ELEC 310 formulas

complex functions

$$z = x + jy, f(z) = u(x, y) + jv(x, y) \quad (1)$$

transfer function, ratio of polynomials with poles and zeros

$$H(z) = \frac{P(z)}{Q(z)} \quad (2)$$

z-transform

$$H(z) = \sum_{k=0}^{\infty} h[k]z^{-k} \quad (3)$$

inverse z-transform, c encloses all poles of $H(z)z^{k-1}$

$$h[k] = \frac{1}{2\pi j}\int_c H(z)z^{k-1}dz \quad (4)$$

c encloses all poles of $H(z)z^{k-1}$

geometric series

$$\frac{C}{1-w} = C\sum_{k=0}^{\infty} w^k \quad (5)$$

$$\frac{C}{(1-w)^2} = C\sum_{k=0}^{\infty}(k+1)w^k = C\sum_{n=1}^{\infty} nw^{(n-1)} \quad (6)$$

$$\frac{C}{(1-w)^3} = C\sum_{k=0}^{\infty}\frac{(k+1)(k+2)}{2}w^k = C\sum_{n=2}^{\infty}\frac{n(n-1)}{2}w^{(n-2)} \quad (7)$$

basic inverse z-transform use geometric series with $w = pz^{-1}$ to obtain transform pair

$$H(z) = \frac{Cz}{z-p} = \frac{C}{1-pz^{-1}} \leftrightarrow h[k] = Cp^k \quad (8)$$

Residue theorem

$$\int_c H(z)dz = 2\pi j\sum_{res} H(z) \quad (9)$$

sum of all residues at poles inside c.

first order pole

$$res_{z=a}F(z) = lim_{z\to a}[(z-a)F(z)] \quad (10)$$

if $F(z) = \frac{P(z)}{Q(z)}$, then

$$res_{z=a}F(z) = \frac{P(a)}{Q'(a)} \quad (11)$$

second order pole

$$res_{z=a}F(z) = lim_{z\to a}\frac{d}{dz}[(z-a)^2F(z)] \quad (12)$$

third order pole

$$res_{z=a}F(z) = lim_{z\to a}\frac{1}{2}\frac{d^2}{dz^2}[(z-a)^3F(z)] \tag{13}$$

mth order pole

$$res_{z=a}F(z) = lim_{z\to a}\frac{1}{m!}\frac{d^{(m-1)}}{dz^{(m-1)}}[(z-a)^mF(z)] \tag{14}$$

frequency response (amplitude and phase), sampling rate f_s

$$H(f) = H(z)|_{z=exp(j2\pi f/f_s)}, -f_s/2 < f < f_s/2 \tag{15}$$

Discrete Fourier Transform (DFT) is frequency response sampled at N_0 samples around the whole unit circle $-f_s/2 < f < f_s/2$ with frequency resolution (step size) $f_0 = f_s/N_0$

$$\begin{aligned}
H[r] &= H(f)|_{f=rf_0=rf_s/N_0}, 0 \le r \le N_0-1 && (16)\\
&= H(z)|_{z=exp(j2\pi f/f_s)}|_{f=rf_s/N_0} && (17)\\
&= H(z)|_{z=exp(j2\pi r/N_0)} && (18)\\
&= \sum_{k=0}^{N_0-1} h[k]z^{-k}|_{z=exp(j2\pi r/N_0)} && (19)\\
&= \sum_{k=0}^{N_0-1} h[k]exp(-j2\pi rk/N_0) && (20)
\end{aligned}$$

inverse DFT

$$h[k] = \frac{1}{N_0}\sum_{r=0}^{N_0-1} H[r]exp(+j2\pi rk/N_0) \tag{21}$$