An Improved Method for the Design of FIR Quadrature Mirror-Image Filter Banks

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***Abstract*— A new method for the design of general finite-duration impulse response (FIR) quadrature mirror-image filter (QMF) banks that eliminates the computation of large matrices is proposed. The design problem is formulated to include low-delay QMF banks, which are highly desirable in some applications. The paper concludes with design results and comparisons that show that conventional QMF banks can be designed with only a fraction of the computational effort required by a method due to Chen and Lee. On the other hand, in the case of low-delay QMF banks, the proposed method can increase the stopband attenuation substantially compared with what can be achieved by existing methods.**

I. Introduction

FINITE-duration impulse response (FIR) quadrature mirror-image filter (QMF) banks have been widely used in one-dimensional (1-D) and two-dimensional (2-D) signal processing [1]–[4], and many theories and techniques have been developed for their analysis and design [5]–[16].

In [6], an iterative algorithm was proposed for the design of QMF banks, which involves calculating the eigenvalues and eigenvectors of a matrix in each iteration. In [11], Chen and Lee introduced an iterative procedure that uses a linearization of the error function in the frequency domain, which speeds up convergence. This design method needs less computation than other QMF design methods [5], [9], [10] and leads to good results. However, the objective function involves two integrals that are evaluated by discretization. This gives rise to two problems. First, the solution obtained actually minimizes the discretized version of the objective function rather than the objective function itself, which can degrade the performance of the QMF bank designed. Second, in order to reduce the performance degradation, the density of sample points needs to be high, which leads to increased computational complexity.

In this paper, a new iterative method based on the method of Chen and Lee [11] for the design of two-channel QMF banks is proposed. Our method differs from the method in [11] in that the perfect reconstruction condition is formulated in the time domain, which leads to reduced computation complexity in the design. On the other hand, unlike the algorithm in [6], the proposed algorithm does not require the calculation of the eigenvalues and eigenvectors of a matrix, and a linear update formula is adopted that improves the speed of convergence. The design problem is formulated to include the design of low-delay QMF banks, which are highly desired in some applications.

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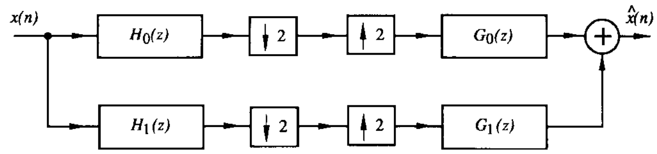

Fig. 1. Two-channel FIR filter bank.

II. Design of General Two-Channel QMF Banks

A. General Objective Function

The input-output relation of the two-channel filter bank illustrated in Fig. 1 is given by

$$\hat{X}(z) = \tfrac{1}{2}[H_0(z)G_0(z) + H_1(z)G_1(z)]X(z) + \tfrac{1}{2}[H_0(-z)G_0(z) + H_1(-z)G_1(z)]X(-z). \quad (1)$$

By assuming that $H_1(z) = H_0(-z)$, $G_1(z) = -H_0(-z)$, and $G_0(z) = H_0(z)$, the aliasing term on the right side is canceled and (1) becomes

$$\hat{X}(z) = \tfrac{1}{2}[H_0^2(z) - H_0^2(-z)]X(z). \quad (2)$$

Perfect reconstruction requires that

$$H_0^2(z) - H_0^2(-z) = z^{-k_d} \quad (3)$$

where $k_d \leq N-1$ is the normalized reconstruction delay, and N is the length of filter H_0 and is assumed to be even. Equation (3) can be expressed in the time domain in terms of the convolution as

$$\boldsymbol{B}_L\boldsymbol{h}_L = \boldsymbol{m}_L \quad (4a)$$

where $\boldsymbol{h}_L = [h(0) \quad h(1)\cdots h(N-1)]^T$, and $\boldsymbol{m}_L = [0\cdots 0 \ 1 \ 0\cdots 0]^T$ is a vector with zero elements except for the $[(k_d+1)/2]$th element, which is unity. $\boldsymbol{B}_L$ is an $(N-1)\times N$ matrix of the form of (4b), shown at the bottom of the next page.

