms that map integers to integers," *Applied*

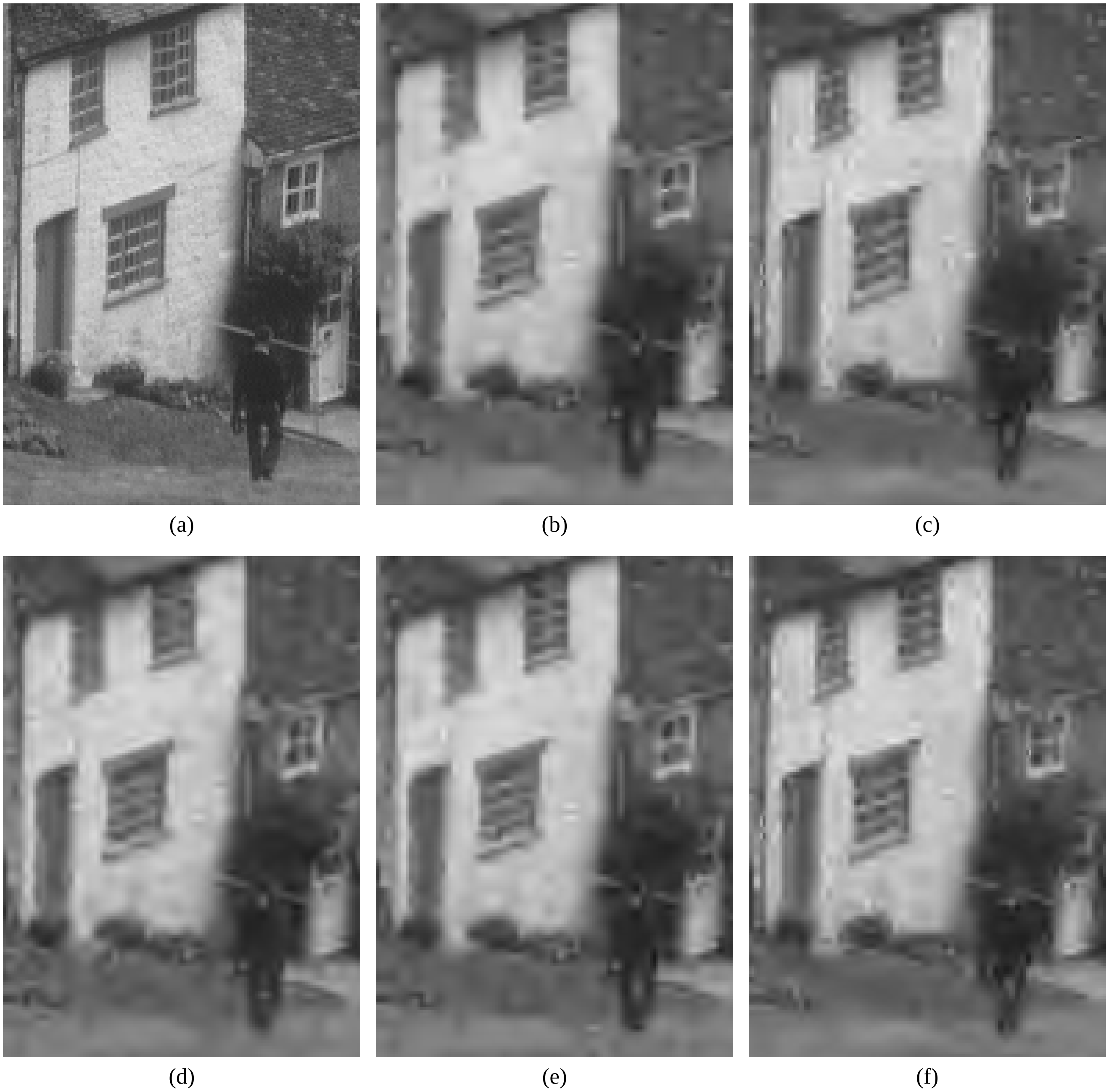

Figure 2: Part of the `gold` image. (a) Original. Lossy reconstruction at a bit rate of 0.125 bpp using the (b) 5/11, (c) 6/14, (d) 13/7, (e) (2+2,2), and (f) TT transforms.

and Computational Harmonic Analysis, vol. 5, no. 3, pp. 332–369, July 1998.

[9] W. Sweldens, "The lifting scheme: A new philosophy in biorthogonal wavelet constructions," in *Proc. of SPIE*, San Diego, CA, USA, Sept. 1995, vol. 2569, pp. 68–79.

[10] A. Zandi, J. D. Allen, E. L. Schwartz, and M. Boliek, "CREW: Compression with reversible embedded wavelets," in *Proc. of IEEE Data Compression Conference*, Snowbird, UT, USA, Mar. 1995, pp. 212–221.

[11] B. Berthelot, E. Majani, and P. Onno, "Transform core experiment: Compare the SNR performance of various reversible integer wavelet transforms," ISO/IEC JTC 1/SC 29/WG 1 N 902, June 1998.

[12] SAIC and University of Arizona, "JPEG-2000 VM0 software," ISO/IEC JTC 1/SC 29/WG 1 N 840, May 1998.

[13] "JPEG-2000 test images," ISO/IEC JTC 1/SC 29/WG 1 N 545, July 1997.