An Improved Incremental/Decremental Delaunay Mesh-Generation Strategy for Image Representation

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Overview

Introduction: uniform vs. nonuniform sampling

Lattice-based (uniform sampling) image representation

- Too few samples in some regions of image
- More samples than needed in others

Triangle-Mesh Models

- Triangle-mesh model: partition image domain. Characterized by:
  - Sample points
  - Triangulation connectivity
  - Function value at sample points
- Mesh-generation: create approximation of image
  - Fixed size (or density)
  - Minimize error (in MSE)
  - Computational efficiency

Triangle-Mesh Model of Image: Example

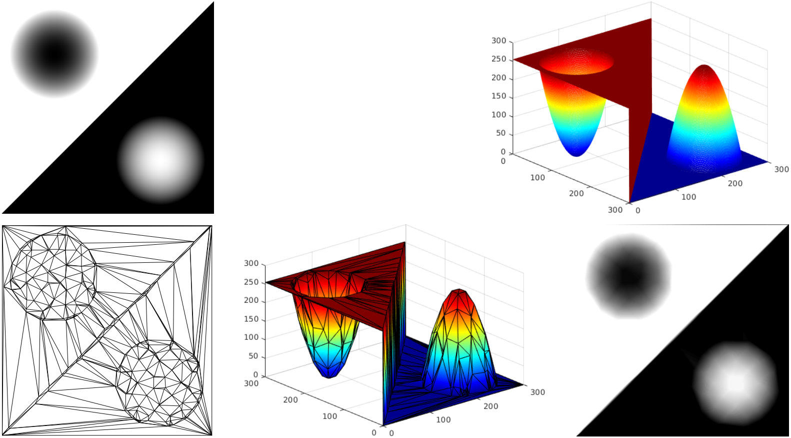


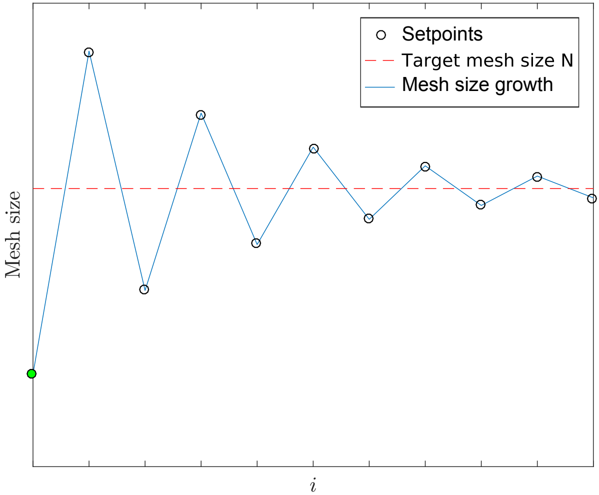
Figure: The original (raster) image. Original image viewed as a surface. A triangulation of the image domain. Resulting triangle mesh. Reconstructed image obtained by rasterizing mesh model.

Mesh-Model Generation Framework

Incremental/decremental mesh generation framework:

- Alternate between insertion and deletion of points
- Combine advantages of mesh-refinement and mesh-simplification methods
- Multiple free parameters: flexible
- Mesh size changes following predefined sequence: growth schedule

Growth Schedule: Example



Mesh-Generation Framework

1. Select initial mesh points.
2. While setpoints remain in the growth schedule:
   1. If mesh size $<$ current setpoint, increase mesh size:
      - Select insertion face f^*
      - Select from face f^* the point p^* to insert
      - Insert selected point p^* into the mesh
   2. Else if mesh size $>$ current setpoint, decrease mesh size:
      - Compute/update error increase for deletion of each point
      - Delete point that causes the least error increase
   3. Select next growth schedule setpoint.
3. Postprocess mesh then stop.