Assuming a passband gain of unity for the lowpass filter H_0, the L_2 objective function

$$\Psi_L = \int_0^{\omega_p} |H_0(e^{j\omega}) - e^{-jk_d\omega/2}|^2\, d\omega + \int_{\omega_s}^{\pi} |H_0(e^{j\omega})|^2\, d\omega \quad (5)$$

can be constructed, where ω_p and ω_s are the passband and stopband edges, respectively.

Taking the perfect reconstruction condition in (4) into account, the design problem is reduced to minimizing Ψ_L in (5) subject to the constraints in (4), which is a difficult constrained optimization problem.

B. Iterative Method

Instead of using constrained optimization to accomplish the design, which is very time consuming in general, we propose an iterative method. The objective function is formed as

$$E_L = E_{L1} + \alpha E_{L2}. \tag{6a}$$

In (6a), error component E_{L1} deals with the perfect reconstruction condition and is given by

$$E_{L1} = (\boldsymbol{B}_L \boldsymbol{f}_L - \boldsymbol{m}_L)^T (\boldsymbol{B}_L \boldsymbol{f}_L - \boldsymbol{m}_L) \tag{6b}$$

where $\boldsymbol{B}_L$ and $\boldsymbol{m}_L$ are defined in (4a) and (4b), and $\boldsymbol{f}_L = [f(0)\ f(1) \cdots f(N-1)]^T$. The error component

$$E_{L2} = \int_0^{\omega_p} |F_0(e^{j\omega}) - e^{-jk_d\omega/2}|^2\, d\omega + \int_{\omega_s}^{\pi} |F_0(e^{j\omega})|^2\, d\omega \tag{6c}$$

deals with the frequency response requirement where $F_0(e^{j\omega}) = \boldsymbol{f}_L^T \boldsymbol{c}_L(\omega)$ with $\boldsymbol{c}_L(\omega) = [1\ e^{-j\omega} \cdots e^{-j\omega(N-1)}]^T$.

It should be noted that whereas both (6a) and the objective function employed in [11] are quadratic expressions, (6a) differs from that in [11] in two ways: First, the perfect reconstruction error term in (6a), namely, E_{L1}, is formulated in the *time domain* rather than the frequency domain. This entirely eliminates the need to evaluate the integral

$$\int_0^{\pi} [H_0(\omega)F_0(\omega) + H_0(\omega+\pi)F_0(\omega+\pi) - 1]^2\, d\omega$$

in the optimization process and, hence, reduces the computation complexity; second, (6a) includes the system delay k_d as an adjustable parameter, and consequently, by minimizing E_L in (6a), both conventional and low-delay filter banks can be designed.

We start the iteration by designing a lowpass filter H_0 with a group delay $k_d/2$, where k_d is the desired system delay, and use its impulse response vector as the initial $\boldsymbol{h}_L$. Then, E_L in (6a) can be formulated as a quadratic function in $\boldsymbol{f}_L$ as

$$\begin{aligned} E_L = &\boldsymbol{f}_L^T(\boldsymbol{B}_L^T\boldsymbol{B}_L + \alpha\boldsymbol{Q}_L)\boldsymbol{f}_L \\ &+ (\alpha\boldsymbol{b}_L^T - 2\boldsymbol{m}_L^T\boldsymbol{B}_L)\boldsymbol{f}_L + \omega_p + 1 \end{aligned} \tag{7}$$

where

$$\begin{aligned} \boldsymbol{Q}_L &= \mathrm{Re}\left[\int_0^{\omega_p} \boldsymbol{c}_L(\omega)\boldsymbol{c}_L^H(\omega)\, d\omega + \int_{\omega_s}^{\pi} \boldsymbol{c}_L(\omega)\boldsymbol{c}_L^H(\omega)\, d\omega\right] \\ &= \boldsymbol{U}_p + \boldsymbol{U}_s \end{aligned} \tag{8a}$$

with $\boldsymbol{U}_p$ and $\boldsymbol{U}_s$ being two Toeplitz matrices defined as shown on the bottom of the next page, and

$$\begin{aligned} \boldsymbol{b}_L &= -2\ \mathrm{Re}\left[\int_0^{\omega_p} \boldsymbol{c}_L(\omega)e^{jk_d\omega/2}\, d\omega\right] \\ &= -2\begin{bmatrix} \dfrac{2}{k_d}\sin\left(\dfrac{k_d\omega_p}{2}\right) \\ \dfrac{2}{k_d-2}\sin\left[\left(\dfrac{k_d}{2}-1\right)\omega_p\right] \\ \vdots \\ \dfrac{2}{k_d-2N+2}\sin\left[\left(\dfrac{k_d}{2}-N+1\right)\omega_p\right] \end{bmatrix}. \end{aligned} \tag{8b}$$

