Binary shape analysis

Mathematical morphology

Required readings:

Sonka 13.1, 13.3

Watershed Transform section (Gonzalez and Woods, excerpt posted on the web site)

Mathematical morphology and image processing

- The analysis of signals and images based on shape (Morphology = "study of shape")
- Uses a set-theoretic approach to modify shapes based on local operators
- Many operations are similar to convolution but using set operations
- Morphological approaches are useful for:
  - Preprocessing
  - Postprocessing
  - Segmentation
  - Feature extraction (shape descriptors)

Set Operations on Binary Images

- A the image ("on" pixels)
- A^c the complement of the image (inverse)
- $A \cup B$ the union of images A and B
- $A \cap B$ the intersection of images A and B
- A - B the difference between A and B (the pixels in A that aren't in B)
- #A the cardinality of A (area of the object)

Morphological operations: translation

- The image A translated by movement vector t is

$A^t = \{c \mid c = a + t \text{ for some } a \text{ in } A\}$

- pick up each pixel in A and move it by the movement vector t
- In common morphological applications, t is a unit vector directed towards N, S, E, W

Dilation

- Consider an image A and a structuring element B, both defined mathematically as sets.
- Dilation of image A by structuring element B is defined as

$$A \otimes B = \{c \mid c = a + b \text{ for some } a \text{ in } A \text{ and } b \text{ in } B\}$$

- We can also express dilation via translation

$$A \oplus B = \bigcup_{t \in B} A_t = \bigcup_{t \in A} B_t$$

Dilation and structuring elements

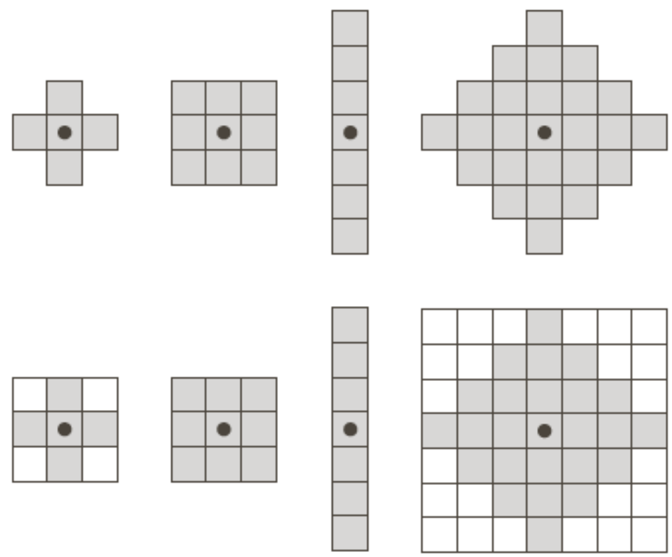
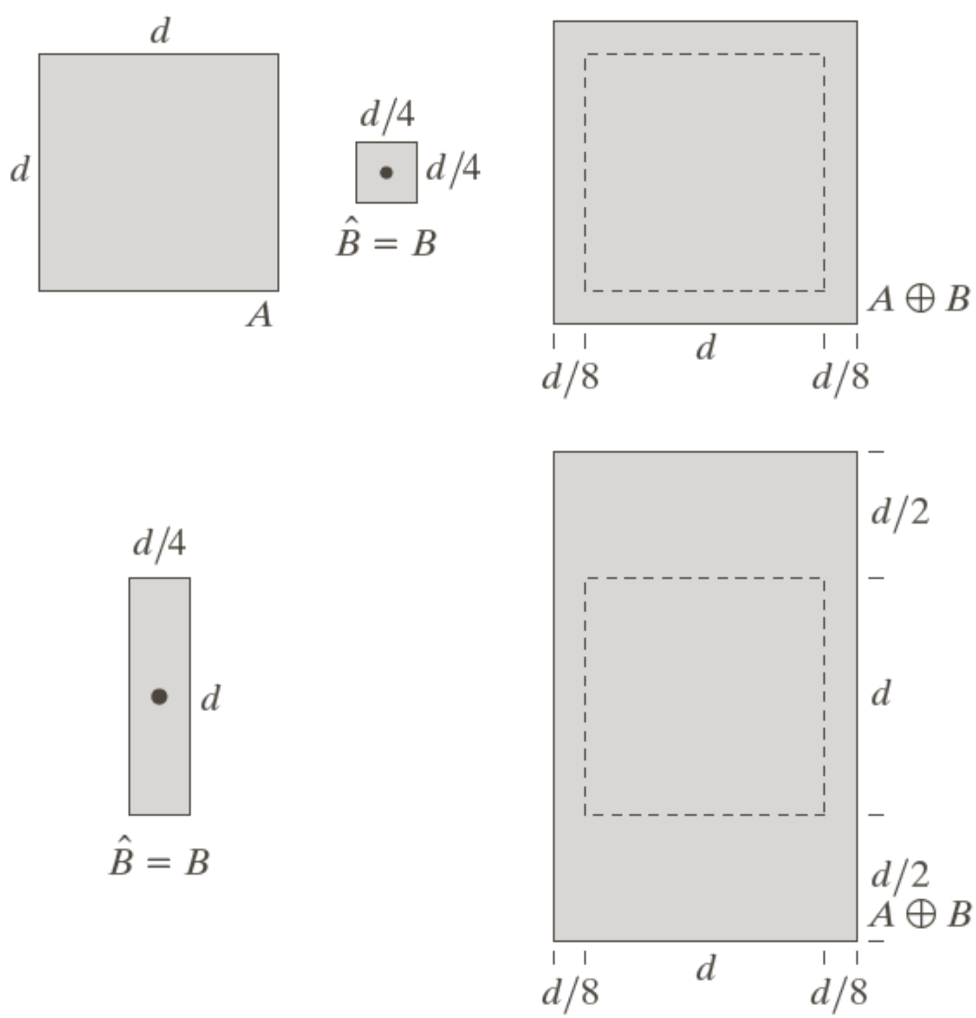


FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

From Gonzalez and Woods, Digital Image Processing, Third edition

Dilation and structuring elements

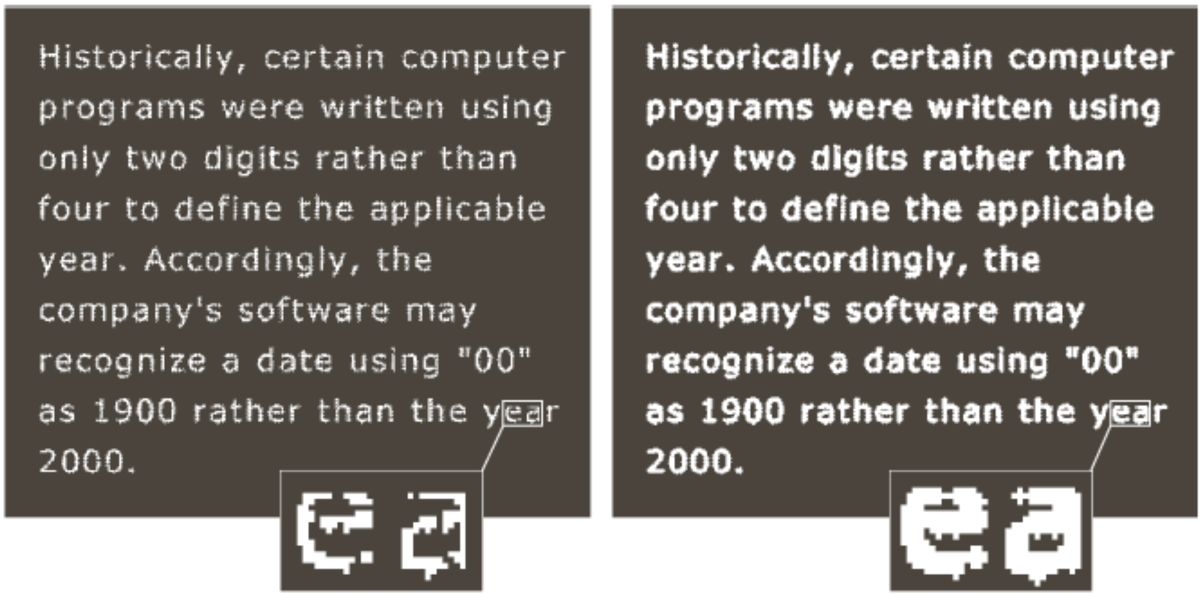


a b c
d e

FIGURE 9.6
(a) Set A.
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

From Gonzalez and Woods, Digital Image Processing, third edition

Applications of dilation: restoration of shape continuity



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Properties of dilation

- Dilation is associative and commutative

- Dilation with a large SE can be implemented via iterative dilations with a small SE

- What SE corresponds a dilation equivalent to two iterations of a 5 point, cross-shaped elementary SE?

Erosion

- Erosion of image A by structuring element B is defined as

$$A \Theta B = \{ x \mid x + b \in A \ for\ every\ b\ in\ B \}$$

- Intuitively, the result of erosion is "everywhere where the structuring element B fist completely inside shape A"
- In terms of translation, the erosion can be defined as

