A FAST FEM–BASED FIELD OPTIMIZATION USING ANALYTICALLY CALCULATED GRADIENTS

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Abstract This paper describes an efficient gradient-based optimization algorithm for the FEM. In contrast to the standard approach, which uses a finite difference calculation to find the gradient of a cost function, the gradient is calculated analytically. Hence only a single FEM analysis is required and no mesh re-adjustment is necessary. This saves not only computer memory but reduces computer time significantly. The analytically calculated gradient is exact and singularities (like in the finite difference technique) do not occur. To illustrate the approach a simple H-plane filter is used heading.

Introduction

Optimization is becoming an indispensable part in the overall design of modern microwave circuits. A large variety of optimization algorithms have been proposed in the past. They can be roughly divided into two groups: deterministic and stochastic algorithms (random search, genetic algorithms, simulated annealing etc.). Depending on the investigated structure the various methods have advantages and disadvantages. In this paper we deal with deterministic algorithms which can also be subdivided into two groups namely gradient-based and non gradient-based algorithms. In contrast to optimization with generic algorithms, gradient based optimization programs (i.e. Newton algorithms) require the gradients of the cost or fitness function with respect to all variables (i.e. geometry parameters). Usually these gradients are calculated by finite differences: the cost function is evaluated two times with one parameter changed and the difference then divided by the increment of the parameter.
If the analysis of a microwave structure is performed by FEM, the problem is formulated in terms of the familiar matrix equation

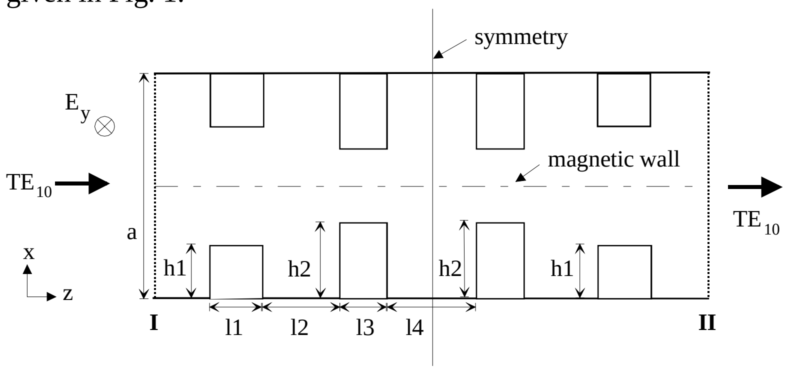
$$[K]\{x\}=\{b\} \qquad (1)$$

Obviously, to determine the gradient of the cost function, two matrix inversions are required and two mesh settings. The combination of both may lead to unacceptably high computation time. Furthermore, using finite differences for gradient calculation of the cost function of high-Q resonating structures may lead to a loss of accuracy (singularities) in the vicinity of resonances.

In this paper we will show, how FEM-based optimization can be accomplished without using finite differences for gradient computation of a cost function. The basic idea has been introduced in [1] using the CIET. In this paper we extend this approach to the FEM and illustrate its application by optimizing an H–plane iris filter.
The key to this new approach is a scattering problem representation of the whole microwave structure of the form given in (1). This matrix representation is common to both the FEM or MoM. [K] is an M x M matrix which depends on the independent variables and represents the structure to be optimized, [b] is the excitation and [x] is the response. For example, the vector [x] contains the expansion coefficients in the MoM or the nodal values in FEM. It can be shown that, as long as the partial derivatives of the matrix [K] and the excitation [b] are known analytically, all sensitivities can be determined analytically. This approach has been successfully tested with the MoM applied to waveguide filters and is now extended to the FEM. The advantages of this optimization approach are obvious: no network representation is needed; only one cost function evaluation instead of two for each gradient calculation; higher accuracy compared to a finite difference scheme in particular in the vicinity of resonances.

