Synthesis of Cross-Coupled Resonator Filters Using an Analytical Gradient-Based Optimization Technique

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***Abstract*—We propose a general approach to the synthesis of cross-coupled resonator filters using an analytical gradient-based optimization technique. The gradient of the cost function with respect to changes in the coupling elements between the resonators is determined analytically. The topology of the structure is strictly enforced at each step in the optimization thereby eliminating the need for similarity transformations of the coupling matrix. For the calculation of group delays, a simple formula is presented in terms of the coupling matrix. A simple recursion relation for the computation of the generalized Chebychev filtering functions is derived. Numerical results demonstrating the excellent performance of the approach are presented.**

***Index Terms*—Bandpass filters, elliptic filters, filters, optimization methods, resonator filters.**

I. Introduction

COUPLED microwave resonators are essential components in modern communication systems. Filtering structures with increasingly stringent requirements can often be met only by using cross-coupled resonators to generate finite transmission zeros.

A general theory of cross-coupled resonator bandpass filters was developed in the 1970s by Atia and Williams [1]–[3]. Low-order filters, up to four, were solved analytically by Kurzrok [4], [5] and Williams [6]. The more general theory presented by Atia and Williams [1] is still widely used in the synthesis of these types of structures. A slightly different approach was advanced by Cameron in a series of papers [7]–[9]. Cameron also gives a scheme to determine the filtering function with arbitrarily placed transmission zeros [7]. Once the system function is obtained the synthesis of the filter proceeds by extracting element values [8] to obtain a coupling matrix. Other excellent techniques were also presented by many researchers, most notably by groups around Rhodes [10]–[15]. The literature on this subject is too extensive to list here; the reader is referred, e.g., to a special issue [16].

The theory of Atia and Williams leads to a coupling matrix which reproduces the system function to be synthesized but which often includes unwanted or unrealizable coupling elements. Repeated similarity transformations are then used to cancel the unwanted couplings [1], [9]. Unfortunately, the process does not always converge [17]. The same approach was also used in a recent publication by Cameron [18] to reduce a potentially full coupling matrix to a folded form; a method of reducing a full matrix to an arbitrary form is still not known. Recently, optimization was also used in synthesizing this type of microwave structure. An interesting approach in which the entries of the coupling matrix were used as independent variables was presented in [17]. A simple cost function along with a standard unconstrained gradient optimization technique was used and excellent results were reported [17].

In this paper, we propose a comprehensive theory of the synthesis problem of these structures. We first present a simple recursion formula to determine the low-pass prototype with arbitrarily placed transmission zeros. The resulting recursion relation is much simpler than that given by Cameron [7], [18]. Once the transmission function is obtained, a coupling matrix which enforces a given topology is synthesized by optimization. Analytical expressions for the gradient of the scattering parameters are derived without recourse to the concept of the adjoint network which was widely used in optimization and sensitivity analysis of linear circuits [19].

II. Computation of Low-Pass Prototype Filtering Function

We start from a low-pass prototype in the frequency variable ω' where the transmission function $S_{21}(\omega')$ is given by

$$|S_{21}(\omega')|^2 = \frac{1}{1+\epsilon^2 F_N^2(\omega')} \tag{1}$$

where ϵ is a constant related to the passband return loss R by $\epsilon = [10^{R/10}-1]^{-1/2}$. The filtering function $F_N(\omega')$ is given by [7]

$$F_N(\omega') = \cosh\left(\sum_{n=1}^{N}\cosh^{-1}(x_n)\right), \quad x_n = \frac{\omega' - 1/\omega'_n}{1-\omega'/\omega'_n}. \tag{2}$$

Here, $s_n = j\omega'_n$ is the location of the nth transmission zero in the complex s-plane [7]. Note that $|F_N(\omega' = \pm 1)| = 1$ for all values of N.

