A Highly-Effective Approach for Generating Delaunay Mesh Models of RGB Color Images

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Overview

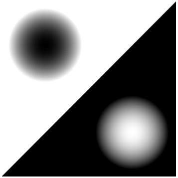
1. Introduction and Background
2. Proposed Method
3. Evaluation of Proposed Method
4. Conclusions

Motivation

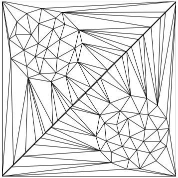
- images are nonstationary
- uniform sampling almost never optimal (e.g., sampling density too high in regions of low variation and too low in regions of high variation)
- in many applications, nonuniform (i.e., content-adaptive) sampling highly beneficial
- triangle meshes popular approach to handling nonuniform sampling
- RGB color images pervasive in many applications
- relatively less work has been done on generating mesh models of color images
- want to improve upon methods for generating mesh model of RGB color images

Triangle-Mesh Models of Images and Mesh Generation

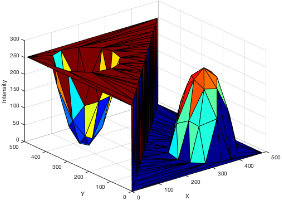
- mesh model of M-component raster image ϕ of width W and height H consists of:
  - set $P = \{p_i\}$ of sample points
  - Delaunay triangulation T of P
  - set $Z = \{z_i\}_{i=0}^{|P|-1}$ of function values, where $z_i = \phi(p_i)$
- sampling density D defined as $D = \frac{|P|}{WH}$
- given image ϕ and desired number N of sample points, find mesh model with $|P| = N$ that best approximates ϕ in terms of mean-squared error (MSE)
- example (single-component case):



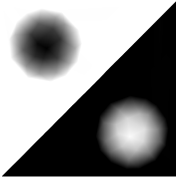
Original Image



Triangulation



Approximating Function



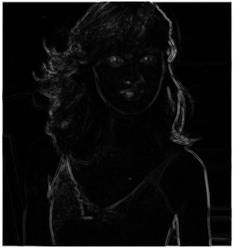
Reconstructed Image

Trivial Extension of Grayscale Methods to RGB Color

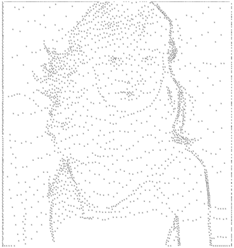
- three highly-effective methods for grayscale images:
  1. error diffusion (**ED**) method: select sample points in one shot using Floyd-Steinberg error diffusion (FSED)
  2. greedy point removal (**GPR**) method: select all sample points in the sampling grid as initial sample points, and then perform mesh-simplification
  3. GPR from subset with ED (**GPRFSED**): select a subset of sample points as initial sample points, and then perform mesh-simplification
- method for grayscale image can be extended to RGB color images as follows:
  1. convert RGB color image into grayscale by standard (RGB-to-luminance) conversion
  2. use grayscale image as input to grayscale method
  3. replace scalar function values with vector (color) values in generated grayscale model to yield color model
- trivially extended versions of above three methods:
  1. **CED**: trivial extension of ED method to RGB color
  2. **CGPR**: trivial extension of GPR method to RGB color
  3. **CGPRFSED**: trivial extension of GPRFSED method to RGB color
- trivial extension method likely to be highly suboptimal

Floyd-Steinberg Error-Diffusion (FSED)

- input: density function d of image of width W and height H, threshold τ, and initial diffused-in error $\tilde{e}$
- output: binary-valued function b indicating position of selected points
- classic FSED sets $\tilde{e}$ to zero, which can cause undesirable startup effect (when τ is large) where very few samples chosen in bottom region of b



Density Function d

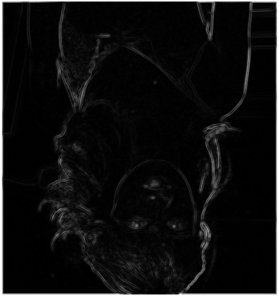


Selected Points b

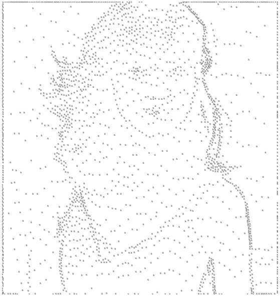
Proposed Strategy for Initial-Condition Selection in FSED

- proposed strategy to obtain initial diffused-in errors $\tilde{e}$:
  1. construct mirrored version d_m of density function d
  2. take d_m as input density function, run FSED (with zero diffused-in error)
  3. record and output errors diffused to last (i.e., top) row