Since $\boldsymbol{B}_L^T\boldsymbol{B}_L + \alpha\boldsymbol{Q}_L$ is positive definite, the global minimum of E_L is achieved if

$$\boldsymbol{f}_L = -\tfrac{1}{2}(\boldsymbol{B}_L^T\boldsymbol{B}_L + \alpha\boldsymbol{Q}_L)^{-1}(\alpha\boldsymbol{b}_L - 2\boldsymbol{B}_L^T\boldsymbol{m}_L). \tag{9}$$

Having obtained $\boldsymbol{f}_L$, a linear formula can be used to update $\boldsymbol{h}_L$ as

$$\boldsymbol{h}_L := (1-\tau)\boldsymbol{h}_L + \tau\boldsymbol{f}_L. \tag{10}$$

The above procedure is repeated until $||\boldsymbol{h}_L - \boldsymbol{f}_L||$ is smaller than a prescribed tolerance.

On the basis of the preceding analysis, an iterative algorithm can now be constructed as follows:

Algorithm 1—Design of Low-Delay QMF Banks:

Step 1) Use a least-squares approach to design a lowpass FIR filter of length N, group delay $k_d/2$, and passband and stopband edges ω_p and ω_s, respectively; then use the impulse response of the filter obtained as the initial $\boldsymbol{h}_L$.

Step 2) Calculate $\boldsymbol{Q}_L$ and $\boldsymbol{b}_L$ using (8a) and (8b), respectively.

Step 3) Use (4) to form $\boldsymbol{B}_L$ and $\boldsymbol{m}_L$, and compute $\boldsymbol{f}_L$ using (9).

Step 4) If $||\boldsymbol{h}_L - \boldsymbol{f}_L|| < \epsilon$, where ϵ is a prescribed tolerance, output $\boldsymbol{f}_L$ as the impulse response of the required design and stop; otherwise, update $\boldsymbol{h}_L$ using (10) with a value for τ between 0.5 and 0.75 and repeat from Step 3.

$$\boldsymbol{B}_L = 2\begin{bmatrix} h(1) & h(0) & 0 & 0 & \cdots & 0 \\ h(3) & h(2) & h(1) & h(0) & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ h(N-1) & h(N-2) & h(N-3) & h(N-4) & \cdots & h(0) \\ 0 & 0 & h(N-1) & h(N-2) & \cdots & h(2) \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & h(N-2) \end{bmatrix} \tag{4b}$$

Several comments on the design of two-channel low-delay QMF banks are now in order.

- With $k_d = N - 1$, the proposed algorithm can be used to design conventional QMF banks. When the filter bank is required to have linear phase response, the number of design parameters is reduced to $N/2$. In Section II-C, a more efficient algorithm will be developed for the design of this important class of filter banks.
- In the design of two-channel QMF banks with low delay, the impulse response is not in general symmetrical, which means that exactly linear phase response cannot be obtained. However, approximately linear phase response can be achieved with respect to the passband, as will be seen from the design results presented in the next section.
- The proposed method can be used to design low-delay QMF filter banks whose lowpass and highpass filters satisfy the mirror-image symmetry relationship, which leads to efficient polyphase implementation. The numbers of multiplications and additions per input in a polyphase implementation can be reduced by 50% relative to those in a direct implementation.
- When a very low reconstruction delay k_d is required, artifacts may occur in the transition region of the designed filter H_0, which have also been observed in [13]. An effective approach to deal with this problem is to modify the objective function to include an additional term $\alpha_1 E_{L3}$, i.e.,

$$E_L = E_{L1} + \alpha E_{L2} + \alpha_1 E_{L3} \tag{11}$$

where

$$E_{L3} = \int_{\omega_{t1}}^{\omega_{t2}} |F_0(e^{j\omega}) - e^{-jk_d\omega/2}|^2 \, d\omega \tag{12}$$