It can be shown that the function $F_N(\omega')$ is a rational function whose denominator is given by the product $\prod_{n=1}^{N}(1-(\omega'/\omega'_n))$ [7]. The function $F_N(\omega')$ can therefore be written as

$$F_N(\omega') = \frac{P_N(\omega')}{\prod_{n=1}^{N}\left(1-\frac{\omega'}{\omega'_n}\right)} = \frac{P_N(\omega')}{D_N(\omega')} \tag{3}$$

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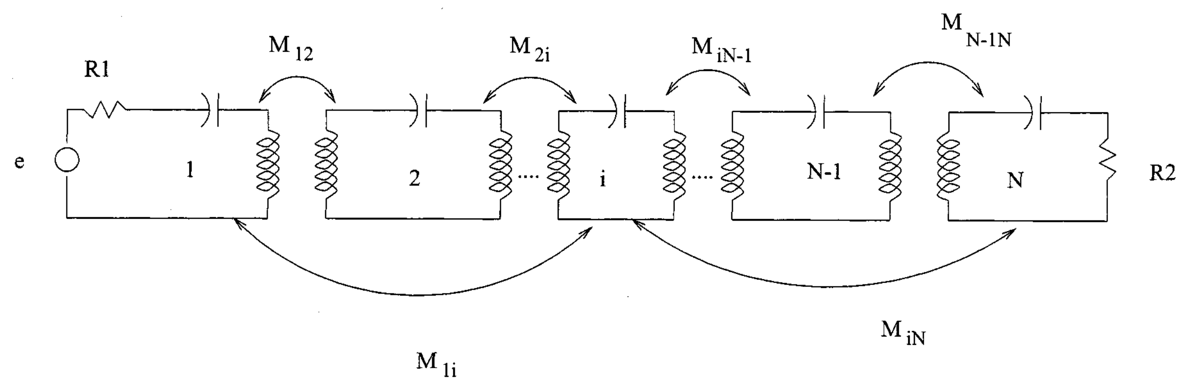


Fig. 1. Model of a general cross-coupled resonator bandpass filter.

where $D_N(\omega) = \prod_{n=1}^{N}(1 - (\omega'/\omega'_n))$. To compute the numerator $P_N(\omega')$ a simple recursion relation is established between $P_{N-1}(\omega')$, $P_N(\omega')$ and $P_{N+1}(\omega')$.

Using the identity $\cosh(\alpha \pm \beta) = \cosh(\alpha)\cosh(\beta) \pm \sinh(\alpha)\sinh(\beta)$, we can write

$$\begin{aligned}
&\frac{P_{N+1}(\omega')}{\left(1 - \dfrac{\omega'}{\omega'_{N+1}}\right) D_N} \\
&\quad = \cosh\left(\sum_{n=1}^{N} \cosh^{-1}(x_n) + \cosh^{-1}(x_{N+1})\right) \\
&\quad = \sinh\left(\sum_{n=1}^{N} \cosh^{-1}(x_n)\right) \sinh(\cosh^{-1}(x_{N+1})) \\
&\qquad + \cosh\left(\sum_{n=1}^{N} \cosh^{-1}(x_n)\right) x_{N+1} \\
&\quad = \sinh\left(\sum_{n=1}^{N} \cosh^{-1}(x_n)\right) \sinh(\cosh^{-1}(x_{N+1})) \\
&\qquad + x_{N+1} \frac{P_N(\omega')}{D_N}.
\end{aligned} \tag{4}$$

Similarly

$$\begin{aligned}
&\frac{\left(1 - \dfrac{\omega'}{\omega'_N}\right) P_{N-1}(\omega')}{D_N} \\
&\quad = \cosh\left(\sum_{n=1}^{N} \cosh^{-1}(x_n) - \cosh^{-1}(x_N)\right) \\
&\quad = -\sinh\left(\sum_{n=1}^{N} \cosh^{-1}(x_n)\right) \sinh(\cosh^{-1}(x_N)) \\
&\qquad + \cosh\left(\sum_{n=1}^{N} \cosh^{-1}(x_n)\right) x_N \\
&\quad = -\sinh\left(\sum_{n=1}^{N} \cosh^{-1}(x_n)\right) \sinh(\cosh^{-1}(x_N)) \\
&\qquad + x_N \frac{P_N(\omega')}{D_N}.
\end{aligned} \tag{5}$$