Analysis

The structure to be optimized is a Ka–band bandpass filter shown in Fig. 1. The same structure has been investigated in Ref. [3] using the Mode Matching Technique (MMT).
The filter consists of a lossless rectangular waveguide of cross section a x b and four inductive H–plane irises. The structure is symmetric and the lengths and heights and distances of the irises are given in Fig. 1.



a = 7.112 mm
l1 = 1.45 mm
l2 = 4.15 mm
l3 = 4.70 mm
h1 = 1.75 mm
h2 = 2.35 mm
h1, h2 are variable

Fig. 1: Bandpass filter for the Ka-band

Assuming TE_{10} modes incidence on the discontinuity, the reflected and transmitted waves and the waves at the discontinuities are described in terms of TE_{m0} modes and thus only 2D-FEM analysis is needed. We are interested in S_{11} and S_{21} at the cutting planes I and II. The fields for this so called H–plane problem can be described by the scalar Helmholtz equation:

$$\nabla E_y + k_0^2 E_y = 0 \tag{2}$$

The associated boundary conditions (b. c.) are:

Dirichlet b. c.: $E_y = 0$ at wave guide walls (perfect conductors) (3)

Neumann b. c.: $\dfrac{\partial E_y}{\partial n} = 0$ at symmetry axis x=a/2 (magnetic wall) with n=normal vector (4)

Considering the homogeneous Neumann b. c. (Fig. 1) the problem can be reduced by half.
At the front and back of the filter, artificial walls have to be installed to cut the domain. They have to be far enough from the irises so that at the input and output only the fundamental mode propagates.

$$\frac{\partial E_y}{\partial z} + jk_z E_y = jk_z \underline{E}_0 \sin(k_x x) \text{ at plane I,} \qquad \frac{\partial E_y}{\partial z} + jk_z E_y = 0 \text{ at plane II,} \quad k_0^2 = k_x^2 + k_z^2 \tag{5}$$

The functional associated with that problem can be found in the literature, Ref. [4].
The next step in the FEM is the division of the domain into a number of two-dimensional elements, i.e. triangular elements (mesh generation). Some further steps have to be taken (they are described in the literature) to get the elemental matrices $\left[K^e\right]$, they are 3 x 3 matrices, e is the triangle number in the mesh. The elements $K_{i,j}^e$ are functions of the x– and z– coordinates of the triangles and the triangle area:

$$K_{i,j}^e = fct(\Delta_e, x_i, x_j) \tag{6}$$

The resulting system of equations is shown in equation (1). The system matrix $\left[K\right]$ is assembled of all $K_{i,j}^e$. Solving this matrix equation yields the field E_y at all triangle nodes. S_{11} at plane I can be calculated from the nodal values at that location:

$$S_{11} = fct(E_y(plane\ I)), \quad S_{21} = \sqrt{1 - S_{11}^2} \tag{7}$$

Optimization

The goal is to optimize the filter response to a desired specification. Here the openings of the irises h_1 and h_2 are the variables. The cost function $F = \sum_{frequency} fct\left|S_{11}^{calculated} - S_{11}^{goal}\right|$ has to be minimized.

This can be done by a gradient based optimization routine like "constr" of the Matlab™ Optimization Toolbox [5]. "constr" is suited to find a minimum of a constraint nonlinear multivariable function, here the cost function F. As mentioned in [1], during the minimization process the gradients of S_{11} are needed (they can be supplied to the routine "constr"). The choice of the starting values of the variables is important because gradient optimizers run easily into local minima. Good starting values can be obtained from a filter synthesis.

Analytical evaluation of the gradients of S_{11}:

The derivatives of S_{11} are functions of the derivatives of the E_y –field at plane I: $\partial\{E_y\}/\partial h_i$.

They can be obtained by solving: $[K]\partial\{E_y\}/\partial h_i = \partial\{b\}/\partial h_i - \partial[K]/\partial h_i \{E_y\}$ (8)

To do this, the gradients of the matrix elements with respect to the heights h_1 and h_2 have to be determined. This is illustrated in the following section.

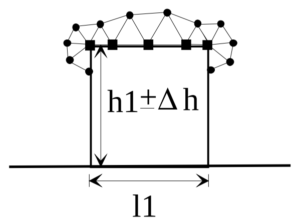


Fig. 2: H-plane stub with triangular mesh around

In Fig. 2 one stub of the filter with triangular mesh is shown. The height h_1 is variable. By changing the height h_1, the nodes marked with a square are moving up or down. The nodes marked with a circle stay fix, all triangles shown in the figure will change their area. According to equation (6) the matrix elements of [K] are functions of the triangle areas and coordinates of the nodes.

Procedure:

1. Substitute the values of the x–coordinates of the nodes marked with square: $x_i \to x_i \frac{h_1}{x_i}$
2. Do the same for the other stubs.
3. Take the derivatives of all matrix elements that are related to the nodes marked with a square or a circle with respect to h1 and h2 (i.e. with Mathematica™).
4. Put the results in matrices $\partial[K]/\partial h_i$.
5. Solve (8) for $\partial\{E_y\}/\partial h_i \to \partial S_{11}/\partial h_i \to \partial F/\partial h_i$.
6. Supply the optimization routine with analytically calculated gradients of $\partial F/\partial h_i$.