$$A \Theta B = \bigcap_{t \in B} A_{-t}$$

Erosion and structuring elements

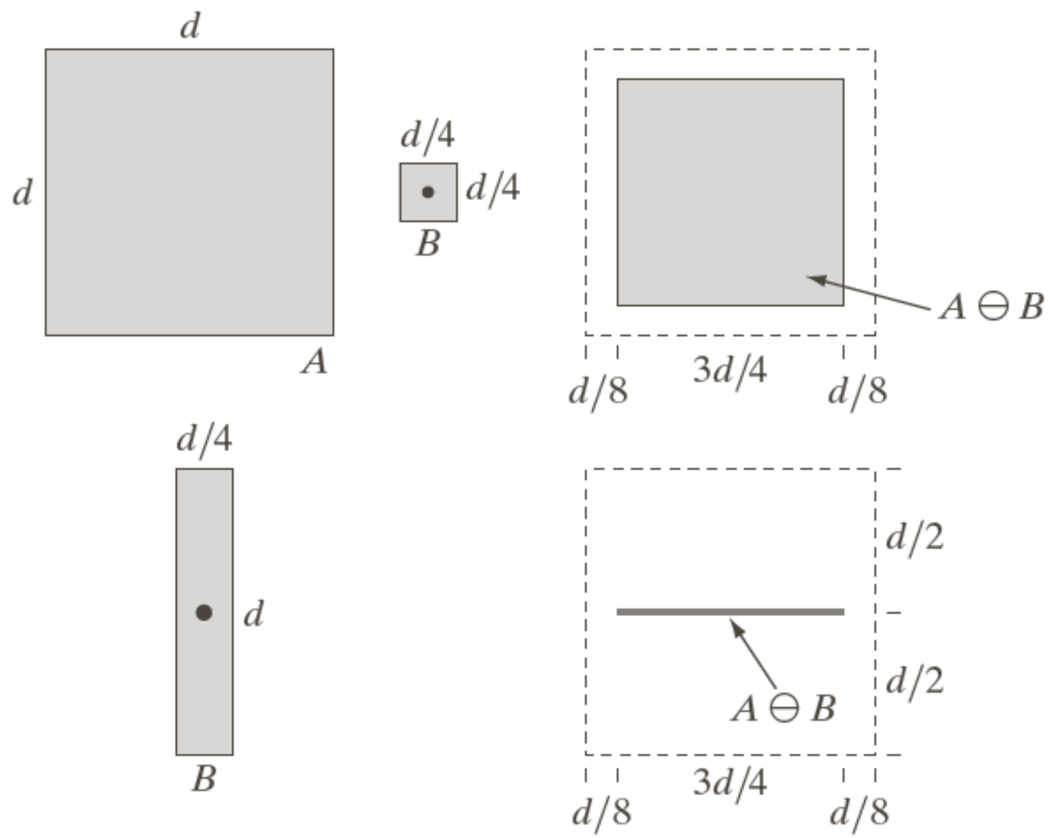


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

From Gonzalez and Woods, Digital Image Processing, third edition

Properties of erosion

- Not commutative
- Not associative

Duality

- Dual operations are ones that can be defined in terms of each other. Duals are not inverse operations

$$(A \oplus B) \ominus B \stackrel{?}{=} A$$

- Dilation and erosion are dual operations:

$$\begin{aligned} (A \oplus B)^c &= A^c \ominus \check{B} \\ (A \ominus B)^c &= A^c \oplus \check{B} \end{aligned}$$