Mesh-Generation Framework: Free Parameters

- Initial mesh
  - Size and method of selecting points
- Growth schedule
  - Type, length, and amplitude
- Insertion face
  - Error (SSE and SAE) and gradient/Laplacian/MMSODD-based
- Insertion point selection
  - Error and gradient/Laplacian/MMSODD-based
- Postprocessing: bad points

Statistical results:

- All parameters fixed to same values
- Face-selection policy varied
- Approximation error computed
- Results ranked for each test case from 1 (best) to 5 (worst)
- Average rank (and standard deviation) computed

Sampling density (%)	**Mean Rank**[1]				
	SSE	SAE	Gradient	Laplacian	MMSODD
0.125	**1.28** (0.60)	1.95 (0.71)	4.03 (1.00)	4.05 (0.81)	3.70 (0.91)
0.250	**1.15** (0.36)	1.85 (0.36)	3.85 (0.83)	4.25 (0.84)	3.90 (0.74)
0.500	**1.08** (0.27)	1.95 (0.32)	3.65 (0.74)	4.50 (0.68)	3.83 (0.84)
1.000	**1.03** (0.16)	1.98 (0.42)	3.73 (0.85)	4.30 (0.72)	3.93 (0.86)
2.000	**1.00** (0.00)	1.98 (0.42)	3.95 (0.81)	4.18 (0.90)	3.83 (0.78)
3.000	**1.00** (0.00)	1.95 (0.32)	4.08 (0.73)	4.13 (1.04)	3.73 (0.78)
Overall	**1.09** (0.32)	1.94 (0.44)	3.88 (0.84)	4.23 (0.85)	3.82 (0.82)

[1] The standard deviation is given in parentheses.

By varying insertion point selection policy, fixing other parameters, then comparing error statistically:

Sampling density (%)	Mean Rank[2]				
	PAE	ALSEM	Gradient	Laplacian	MMSODD
0.125	2.95 (0.93)	**1.15** (0.58)	*2.90* (1.13)	4.73 (0.60)	3.28 (0.88)
0.250	3.30 (0.99)	**1.20** (0.61)	2.90 (1.17)	4.73 (0.75)	*2.85* (0.74)
0.500	3.48 (0.78)	**1.33** (0.89)	3.00 (1.15)	4.70 (0.82)	*2.43* (0.78)
1.000	3.53 (0.78)	**1.25** (0.71)	2.93 (1.14)	4.58 (1.08)	*2.20* (0.79)
2.000	3.60 (0.84)	**1.10** (0.30)	3.00 (1.04)	4.55 (1.11)	*2.00* (0.60)
3.000	3.63 (0.84)	**1.13** (0.33)	2.93 (1.00)	4.50 (1.18)	*2.08* (0.80)
Overall	3.41 (0.89)	**1.19** (0.60)	2.94 (1.10)	4.63 (0.94)	*2.47* (0.89)

[2] The standard deviation is given in parentheses.

Methods Development – Growth Schedule

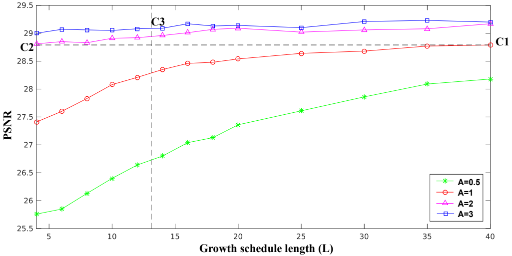
By varying growth schedule type, fixing other parameters, then comparing error statistically:

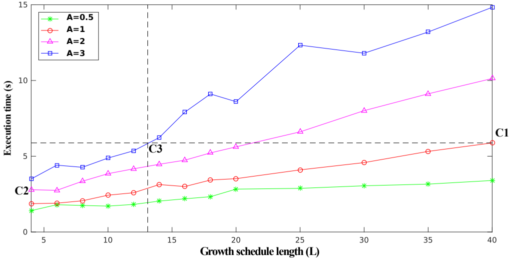
Sampling density (%)	**Mean Rank**[3]			
	A'	B'	C'	I'
0.125	**1.05** (0.22)	2.95 (0.31)	3.98 (0.15)	3.98 (0.15)
0.250	**1.07** (0.34)	2.90 (0.37)	4.00 (0.00)	4.00 (0.00)
0.500	**1.02** (0.15)	2.90 (0.37)	4.00 (0.00)	4.00 (0.00)
1.000	**1.00** (0.00)	2.86 (0.47)	4.00 (0.00)	4.00 (0.00)
2.000	**1.00** (0.00)	2.83 (0.54)	3.86 (0.65)	3.86 (0.65)
3.000	**1.00** (0.00)	2.86 (0.52)	3.79 (0.78)	3.79 (0.78)
Overall	**1.02** (0.18)	2.88 (0.44)	2.00 (0.30)	3.94 (0.42)

[3] The standard deviation is given in parentheses.