Eliminating the quantity $\sinh(\sum_{n=1}^{N} \cosh^{-1}(x_n))$ from these last two equations and using simple hyperbolic identities, we get the following recursion relation:

$$\begin{aligned}
P_{N+1}(\omega') = &-P_{N-1}(\omega')\left(1 - \frac{\omega'}{\omega'_N}\right)^2 \frac{(1 - 1/\omega'^2_{N+1})^{1/2}}{(1 - 1/\omega'^2_N)^{1/2}} \\
&+ P_N(\omega')\left[\omega' - \frac{1}{\omega'_{N+1}} + \left(\omega' - \frac{1}{\omega'_N}\right) \right. \\
&\left. \cdot \frac{(1 - 1/\omega'^2_{N+1})^{1/2}}{(1 - 1/\omega'^2_N)^{1/2}}\right].
\end{aligned} \tag{6}$$

The polynomials $P_0(\omega,)$ and $P_1(\omega')$ are given by

$$P_0(\omega') = 1, \quad P_1(\omega') = \omega' - \frac{1}{\omega'_1}. \tag{7}$$

From (6) it is obvious that $P_{N+1}(\omega')$ is a polynomial of degree $N + 1$ if $P_N(\omega')$ and $P_{N-1}(\omega')$ are polynomials of degree N and $N - 1$, respectively.

III. Basic Model and its Governing Equations

We propose to synthesize a network consisting of N coupled lossless resonators as shown in Fig. 1. The resonant frequency of resonator i is $f_i = f_0 + \delta_i$ where f_0 is the center frequency of the filter and corresponds to the angular frequency ω_0. The frequency-independent coupling coefficient between resonators i and j is denoted by $M_{ij} = M_{ji}$. A voltage source of internal resistance R_1 and of magnitude equal to unity excites the structure at resonator 1. The load at the output is a resistor R_2 connected to resonator N. The normalized angular frequency ω' is related to ω_0 and the bandwidth $\Delta\omega$ by $\omega' = (\omega_0/\Delta\omega)((\omega/\omega_0) - (\omega_0/\omega))$. For narrow-band filters, the shift in the resonant frequencies of the resonators is absorbed in frequency-indepedent diagonal elements of the coupling matrix $[M]$.

In the remainder of the paper, we set $\omega_0 = 1$ and $\Delta\omega = 1$; these quantities act as scaling factors on the network parameters [1, footnote, p. 34]. Following the analysis in [1], the loop currents, which are grouped in the vector $[I]$, are governed by the following matrix equation:

$$[\omega' U - jR + M][I] = [A][I] = -j[e], \quad j^2 = -1. \tag{8}$$

Here, $[U]$ is the identity matrix, R is a matrix whose only nonzero entries are $R_{11} = R_1$ and $R_{NN} = R_2$, and M is a symmetric square coupling matrix. The excitation vector $[e]$ is given by $[e]^t = [1, 0, 0, \cdots, 0]$ where t is the transposition operator. The discussion of the limitations of this model to narrow-band filters is well presented in [1] and is not repeated here.

From (8), we see that the vector current $[I]$ is given by the formal solution

$$[I] = -j[A^{-1}][e]. \tag{9}$$

Using this equation, the scattering parameters are given by

$$S_{21} = 2\sqrt{R_1R_2}I_N = -2j\sqrt{R_1R_2}[A^{-1}]_{N1} \tag{10}$$

and

$$S_{11} = 1 - 2R_1I_1 = 1 + 2jR_1[A^{-1}]_{11}. \tag{11}$$

At this point, the synthesis problem can be formulated simply: determine the coupling matrix $[M]$ and the resistors R_1 and R_2 such that the scattering parameters given by (12) and (13) reproduce the insertion and return loss given by the prototype.