and $[\omega_{t1}, \omega_{t2}]$ is an interval in the transition region where the artifacts occur. It can be readily shown that with this modification, Algorithm 1 can still be used after certain modifications in (7)–(10) as follows. The objective function in (7) assumes the form

$$\begin{aligned} E_L = &\boldsymbol{f}_L^T(\boldsymbol{B}_L^T\boldsymbol{B}_L + \alpha\boldsymbol{Q}_L + \alpha_1\boldsymbol{Q}_t)\boldsymbol{f}_L \\ &+ (\alpha\boldsymbol{b}_L^T + \alpha_1\boldsymbol{b}_t^T - 2\boldsymbol{m}_L^T\boldsymbol{B}_L)\boldsymbol{f}_L + c \end{aligned} \tag{13a}$$

where c is a constant, $\boldsymbol{Q}_t = \{q_{ij}\} \in R^{N\times N}$ with

$$q_{ij} = q_{ji}$$
$$q_{ij} = \begin{cases} \omega_{t2} - \omega_{t1} & \text{for } i = j \\ \phi(\omega_{t2}, \omega_{t1}, |i-j|) & \text{for } i \neq j \end{cases}$$

and

$$\phi(\omega_{t2}, \omega_{t1}, k) = \frac{1}{k}[\sin k\omega_{t2} - \sin k\omega_{t1}]. \tag{13b}$$

On the other hand, vector $\boldsymbol{b}_L$ in (8b) becomes

$$\begin{aligned} \boldsymbol{b}_t &= -2 \text{ Re}\left[\int_{\omega_{t1}}^{\omega_{t2}} \boldsymbol{c}_L(\omega)e^{jk_d\omega/2}\, d\omega\right] \\ &= -2\begin{bmatrix} \phi\left(\omega_{t2}, \omega_{t1}, \frac{k_d}{2}\right) \\ \phi\left(\omega_{t2}, \omega_{t1}, \frac{k_d}{2} - 1\right) \\ \vdots \\ \phi\left(\omega_{t2}, \omega_{t1}, \frac{k_d}{2} - N + 1\right) \end{bmatrix} \end{aligned}$$

where $\phi(\omega_{t2},\ \omega_{t1},\ k)$ is defined by (13b).

The global minimum of E_L is achieved if

$$\begin{aligned} \boldsymbol{f}_L = &-\tfrac{1}{2}(\boldsymbol{B}_L^T\boldsymbol{B}_L + \alpha\boldsymbol{Q}_L + \alpha_1\boldsymbol{Q}_t)^{-1} \\ &\cdot (\alpha\boldsymbol{b}_L + \alpha_1\boldsymbol{b}_t - 2\boldsymbol{B}_L^T\boldsymbol{m}_L). \end{aligned} \tag{14}$$

C. Design of Linear-Phase QMF Banks

For conventional QMF banks, the system delay k_d in (3) is equal to $N - 1$, and filter H_0 has a symmetrical impulse response, which guarantees a linear phase response. Consequently, the number of parameters involved in the design is reduced to $N/2$, and the perfect reconstruction condition in the time domain can be expressed as

$$\boldsymbol{Bh} = \boldsymbol{m} \tag{15}$$

where

$$\boldsymbol{B} = [\boldsymbol{d}_1 + \boldsymbol{d}_N \ \ \boldsymbol{d}_2 + \boldsymbol{d}_{N-1} \ \cdots \ \boldsymbol{d}_{N/2} + \boldsymbol{d}_{N/2+1}] \tag{16a}$$

$$\boldsymbol{U}_p = \begin{bmatrix} \omega_p & \sin\omega_p & \frac{\sin 2\omega_p}{2} & \cdots & \frac{\sin(N-1)\omega_p}{N-1} \\ \sin\omega_p & \omega_p & \sin\omega_p & \cdots & \frac{\sin(N-2)\omega_p}{N-2} \\ \vdots & & \ddots & & \vdots \\ \frac{\sin(N-1)\omega_p}{N-1} & \cdots & \cdots & \cdots & \omega_p \end{bmatrix}$$