The optimization process is shown in the following block diagram:

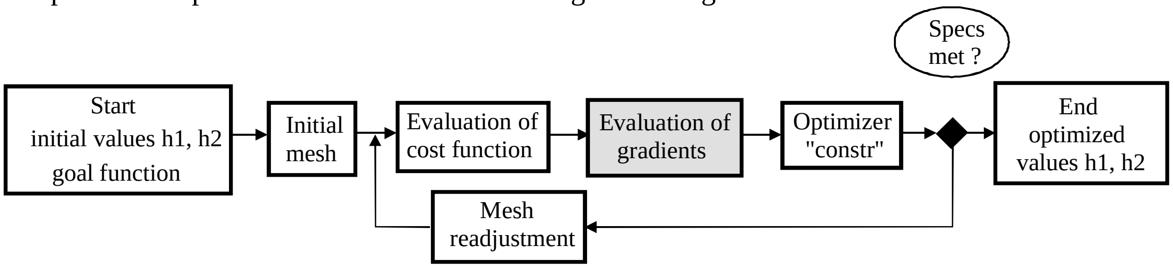


Fig.3: Optimization process

The triangular mesh can be generated automatically with the Partial Differential Equation Toolbox of Matlab.

Results

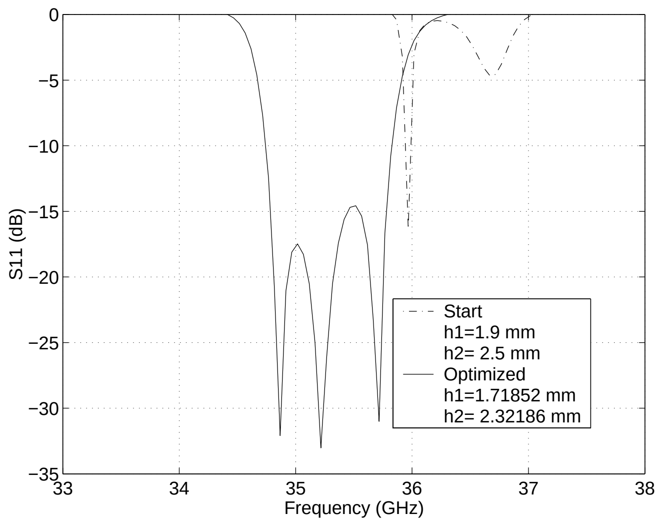


Fig. 4: Optimization results

Fig. 4 shows the results of the optimization of the H–plane filter with 4 inductive irises (see Fig. 1). The variables are the openings of the irises h_1 and h_2, respectively. Although a filter synthesis for this type of filter can deliver better starting values than the ones shown in Fig.4, the objective was to illustrate the excellent performance of the optimization technique even for far-off start values.

For the filter analysis a triangular mesh with 2016 triangles and 1117 nodes at 100 frequency points have been used to guarantee sufficient accuracy. The optimization process converged to the final solution after 23 iterations.

Conclusion

An efficient gradient-based optimization for the FEM has been introduced. The method has been tested by optimizing an H-plane waveguide filter. The gradient of the cost functions is determined from a single FEM analysis of the filter without finite differencing or mesh readjustment. This results in much shorter computation time and significantly less computer memory. Although the approach introduced here is only for the 2-D FEM it can be extended to 3-D FEM.

References

[1] S. Amari, P. Harscher, R. Vahldieck, J. Bornemann, ‘Novel analytic gradient evaluation techniques for optimization of microwave structures’, IEEE MTT-S, 1999, page 31-34.

[2] A. Weisshaar, M. Mongiardo, A. Tripathi, V. K. Tripathi, ‘CAD-oriented full-wave equivalent circuit models for waveguide components and circuits’, IEEE, Transactions on Microwave and Technique, Vol. 44, No. 12, 1996.

[3] J. Uher, J. Bornemann, U. Rosenberg, ‘Waveguide components for antenna feed systems’, Artech House, Norwood, MA, 1993.

[4] Matlab Optimization Toolbox, User’s Guide 5, The Math Works Inc., Natick, MA, 1996.