- Intuitively, dilating the foreground is the same as eroding the background with a 'flipped' structuring element

Simple applications of dilation and erosion

- Can find all of the boundary pixels by dilating the object and subtracting the original:

$$\mathrm{Bound}_{\mathrm{ext}}(A) = (A \oplus B) - A$$

where B is a 3X3 structuring element containing all 1s

- Or by eroding the original and subtracting that from the original:

$$\mathrm{Bound}_{\mathrm{int}}(A) = A - (A \ominus B)$$

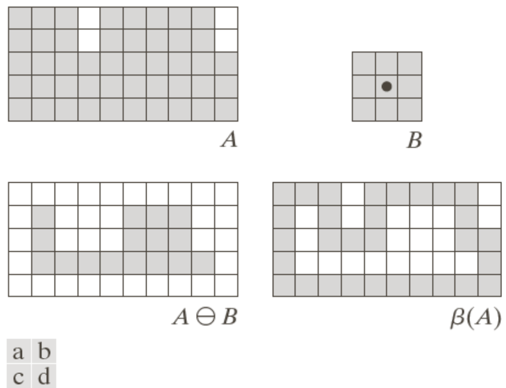


FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

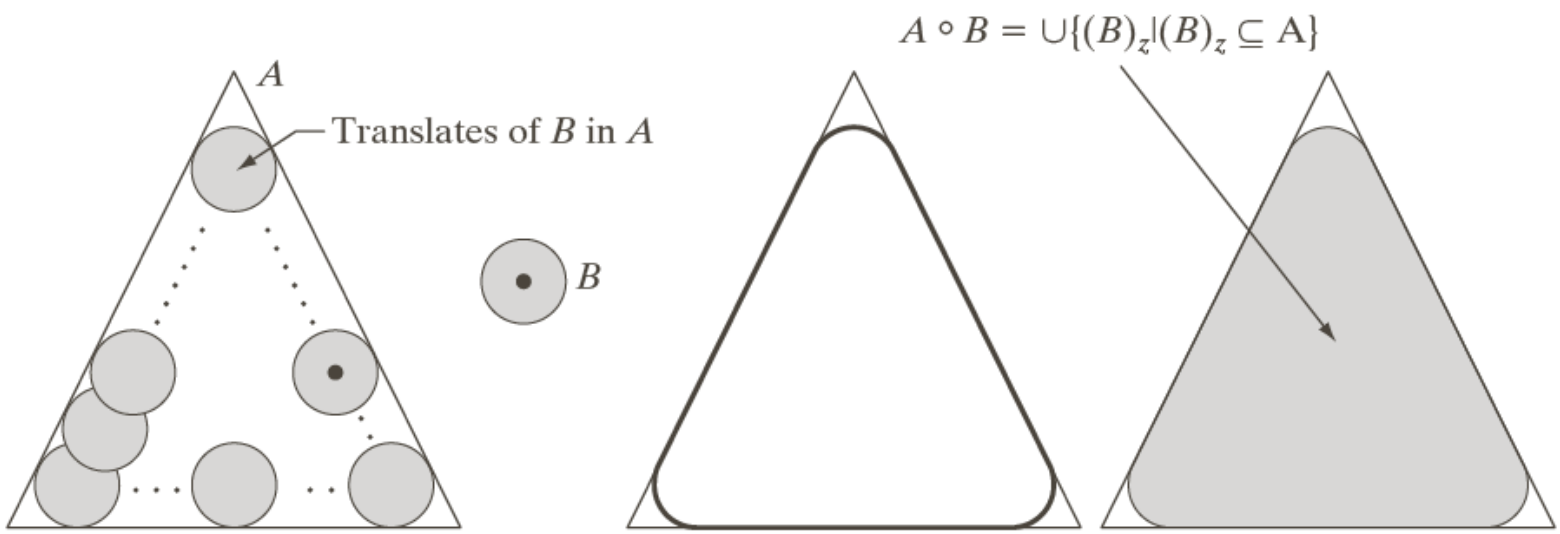
From Gonzalez and Woods, Digital Image Processing, third edition