$$\boldsymbol{U}_s = \begin{bmatrix} \pi - \omega_s & -\sin\omega_s & -\frac{\sin 2\omega_s}{2} & \cdots & -\frac{\sin(N-1)\omega_s}{N-1} \\ -\sin\omega_s & \pi - \omega_s & -\sin\omega_s & \cdots & -\frac{\sin(N-2)\omega_s}{N-2} \\ \vdots & & \ddots & & \vdots \\ -\frac{\sin(N-1)\omega_s}{N-1} & \cdots & \cdots & \cdots & \pi - \omega_s \end{bmatrix}$$

with

$$[\boldsymbol{d}_1\ \boldsymbol{d}_2\ \cdots\ \boldsymbol{d}_N] = 2 \cdot \begin{bmatrix} h(1) & h(0) & 0 & 0 & \cdots & \cdots & 0 \\ h(3) & h(2) & h(1) & h(2) & \cdots & \cdots & 0 \\ h(5) & h(4) & h(3) & h(4) & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & & \vdots \\ \vdots & \vdots & \vdots & \vdots & & & \vdots \\ h(N-1) & h(N-2) & \cdots & \cdots & \cdots & \cdots & h(0) \end{bmatrix} \tag{16b}$$

and

$$\boldsymbol{h} = [h(0)\quad h(1)\quad \cdots\quad h(N/2-1)]^T \tag{17}$$

$$\boldsymbol{m} = [0\quad \cdots\quad 0\quad 1]^T \tag{18}$$

and $h(i)$, $i = 0, \cdots, N-1$ is the impulse response of filter H_0. After some algebraic manipulation, matrix $\boldsymbol{Q}$ and vectors $\boldsymbol{b}$, $\boldsymbol{f}$, which are anologous to matrix $\boldsymbol{Q}_L$ and vectors $\boldsymbol{b}_L$, $\boldsymbol{f}_L$, respectively, can be deduced as

$$\boldsymbol{Q} = \{q_{ij}\}$$

where

$$\begin{aligned} q_{ij} &= 4\int_0^{\omega_p} \cos\left[\left(i-1-\frac{N-1}{2}\right)\omega\right] \cdot \cos\left[\left(j-1-\frac{N-1}{2}\right)\omega\right] d\omega \\ &\quad + 4\int_{\omega_s}^{\pi} \cos\left[\left(i-1-\frac{N-1}{2}\right)\omega\right] \cdot \cos\left[\left(j-1-\frac{N-1}{2}\right)\omega\right] d\omega \\ &= 2\begin{cases} \phi(\omega_p, \omega_s, a_1) + \pi - \omega_s + \omega_p & \text{for } i = j \\ \phi(\omega_p, \omega_s, a_1) + \phi(\omega_p, \omega_s, a_2) & \text{for } i \neq j \end{cases} \end{aligned} \tag{19}$$

with $a_1 = i + j - N - 1$ and $a_2 = i - j$

$$\boldsymbol{b} = -8 \begin{bmatrix} \sin\left(\frac{N-1}{2}\omega_p\right) \Big/ (N-1) \\ \sin\left(\frac{N-3}{2}\omega_p\right) \Big/ (N-3) \\ \vdots \\ \sin\left(\frac{1}{2}\omega_p\right) \end{bmatrix} \tag{20}$$

and

$$\boldsymbol{f} = -\tfrac{1}{2}(\boldsymbol{B}^T\boldsymbol{B} + \alpha\boldsymbol{Q})^{-1}(\alpha\boldsymbol{b} - 2\boldsymbol{B}^T\boldsymbol{m}). \tag{21}$$

The formula for updating $\boldsymbol{h}$ is given by

$$\boldsymbol{h} := (1-\tau)\boldsymbol{h} + \tau\boldsymbol{f}. \tag{22}$$

A step-by-step description of the design procedure is given in terms of the following algorithm:

Algorithm 2–Design of Linear-Phase QMF Banks:

Step 1) Use a conventional method (e.g., the window method) to design a linear-phase lowpass FIR filter of length N, and use its impulse response as the initial $\boldsymbol{h}$.

Step 2) Calculate $\boldsymbol{Q}$ and $\boldsymbol{b}$ using (19) and (20), respectively, using specified values for ω_p and ω_s.

TABLE I

COMPARISON OF THE PROPOSED METHOD

	Proposed	Method of [13]
MFLOPS	1.26	-
A_a (dB)	60.96	36.97
A_p (dB)	0.0532	0.0537
PRE (dB)	1.8×10^{-3}	1.2×10^{-3}
SNR_s (dB)	78.5	75.8
SNR_r (dB)	77.6	77.4

Step 3) Use (16) and (18) to form $\boldsymbol{B}$ and $\boldsymbol{m}$, respectively, and compute $\boldsymbol{f}$ using (21).