We propose to solve this problem by optimization for the following reasons.

1) We can strictly enforce the desired topology; this eliminates the need for similarity transformations.
2) We can synthesize both symmetric and asymmetric responses. If the structure is symmetric, this information can be used to reduce the numerical effort.
3) We can synthesize filters of arbitrary even or odd orders.
4) We can constrain specific coupling elements to be of a given sign or within a magnitude range if the intended implementation calls for such a constraint.
5) The resulting solution, if one is obtained, is not affected by the problem of round off errors which plagues extraction methods.
6) If an exact solution is not found, an approximate one which maybe acceptable, is always given. This happens when the desired prototype response is not within the range of the chosen topology.
7) We could formulate the problem as a set of nonlinear equations similarly to the technique presented by Orchard [20], for example. It is, however, much easier to find a minimum than a zero when the number of variables is large [21, p. 272].

IV. Cost Function

Keeping in mind that the filtering functions under consideration, generalized Chebychev prototypes, are rational functions of frequency; they are uniquely specified by the location of their poles and zeros and an additional scaling constant. Since the zeros of the filtering function are identical to those of S_{11} and its poles coincide with the zeros of S_{21}, the original analytic structure is recovered from the vanishing of S_{11} and S_{21} at the corresponding frequency points. To determine the scaling constant, we evaluate the return loss at $\omega' = \pm 1$ to get $|S_{11}(\omega' = \pm 1)| = (\epsilon/(\sqrt{1+\epsilon^2}))$. Consequently, the following cost function is used in this work:

$$\begin{aligned} K = &\sum_{i=1}^{N} |S_{11}(\omega'_{zi})|^2 + \sum_{i=1}^{P} |S_{21}(\omega'_{pi})|^2 \\ &+ \left(|S_{11}(\omega' = -1)| - \frac{\epsilon}{\sqrt{1+\epsilon^2}}\right)^2 \\ &+ \left(|S_{11}(\omega' = 1)| - \frac{\epsilon}{\sqrt{1+\epsilon^2}}\right)^2. \end{aligned} \tag{12}$$

Here, ω'_{zi} and ω'_{pi} are the zeros and poles of the filtering function F_N, respectively. It is assumed that F_N has P poles and N zeros.

Except for the last two terms, this cost function is identical to that given in [17]. However, this seemingly trivial difference allows us to use the analytical gradient of the cost function.

V. Gradient Calculation

It this work, the entries of the coupling matrix will be used as independent variables in the optimization process. The same approach was used in [17]. To make the process more efficient, both the values of the error function and its gradient are used.

We first note that the gradient of the error function with respect to an independent variable x involves the derivatives $((\partial|S_{11}|)/\partial x)$ and $((\partial|S_{21}|)/\partial x)$. It can be shown that [22]

$$\frac{\partial|S_{11}|}{\partial x} = Re\left[\frac{|S_{11}|}{S_{11}}\frac{\partial S_{11}}{\partial x}\right] \tag{13}$$

with a similar expression for S_{21}.

Using the expressions of S_{11} and S_{21} in terms of the coupling matrix [equations (10) and (11)] we get

$$\frac{\partial S_{11}}{\partial x} = -2R_1\frac{\partial I_1}{\partial x}, \quad x \neq R_1 \tag{14}$$

and

$$\frac{\partial S_{21}}{\partial x} = 2\sqrt{R_1R_2}\frac{\partial I_N}{\partial x}, \quad x \neq R_1, R_2. \tag{15}$$

To calculate these derivatives, we take the derivative of the matrix equation $[I] = -j[A^{-1}][e]$ to get

$$\frac{\partial [I]}{\partial x} = -j\frac{\partial [A^{-1}]}{\partial x}[e] - j[A^{-1}]\frac{\partial [e]}{\partial x} = -j\frac{\partial [A^{-1}]}{\partial x}[e]. \tag{16}$$

The last term is zero since $[e]$ is a constant vector. Here, the derivative of a matrix is a matrix whose entries are the derivatives of the corresponding entries in the original matrix.