Opening

- erosion followed by dilation

$$A \circ B = (A \ominus B) \oplus B$$

- Why dilate after erosion?
- erosion finds all the places where the structuring element fits inside the shape; it only marks these positions at the origin of the element.
- with a dilation, we "fill back in" the full structuring element at places where the element fits inside the object.
- So, an opening is the union of all translated copies of the structuring element that can fit inside the object.
- Openings can be used to remove small objects, protrusions from objects, and connections between objects.



a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

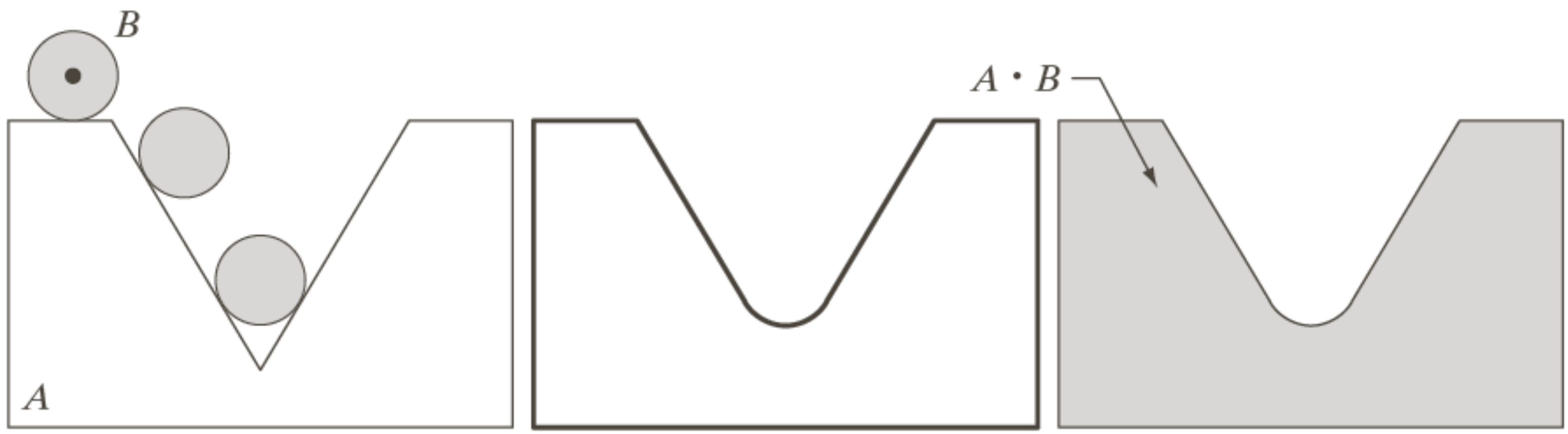
From Gonzalez and Woods, Digital Image Processing, third edition