Step 4) If $\|\boldsymbol{h} - \boldsymbol{f}\| < \epsilon$, where ϵ is a prescribed tolerance, output $\boldsymbol{f}$ as the impulse response of the required design and stop; otherwise, update $\boldsymbol{h}$ using (22) with a value for τ close to 0.5 and repeat from Step 3.

D. Filter Banks with Regularity

It is known that the smoothness of a wavelet (or scaling) function, which is important in many engineering applications, largely depends on the flatness of the associated analysis lowpass filter at $\omega = \pi$ [17]. If we use the number of zeros of $H_0(e^{j\omega})$ at $\omega = \pi$ as the degree of regularity (DoR) of the filter bank, the algorithm developed in this paper can be readily modified to incorporate the constraints

$$\left.\frac{d^k}{d\omega^k} H_0(e^{j\omega})\right|_{\omega=\pi} = 0 \quad \text{for } k = 0, 1, \cdots, K-1$$

in order to design a low-delay QMF bank with DoR $= K$. In effect, with the above constraints, each iteration of the modified design method solves a quadratic programming problem with a set of equality constraints.

III. EXAMPLES

In this section, we present results obtained by applying the algorithms developed to two design examples. The proposed method is then compared with some existing methods in terms of design efficiency and the performance of the filter banks obtained.

A. Low-Delay QMF Banks

As Example 1, a QMF bank with low reconstruction delay was designed with Algorithm 1 using the design parameters $N = 32$, $k_d = 15$, $\alpha = 5 \times 10^{-3}$, $\alpha_1 = 3 \times 10^{-5}$, $\omega_p = 0.35\pi, \omega_s = 0.72\pi, \omega_{t1} = 0.35\pi, \omega_{t2} = 0.45\pi, \tau = 0.5$, and $\epsilon = 10^{-3}$. For comparison purposes, we refer to [13, Example 6.6.1], which was designed with another time-domain approach. Comparisons were carried out in terms of

- number of iterations (NI);
- number of floating-point operations in millions (MFLOPS);
- minimum stopband attenuation

$$A_a = \min_{\omega_s \le \omega \le \pi} [-20 \log_{10} |H_0(e^{j\omega})|];$$

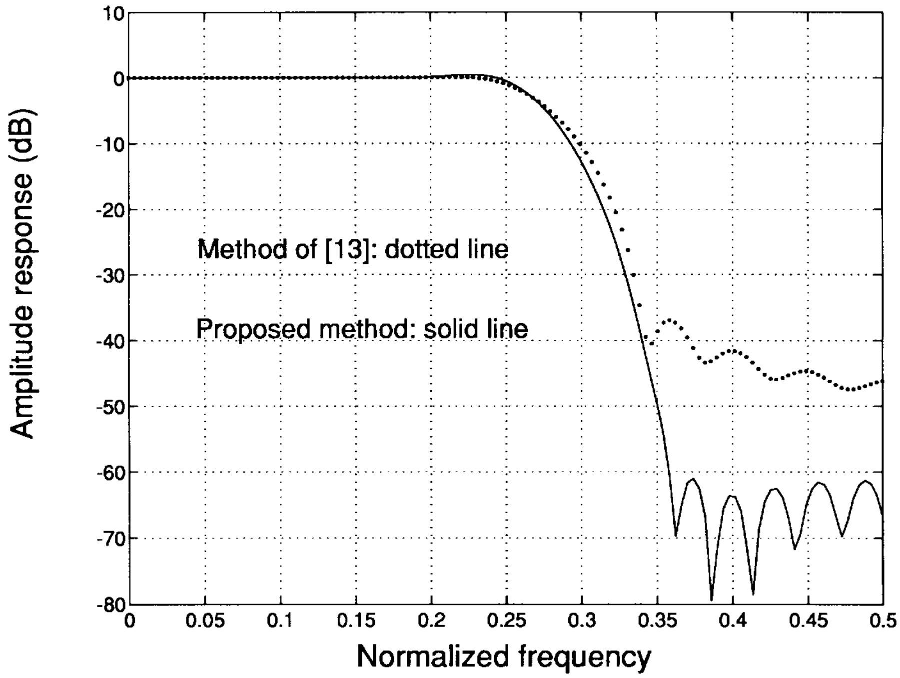

Fig. 2. Amplitude responses of lowpass filters in Example 1.