Density Function d



Mirrored Density Function d_m

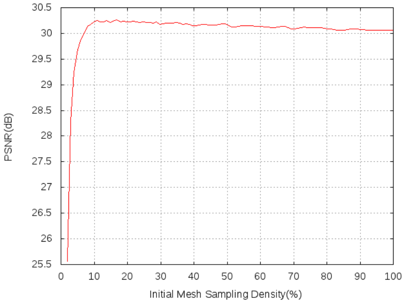


Selected Points p

Proposed Mesh-Generation Method: CMG

- input: M-component (e.g., $M = 3$ for RGB) image of width W and height H, desired number N of sample points, initial mesh-size control parameter γ
- output: mesh model having set P of sample points, where $|P| = N$
- proposed method, called CMG(γ), works as follows:
  1. select initial mesh having $N_0 = \min\{\gamma N, WH\}$ sample points using FSED with our new initial-condition selection scheme and density function d, where
     - density function d obtained by taking pixel-wise maximum of MMSODD across all image components
  2. construct Delaunay triangulation T of P
  3. while mesh size $> N$:
     - delete sample point that results in least increase in approximation error
- recommend choosing γ as either 1 or 4 (i.e., CMG(1) and CMG(4), respectively) to tradeoff between computational cost and mesh quality

- consider impact of initial mesh size on mesh quality for given target sampling density $D = \frac{N}{WH}$
- highest mesh quality typically obtained when initial mesh has sampling density $D_0 = \gamma D$ with $\gamma \in [4, 5.5]$
- this observation led us to recommend using CMG with $\gamma = 4$ (i.e., CMG(4))



Mesh Quality Versus D_0 for peppers Image with $D = 2\%$

Evaluation of Proposed Method

- test data:
  - 45 RGB images
  - taken mostly from standard data sets, including: JPEG-2000, USC-SIPI, CIPR-Canon, and Kodak
- compare to CED, CGPRFSED, and CGPR (i.e., color-extended versions of grayscale mesh generators ED, GPRFSED, and GPR, respectively)
- most meaningful comparisons to be made:
  - CMG(1) versus CED
  - CMG(4) versus CGPRFSED
  - CMG(4) versus CGPR

Comparison of Mesh Quality — Summary Results

Samp. Density (%)	Average Rank*				
	CED	CMG(1)	CGPRFSED	CMG(4)	CGPR
0.5	4.88 (0.32)	4.12 (0.32)	2.83 (0.37)	**1.36** (0.48)	1.81 (0.70)
1.0	4.90 (0.29)	4.10 (0.29)	2.50 (0.50)	**1.12** (0.39)	2.38 (0.65)
2.0	4.90 (0.29)	4.10 (0.29)	2.24 (0.48)	**1.05** (0.21)	2.71 (0.50)
3.0	4.81 (0.39)	4.17 (0.43)	2.19 (0.45)	**1.05** (0.21)	2.79 (0.51)
4.0	4.81 (0.39)	4.19 (0.39)	2.14 (0.41)	**1.07** (0.26)	2.79 (0.51)
Overall	4.86 (0.34)	4.13 (0.35)	2.38 (0.51)	**1.13** (0.35)	2.50 (0.69)

*Average across 45 test images. Standard deviations are given parentheses.

- CMG(1) beats CED in 84.4% of all test cases by up to 7.08 dB
- CMG(4) beats CGPRFSED and CGPR respectively, in 97.8% and 89% of all test cases by up to 7.05 dB and 5.15 dB

Image	Samp. Density (%)	PSNR (dB)				
		CED	CMG(1)	CGPRFSED	CMG(4)	CGPR
lena	0.5	17.48	19.18	25.63	26.04	**26.09**
	1.0	21.31	22.12	28.02	**28.38**	28.09
	2.0	25.54	25.77	30.33	**30.48**	30.13
	3.0	27.42	27.73	31.44	**31.68**	31.29
	4.0	28.82	28.83	32.13	**32.49**	32.04
pens	0.5	13.96	15.78	22.49	**24.05**	23.60
	1.0	17.24	19.27	25.95	**26.77**	26.40
	2.0	21.98	23.48	29.08	**29.43**	29.12
	3.0	25.05	25.91	30.59	**31.16**	30.77
	4.0	27.05	27.92	31.97	**32.45**	32.01
bluegirl	0.5	19.73	21.17	27.10	29.37	**29.68**
	1.0	22.49	25.30	31.99	**32.67**	32.54
	2.0	25.29	29.40	34.97	**35.38**	34.98
	3.0	29.49	31.90	36.39	**36.85**	36.33
	4.0	32.67	33.40	37.29	**37.86**	37.23

- CMG(1) beats CED in 15/15 of test cases by up to 4.11 dB
- CMG(4) beats CGPRFSED in 15/15 of test cases by up to 2.88 dB
- CMG(4) beats CGPR in 13/15 of test cases by up to 0.63 dB

Comparison of Mesh Quality — Subjective Quality

CED

CMG(1)

CGPRFSED

CMG(4)

