

Example B.1 (Simple pole). Find the partial fraction expansion of the function

$$f(z) = \frac{3}{z^2 + 3z + 2}. \quad \leftarrow \text{Strictly proper}$$

Solution. First, we rewrite f with the denominator polynomial factored to obtain

$$f(z) = \frac{3}{(z+1)(z+2)}. \quad \leftarrow \text{Simple (i.e., 1st order) poles at } -1 \text{ and } -2$$

From this, we know that f has a partial fraction expansion of the form

$$f(z) = \frac{A_1}{z+1} + \frac{A_2}{z+2}, \quad \textcircled{1}$$

where A_1 and A_2 are constants to be determined. Now, we calculate A_1 and A_2 as follows:

$$\left. \begin{aligned} A_1 &= (z+1)f(z)|_{z=-1} \\ &= \frac{3}{z+2} \Big|_{z=-1} \\ &= 3 \quad \text{and} \\ A_2 &= (z+2)f(z)|_{z=-2} \\ &= \frac{3}{z+1} \Big|_{z=-2} \\ &= -3. \end{aligned} \right\} \textcircled{2}$$

Thus, the partial fraction expansion of f is given by

$$f(z) = \frac{3}{z+1} - \frac{3}{z+2}. \quad \leftarrow \text{from } \textcircled{1} \text{ and } \textcircled{2}$$

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