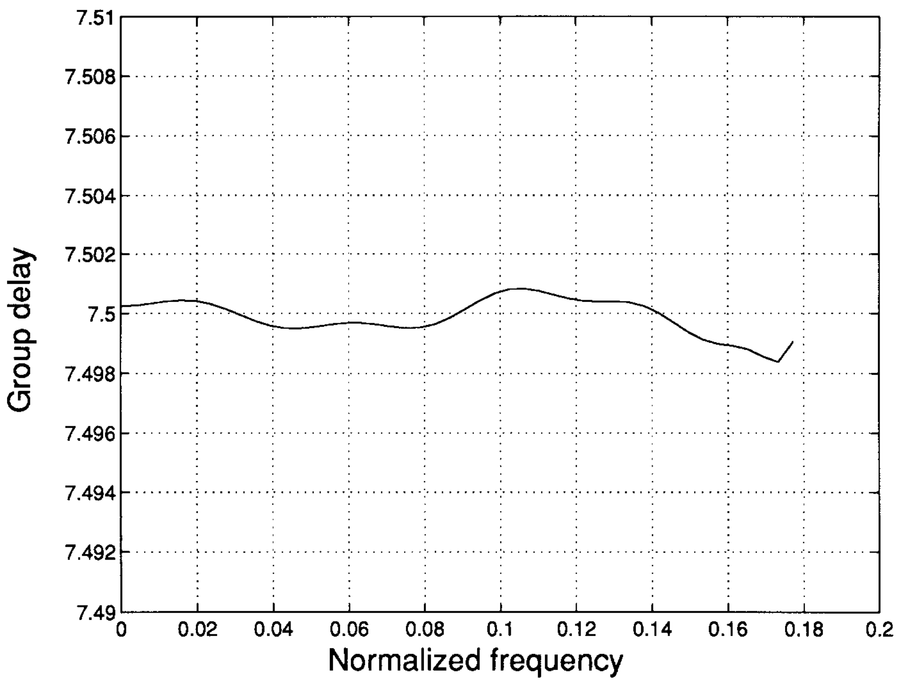

Fig. 3. Normalized group-delay characteristic of lowpass filter in Example 1.

- peak-to-peak passband ripple

$$A_p = \max_{0 \le \omega \le \omega_p} [20 \log_{10} |H_0(e^{j\omega})|] - \min_{0 \le \omega \le \omega_p} [20 \log_{10} |H_0(e^{j\omega})|]$$

where ω_p is the passband edge;
- peak reconstruction error

$$\text{PRE} = \max_{\omega} |20 \log_{10}[|H_0^2(\omega) - H_0^2(\omega + \pi)|]|$$

- signal-to-noise ratio

$$\text{SNR} = 10 \log_{10} \left(\frac{\text{energy of the signal}}{\text{energy of the reconstruction noise}} \right) = 10 \log_{10} \left\{ \frac{\Sigma x^2(n)}{\Sigma[x(n) - \hat{x}(n + k_d)]^2} \right\}.$$

The results are summarized in Table I, where SNR_s and SNR_r denote the SNR with a step input and a random input, respectively. The amplitude responses of the lowpass