Taking the derivative of the identity $[A][A^{-1}] = [U]$, where $[U]$ is the identity matrix, we get

$$\frac{\partial [A^{-1}]}{\partial x} = -[A^{-1}]\frac{\partial [A]}{\partial x}[A^{-1}]. \tag{17}$$

Combining equations (16) and (17), we get

$$\frac{\partial [I]}{\partial x} = j[A^{-1}]\frac{\partial [A]}{\partial x}[A^{-1}][e]. \tag{18}$$

Let us define the topology matrix $[P]$ of the network by $P_{ij} = 1$ if $M_{ij} \neq 0$ and $P_{ij} = 0$ if $M_{ij} = 0$. The topology of the network can be specified beforehand and will be enforced at each step in the optimization.

When the generic variable x is replaced by a generic element of the coupling matrix $M_{pq} = M_{qp}$ in (18) which is then used in equations (14) and (15), we get the simple results

$$\frac{\partial S_{11}}{\partial M_{pq}} = -4jR_1 P_{pq}[A^{-1}]_{1p}[A^{-1}]_{q1} \tag{19}$$

and

$$\frac{\partial S_{21}}{\partial M_{pq}} = 2j\sqrt{R_1 R_2} P_{pq} \cdot \left([A^{-1}]_{Np}[A^{-1}]_{q1} + [A^{-1}]_{Nq}[A^{-1}]_{p1}\right). \tag{20}$$

Here, the symmetry of the matrices $[A]$ and $[A^{-1}]$ was used. The gradient of the scattering parameters with respect to the diagonal elements of the coupling matrix is obtained from the previous expressions by simply setting $p = q$ and dividing by a factor of two. The factor of two accounts for the fact that the diagonal elements of a symmetric matrix occur only once whereas off-diagonal elements occur twice. Thus

$$\frac{\partial S_{11}}{\partial M_{pp}} = -2jR_1 P_{pp}[A^{-1}]_{p1}[A^{-1}]_{p1} \tag{21}$$

and

$$\frac{\partial S_{21}}{\partial M_{pp}} = 2j\sqrt{R_1 R_2} P_{pp}[A^{-1}]_{Np}[A^{-1}]_{p1}. \tag{22}$$

Although the values of the terminations R_1 and R_2 can be determined from the theory of Atia and Williams [1], we prefer to determine them along with the coupling coefficients using optimization.

Let us assume that the ratio of the two resistors is specified as $R_2 = rR_1$. The resistor R_1 will be used as an independent variable. The computation of the gradient of S_{11} and S_{21} with respect to R_1 follows the discussion above; we only give the final result as

$$\frac{\partial S_{11}}{\partial R_1} = 2j[A^{-1}]_{11} + 2R_1 \cdot \left([A^{-1}]_{11}[A^{-1}]_{11} + r[A^{-1}]_{N1}[A^{-1}]_{N1}\right) \tag{23}$$

and

$$\frac{\partial S_{21}}{\partial R_1} = -2j\sqrt{r}[A^{-1}]_{N1} + 2R_1\sqrt{r} \cdot \left([A^{-1}]_{N1}[A^{-1}]_{11} + r[A^{-1}]_{NN}[A^{-1}]_{N1}\right). \tag{24}$$