Methods Development – Growth Schedule Parameters

By varying the amplitude (A) and length (L) parameters of growth schedule A', fixing other parameters, then comparing effect:





By fixing parameters and changing initial mesh:

Image	S. density (%)	PSNR (dB)							
		ED	Subset[4]			Random	Uniform Grid	Jittered Grid	Poisson Disk
			G.	L.	M.				
bull	0.125	33.75	33.70	33.46	33.51	33.46	33.48	33.59	**33.81**
	0.250	**38.45**	38.21	38.36	38.31	38.18	38.39	38.27	38.00
	0.500	41.55	41.51	**41.61**	41.55	41.53	41.60	41.51	40.79
	1.000	43.55	43.45	43.43	43.49	43.55	43.55	**43.56**	42.41
	2.000	45.44	45.39	45.35	45.40	45.45	45.40	45.43	**45.51**
	3.000	**46.92**	46.81	46.82	46.79	46.86	46.85	46.82	46.89
ct	0.125	28.52	28.48	**28.86**	28.38	28.46	28.32	28.55	28.14
	0.250	32.64	32.78	**32.86**	32.68	32.64	32.67	32.73	32.84
	0.500	37.30	37.35	37.51	37.41	37.50	37.50	37.27	**37.54**
	1.000	41.27	41.35	41.28	41.26	**41.48**	41.44	41.43	41.41
	2.000	45.37	**45.41**	45.24	45.28	45.27	45.36	45.31	45.00
	3.000	**47.87**	47.86	47.82	47.84	47.85	47.84	47.85	47.37
lena	0.125	21.77	21.46	21.68	21.79	21.84	21.75	21.63	**22.16**
	0.250	24.17	23.98	24.18	24.04	**24.36**	24.21	24.29	**24.36**
	0.500	26.53	26.34	26.45	26.54	26.60	26.53	26.61	**26.73**
	1.000	28.99	28.69	29.01	29.01	29.05	29.01	29.03	**29.24**
	2.000	31.69	31.55	31.62	31.65	31.62	31.70	**31.72**	31.58
	3.000	33.24	33.05	33.13	33.06	33.20	**33.25**	33.13	32.88

[4] Gradient, Laplacian, and MMSODD

Two methods proposed: low complexity IID1 and higher complexity IID2

- Initial mesh: error diffusion with same size as target mesh (new)
- Growth schedule: type A' (new), amplitude = 3, and
  - for IID1: length = 4
  - for IID2: length = 6
- Insertion face: SSE
- Insertion point selection:
  - for IID1: MMSODD (new)
  - for IID2: ALSEM
- Postprocessing: Bad point replacement

Evaluation of Proposed Methods – Mesh Quality

Image	Sampling density (%)	PSNR (dB)					
		IID1	IID2	ID1	ID2	IDDT	GPR
bull	0.125	31.14	34.18	**34.90**	*34.56*	33.85	33.51
	0.250	37.63	**39.15**	*38.87*	38.76	37.51	38.18
	0.500	41.45	**42.24**	41.84	**42.24**	40.42	41.89
	1.000	43.81	*44.22*	43.91	**44.27**	42.50	43.97
	2.000	45.73	*46.09*	45.79	**46.13**	44.46	45.83
	3.000	47.08	**47.37**	47.15	*47.37*	45.78	47.14
	4.000	48.23	**48.44**	48.26	*48.44*	46.97	48.24
ct	0.125	27.71	**28.88**	*28.62*	28.60	27.52	28.22
	0.250	32.30	*33.09*	**33.27**	32.99	32.43	32.38
	0.500	37.77	37.87	**38.20**	*37.88*	37.44	37.44
	1.000	**42.07**	41.79	*41.97*	41.74	41.37	41.45
	2.000	45.62	45.59	**45.83**	*45.69*	45.25	45.32
	3.000	47.96	48.10	**48.30**	*48.17*	47.74	47.88
	4.000	49.91	49.99	**50.16**	*50.07*	49.63	49.80
lena	0.125	20.43	**22.76**	22.03	*22.50*	20.39	22.08
	0.250	23.73	**24.90**	24.68	*24.84*	23.18	24.38
	0.500	26.75	**27.19**	26.93	*27.10*	25.82	26.59
	1.000	29.40	*29.58*	29.44	**29.62**	28.46	29.09
	2.000	32.10	**32.22**	32.15	*32.17*	31.05	31.78
	3.000	33.63	**33.73**	33.59	*33.64*	32.50	33.37
	4.000	34.66	34.71	34.62	**34.72**	33.49	34.42