Closing

- Dilation followed by erosion

$$A \bullet B = (A \oplus B) \ominus B$$

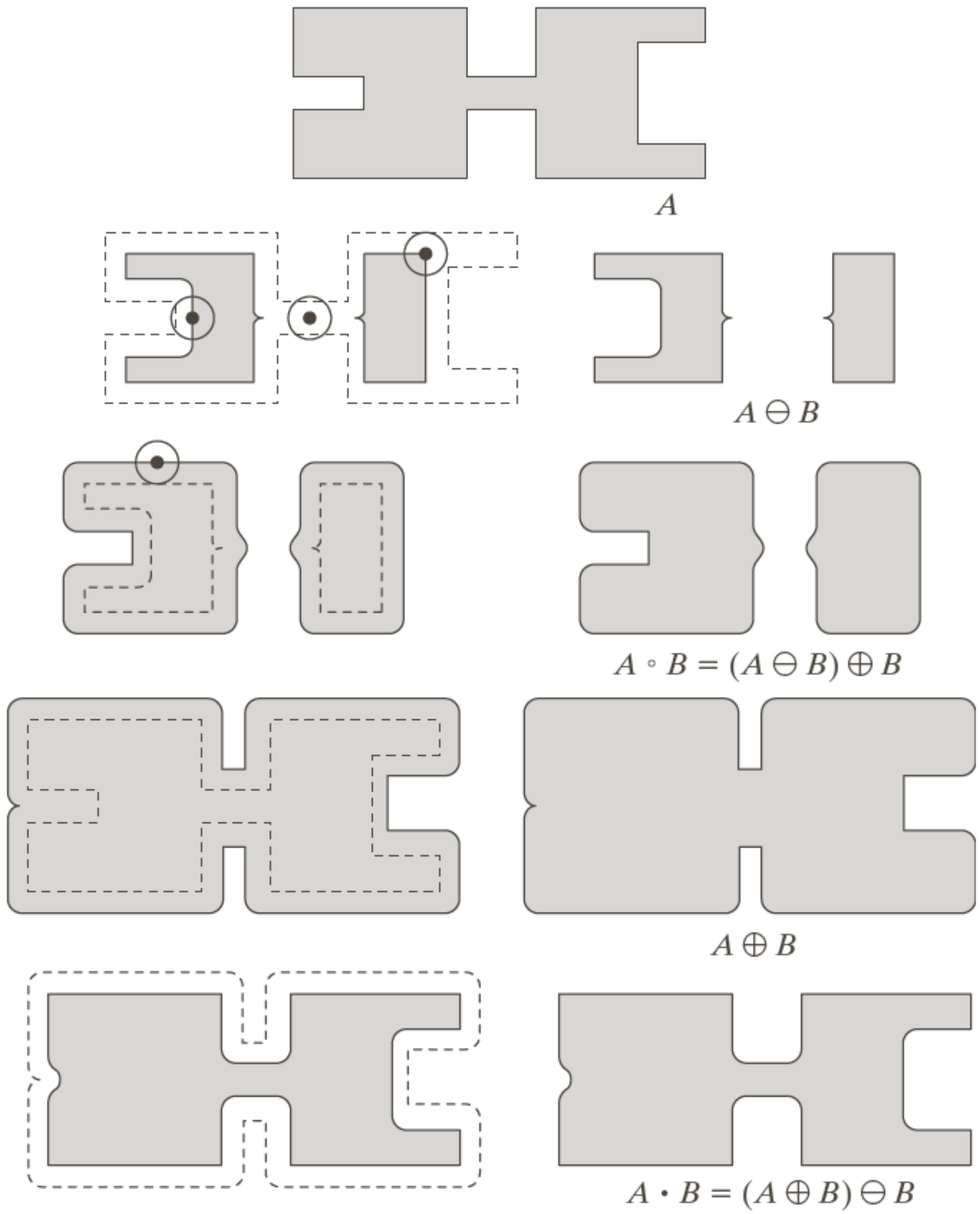
- closing *fills in all places where the* structuring element will not fit in the image *background.*
- Used to remove small holes, gaps, etc.