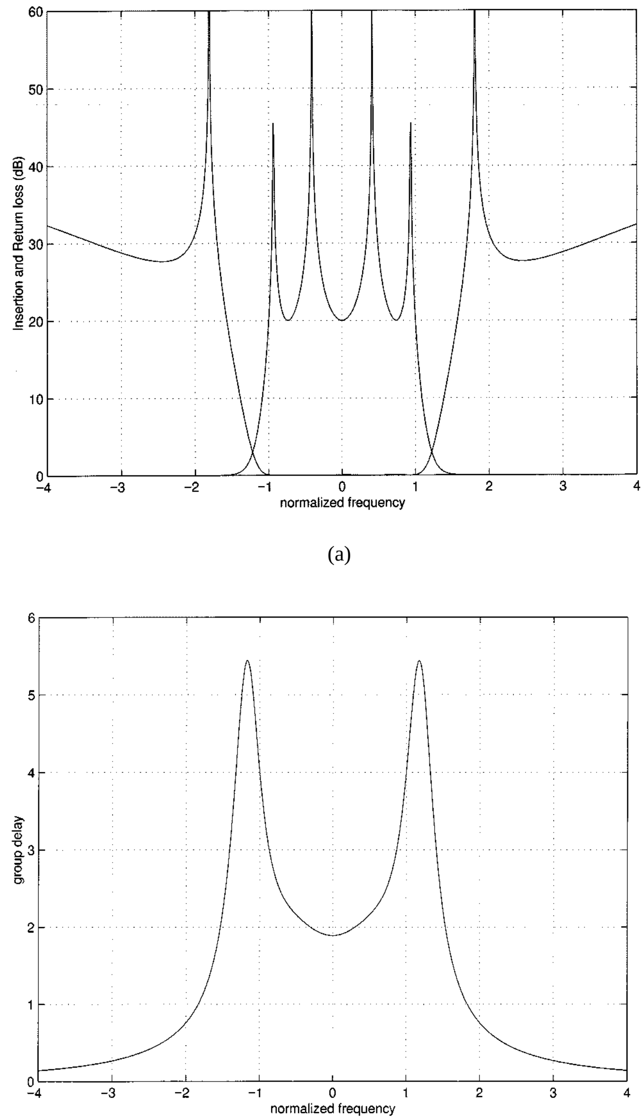
Expressions of the logarithmic derivatives of the transmission coefficient of multicoupled cavity filters were also given in [23] without derivation. The derivation presented here is more general and can be used even in cases where the adjoint network method is not applicable [24].

VI. Computation of Group Delay

It can be shown that the group delay τ_g is given by [22]

$$\tau_g = -\mathrm{Im}\left[\frac{1}{S_{21}}\frac{\partial S_{21}}{\partial \omega'}\right]. \tag{25}$$

Recall that ω' is a normalized and transformed frequency variable; the actual value of the group delay should take this transformation into consideration.



(a)

(b)

Fig. 2. Response of filter 1: (a) insertion and return loss and (b) group delay.

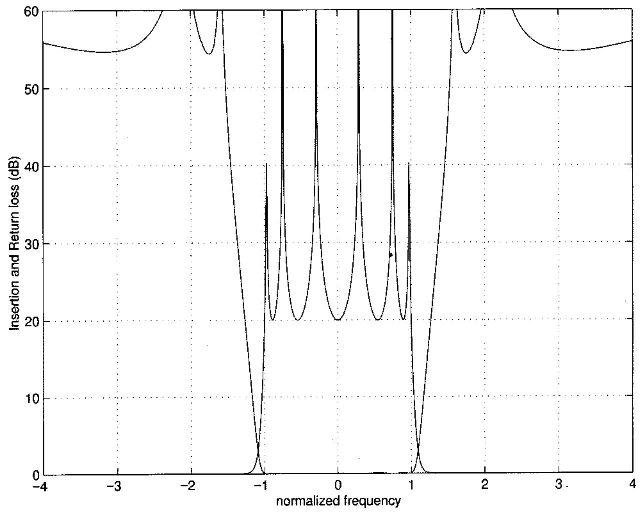
Following similar steps to those in the previous section and using the fact that the derivative of the matrix $[A]$ with respect to ω' is equal to the identity matrix, we get the equation

$$\tau_g = \mathrm{Im}\left[\frac{\sum_{k=1}^{N}[A^{-1}]_{Nk}[A^{-1}]_{k1}}{[A^{-1}]_{N1}}\right]. \tag{26}$$