Table: Comparison of the mesh quality of the methods: averaged rankings for 50 images

Sampling density (%)	**Mean Rank** (standard deviation in parentheses)					
	IID1	IID2	ID1	ID2	IDDT	GPR
0.125	4.92 (0.57)	**1.36** (0.94)	3.72 (0.86)	2.08 (0.57)	5.64 (1.16)	3.28 (0.83)
0.250	4.74 (0.69)	**1.38** (0.81)	3.68 (1.04)	2.06 (0.68)	5.70 (1.04)	3.44 (0.93)
0.500	4.02 (0.84)	**1.42** (0.84)	3.70 (1.36)	2.04 (0.70)	5.78 (0.91)	4.02 (0.94)
1.000	3.58 (0.95)	**1.38** (0.67)	3.46 (1.28)	2.02 (0.74)	5.70 (1.20)	4.46 (0.99)
2.000	3.12 (0.77)	**1.32** (0.65)	3.38 (1.18)	1.96 (0.73)	5.68 (1.20)	4.64 (1.05)
3.000	3.10 (0.81)	**1.40** (0.83)	3.22 (1.23)	1.98 (0.65)	5.68 (1.20)	4.70 (0.99)
4.000	3.18 (0.80)	**1.38** (0.67)	3.20 (1.26)	1.96 (0.67)	5.68 (1.20)	4.70 (0.99)
Overall	3.81 (1.06)	**1.38** (0.77)	3.48 (1.19)	2.01 (0.67)	5.69 (1.13)	4.18 (1.11)

Evaluation of Proposed Methods – Mesh Quality

IID1 ID1

Figure: Comparison of (part of) the image approximation obtained for the lena image with 2% sampling density

Evaluation of Proposed Methods – Mesh Quality

IID2

ID2

Figure: Comparison of (part of) the image approximation obtained for the lena image with 2% sampling density

Evaluation of Proposed Methods – Computational Cost

Table: Comparison of the computational cost of the methods

Image	**Sampling density(%)**	**Execution Time (s)**					
		IID1	IID2	ID1	ID2	IDDT	GPR
`bull`	0.125	4.0	8.8	5.0	11.5	3.2	125.4
	0.250	5.4	11.1	8.0	16.3	4.3	117.5
	0.500	7.2	16.0	16.1	26.7	7.5	116.1
	1.000	9.9	22.6	24.0	38.5	11.4	115.2
	2.000	14.8	31.4	30.3	45.4	16.1	112.3
	4.000	24.4	41.2	29.6	45.2	22.0	109.7
`ct`	0.125	1.1	2.7	1.4	3.4	0.9	38.9
	0.250	1.2	3.2	1.8	4.0	1.1	39.2
	0.500	1.8	4.0	2.4	5.2	1.5	36.6
	1.000	2.7	5.4	3.0	6.0	1.9	37.4
	2.000	4.2	7.2	4.1	7.6	2.6	36.8
	4.000	7.1	12.2	6.7	11.1	4.0	35.3
`lena`	0.125	1.1	3.0	1.6	3.6	1.2	39.1
	0.250	1.2	2.9	1.7	4.1	1.2	39.1
	0.500	1.6	3.9	2.5	4.9	1.4	38.7
	1.000	2.5	5.3	3.3	6.3	2.0	37.5
	2.000	4.1	7.8	4.7	8.6	2.7	37.3
	4.000	7.1	12.7	7.6	11.9	4.3	36.8

Conclusions

- Proposed methods outperform method of similar complexity: improvement obtained in terms of approximation quality, computational cost, or both
- Better efficiency achieved: higher quality image approximations for a given computational cost
- Trade off between computational cost and mesh quality: useful in wide range of applications
- Growth schedule and initial mesh choices: significant impact on performance

Thank you