CGPR

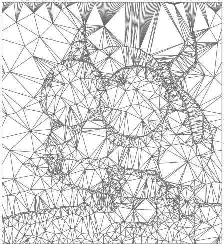
Conclusions

- proposed CMG method for generating mesh models of RGB color images
- proposed method can also handle images with arbitrary number of components
- proposed method outperforms competing schemes with similar and higher complexity
- CMG(1) outperforms CED in mesh quality, with similar computational and memory costs
- CMG(4) outperforms CGPRFSED in mesh quality, with similar in computational and memory costs
- CMG(4) yields meshes with quality better than CGPR in mesh quality, while requiring substantially less computational and memory costs
- proposed approach allows tradeoff to be made between mesh quality and computational cost, which is useful in a wide range of applications

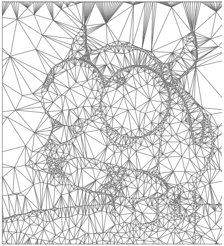
Questions?

Supplemental Slides

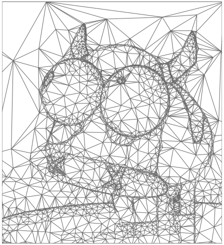
Comparison of Mesh Quality — Triangulations



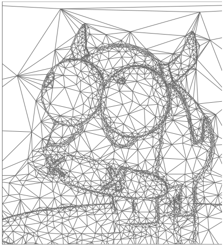
CED



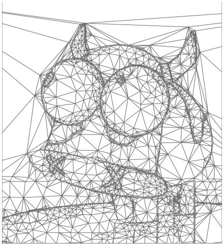
CMG(1)



CGPRFSED



CMG(4)



CGPR

Comparison of Computational Costs

Image	Samp. Density (%)	Time (s) CED	CMG(1)	CGPRFSED	CMG(4)	CGPR
lena	0.5	0.16	0.35	0.68	1.72	29.97
	1.0	0.18	0.46	1.03	2.09	29.62
	2.0	0.23	0.58	2.31	2.79	29.33
	3.0	0.30	0.73	3.15	3.49	29.19
	4.0	0.38	0.80	4.32	5.04	29.00

- CMG(4) requires 6 to 17 times less time than CGPR
- CMG(1) requires similar time to CED
- CMG(4) requires similar time to CGPRFSED

Comparison of Memory Costs

Comparison of the maximum mesh size for the various methods

Method	Maximum Mesh Size	Relative Maximum Mesh Size		
		General	$D = 0.5\%$	$D = 4\%$
CED	DWH	1	1	1
CMG(1)	DWH	1	1	1
CGPRFSED	$4DWH$	4	4	4
CMG(4)	$4DWH$	4	4	4
CGPR	WH	$1/D$	200	25

- CMG(4) requires $\frac{25}{4} \approx 6.2$ to $\frac{200}{4} = 50$ times less memory than CGPR
- CMG(1) requires same memory as CED
- CMG(4) requires same memory as CGPRFSED

Evaluation of Proposed FSED Initial-Condition Selection Strategy

Samp. Density (%)	Win Ratio (%)	
	CMG(1)	CMG(4)
0.5	77.8	82.2
1.0	80.0	68.0
2.0	68.9	53.3
3.0	64.4	48.9
4.0	66.7	44.4
Overall	71.6	59.6

Image	Samp. Density (%)	PSNR(dB)			
		CMG(1)		CMG(4)	
		proposed	classic	proposed	classic
lena	0.5	**19.18**	18.00	**26.04**	25.83
	1.0	**22.12**	21.66	**28.38**	28.23
	2.0	**25.77**	25.56	30.48	**30.50**
	3.0	**27.73**	27.39	31.68	**31.71**
	4.0	**28.83**	28.57	32.49	32.49

- when $\gamma = 1$, proposed strategy beats classic strategy in 71.6% of all test cases, by up to 3.87 dB.
- when $\gamma = 4$, proposed strategy beats classic strategy at sampling densities of 1.0% and lower in 75.6% of test cases by up to 2.29 dB, and behavies similar to classic strategy at higher sampling densities.

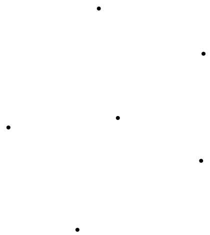
Triangulation

Triangulation

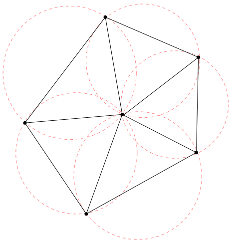
A triangulation of a set P of points in $\mathbb{R}^2$ is a set T of (non-degenerate) triangles satisfying the following conditions:

1. the set of all vertices of triangles in T is P;
2. the union of all triangles in T is the convex hull of P; and
3. the interiors of any two triangles in T are disjoint.

- Preferred-directions Delaunay Triangulation (PDDT) is employed in our work



(a) P



(b) PDDT of P