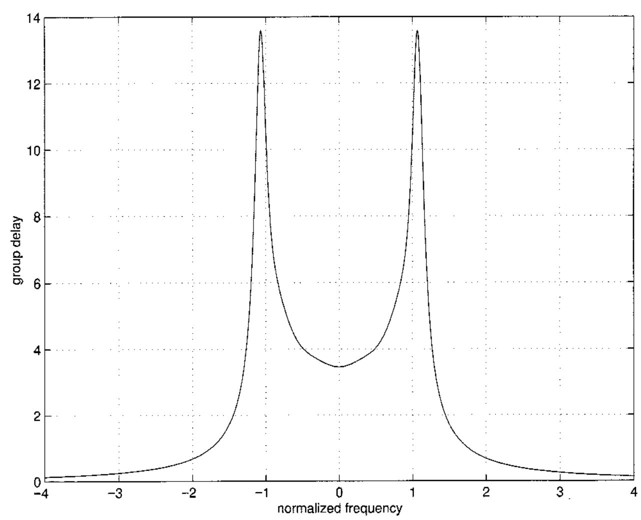
VII. Numerical Results

The theory presented here is first applied to the synthesis of an equally terminated fourth-order filter with two transmission zeros at $\omega' = \pm 1.81$ and a passband return loss of 20 dB (filter 1).

The network used involves direct coupling of each resonator i to $i+1$ but only resonators 1 and 4 are cross-coupled with

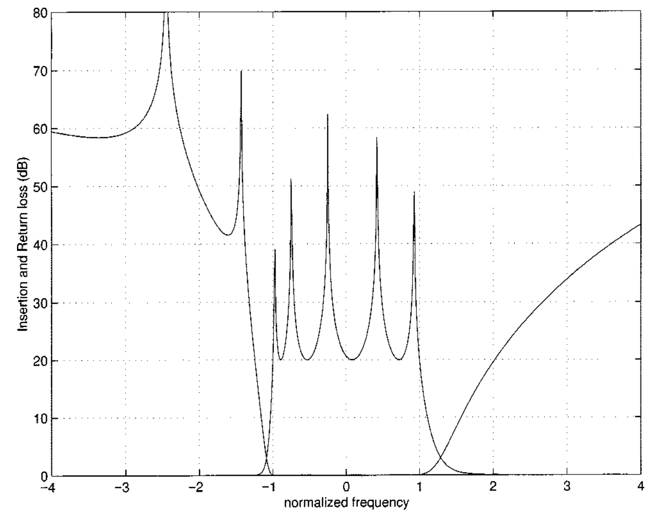


(a)

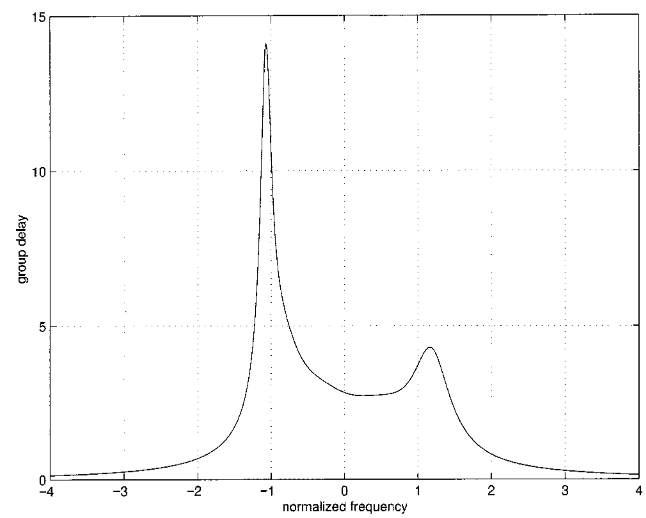


(b)

Fig. 3. Response of filter 2: (a) insertion and return loss and (b) group delay.