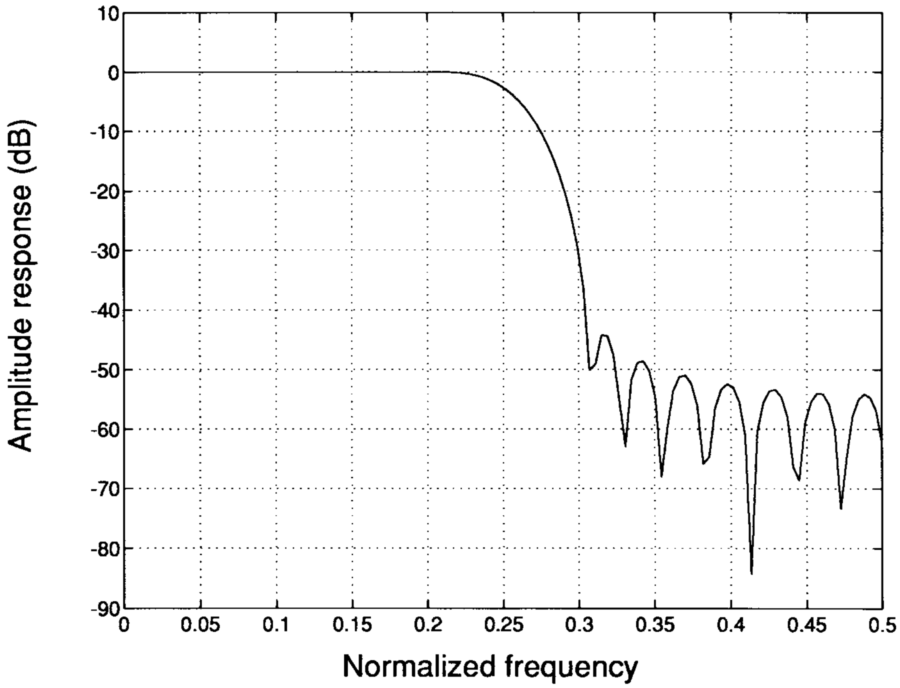

Fig. 4. Amplitude response of lowpass filter H_0 for both designs in Example 2.

analysis filters designed with the proposed method and the method in [13] are depicted in Fig. 2. As can be seen from Table I and Fig. 2, the proposed method yields an improved design in terms of increased stopband attenuation relative to that in the example reported in [13]. The nomalized group-delay characteristic of the designed filter H_0, i.e., $\tau_n = -d\theta(\omega)/Td\omega$ s, where $\theta(\omega)$ is the phase response of the filter and T is the sampling interval, is plotted in Fig. 3. As can be observed, the normalized group delay is approximately equal to $k_d/2 = 7.5$ s, i.e., the filter has approximately linear phase with respect to the passband. In addition, unlike the low-delay filter banks desinged in [13], the quadrature mirror-image structure of the proposed design is amenable to an efficient polyphase-type implementation [4], [7], which requires only $N/2$ additions and $N/2$ multiplications per input as compared with N additions and N multiplications per input for the low-delay filter banks in [13]. We also expect our approach to be computationally less demanding than that in [13] in view of the efficiency achieved through the linearization step.

B. Conventional QMF Banks

As Example 2, Algorithm 2 was applied to design a conventional two-channel QMF bank using the design parameters $N = 32$, $\alpha = 0.1$, $\omega_p = 0.3\pi$, $\omega_s = 0.6\pi$, $\tau = 0.7$, and $\epsilon = 10^{-3}$. The initial $\boldsymbol{h}$ was obtained by using the window method. For comparison purposes, the method of Chen and Lee [11] was used to design a corresponding QMF bank with the parameters $N = 32$, $\alpha = 1$, $\omega_s = 0.6\pi$, $\tau = 0.7$, and $\epsilon = 10^{-3}$. The initial $\boldsymbol{h}$ was the same as before.

Both the proposed method and the method in [11] were programmed using Matlab and run on a Sun SPARC station. The number of frequency sampling points was set to $8N = 256$ when implementing the method in [11]. The results obtained are summarized in Table II. As can be observed, the proposed method has resulted in almost the same design as the method in [11] with only about 5% the computation. The amplitude response of the lowpass filter H_0 for both designs is illustrated in Fig. 4.

TABLE II
COMPARISON OF THE PROPOSED METHOD

	Proposed	Chen-Lee
NI	3	5
MFLOPS	0.060	1.12
A_a (dB)	34.9	35.27
A_p (dB)	0.0114	0.0131
PRE (dB)	0.0145	0.0152
SNR_s (dB)	82.9	83.4
SNR_r (dB)	69.5	67.8

IV. CONCLUSIONS

A new iterative method for the design of conventional and low-delay QMF banks has been developed by formulating the perfect reconstruction condition in the time domain. Several design examples have shown that in conventional QMF bank designs, our method can achieve the same designs with only 5% the computation required by the method in [11], and in low-delay QMF bank designs, the proposed method can increase the stopband attenuation by as much as 20 dB over that in [13].

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