(a)



(b)

Fig. 4. Response of filter 3: (a) insertion and return loss and (b) group delay.

a coupling coefficient $M_{14} = M_{41}$. The initial guess of the coupling matrix corresponds to setting all direct couplings to 0.5; all the remaining entries to zero and $R_1 = R_2 = 1$.

The nonzero entries of the obtained coupling matrix are: $M_{12} = M_{34} = 0.8577$, $M_{23} = 0.7856$, $M_{14} = -0.2174$, and $R_1 = R_2 = 1.0442$. The insertion and return loss of the synthesized filter are shown in Fig. 2(a). These are identical to those of the prototype within plotting accuracy. This demonstrates the accuracy of the approach.

The group delay was also computed using equation (26) and is shown in Fig. 2(b). Note that this is not the actual group delay as the normalized frequency was used in its computation. The two are related by a simple frequency transformation [15]. The calculated group delay agrees very well with that determined directly from the prototype.

The next example is an equally terminated sixth-order filter with four transmission zeros located at $\omega' = \pm 1.592\,692$ and $\pm 2.132\,335$. The passband return loss is 20 dB (filter 2). We introduce cross couplings between resonators 1 and 6, and 2 and 5; the remaining resonators are coupled only to their nearest neighbors.

The starting guess corresponds to all cross couplings (M_{16} and M_{25}) set to zero, all direct couplings to 0.5, and the terminations $R_1 = R_2$ to unity and constrained to positive values in the optimization routine. The insertion and return loss of the synthesized filter are shown in Fig. 3(a) along with the prototype; the two coincide within plotting accuracy. The group delay of the filter was also computed and its shown in Fig. 3(a). It agrees with that computed directly from the prototype response function. The nonzero entries of the coupling matrix are $M_{56} = M_{12} = 0.8298$, $M_{45} = M_{23} = 0.5789$, $M_{34} = 0.7060$, $M_{25} = -0.1658$, $M_{16} = 0.019\,38$, and $R_1 = R_2 = 0.990$. The minimum value of the cost function is zero (machine accuracy) for both filters.

Finally, to show performance of the approach in synthesizing filters with asymmetrically located transmission zeros, we consider a fifth-order filter with two transmission zeros at $\omega' =$

$-2.452\,692$ and $\omega' = -1.432\,335$ and a passband return loss of 20 dB (filter 3). The starting guess consists of all direct couplings set to 0.5, the terminations to unity, and all the remaining entries of the coupling matrix to zero. Resonator 1 is coupled to 3 which is also coupled to resonator 5. The nonzero entries of the calculated coupling matrix are as follows: $M_{11} = -0.0469$, $M_{22} = 0.2742$, $M_{33} = -0.1422$, $M_{44} = 0.5817$, $M_{55} = -0.0469$, $M_{12} = 0.8373$, $M_{23} = 0.6059$, $M_{34} = 0.5149$, $M_{45} = 0.7435$, $M_{13} = -0.2328$, $M_{35} = -0.4500$ and $R_1 = R_2 = 1.0291$. The corresponding return and insertion loss are shown in Fig. 4(a). Both the response of the prototype as computed from (6) and that calculated directly from the coupling matrix are superimposed. The excellent agreement between the two, the difference is not visible in the figure, shows the accuracy of the synthesis approach. The group delay of the synthesized filter was also computed from (26) and is shown in Fig. 4(b). It agrees very well with that computed directly from the response of the prototype.

VIII. Conclusions

A theory for the synthesis of cross-coupled resonator filters was presented. The coupling matrix required to reproduce a given prototype response function is synthesized by gradient-based optimization. Analytical expressions of the gradient of the cost function as well as a formula for the group delay were derived. Numerical results obtained from this new approach agree well with those of the prototypes.

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