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.

From Gonzalez and Woods, Digital Image Processing, third edition



a
b c
d e
f g
h i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

From Gonzalez and Woods, Digital Image Processing, third edition

a b
d c
e f

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of A.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

From Gonzalez and Woods, Digital Image Processing, third edition

Properties of opening and closing

- Opening and closing are also duals:

$$(A \circ B)^c = A^c \bullet \breve{B}$$

- Intuitively, opening to remove small foreground objects is equivalent to closing small holes in the background
- Idempotence : nothing changes after the first application

$$A \circ B \circ B = A \circ B$$

$$A \bullet B \bullet B = A \bullet B$$

Hit or miss operator

- Can be considered as the morphological equivalent of template matching (in binary images)
- Needs two structuring elements: one that must 'hit', the other that must 'miss'
- Example: detection of an upper right corner

  J (hit) finds points with connected left and lower neighbors

  K (miss) finds points without upper, upper right

and right neighbors

0	0	0
1	1	0
0	1	0

J

0	1	1
0	0	1
0	0	0

K

	1	
0	1	1
0	0	

	1	
1	1	0
	0	0

	0	0
1	1	0
	1	

0	0	
0	1	1
	1	

Figure 2 Four structuring elements used for corner finding in binary images using the hit-and-miss transform. Note that they are really all the same element, but rotated by different amounts.

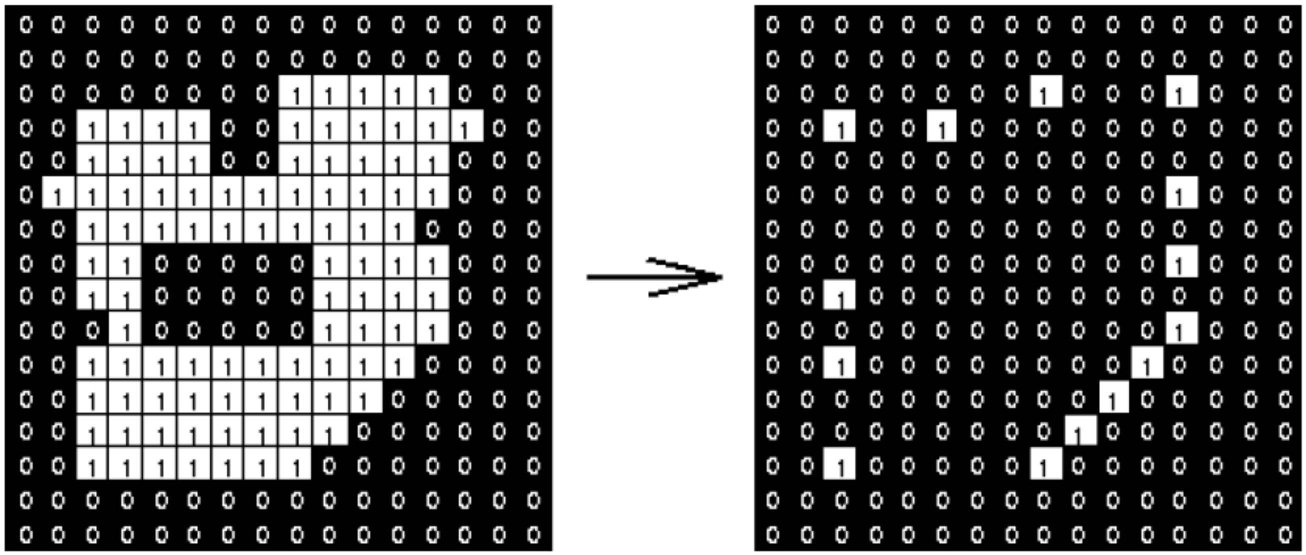
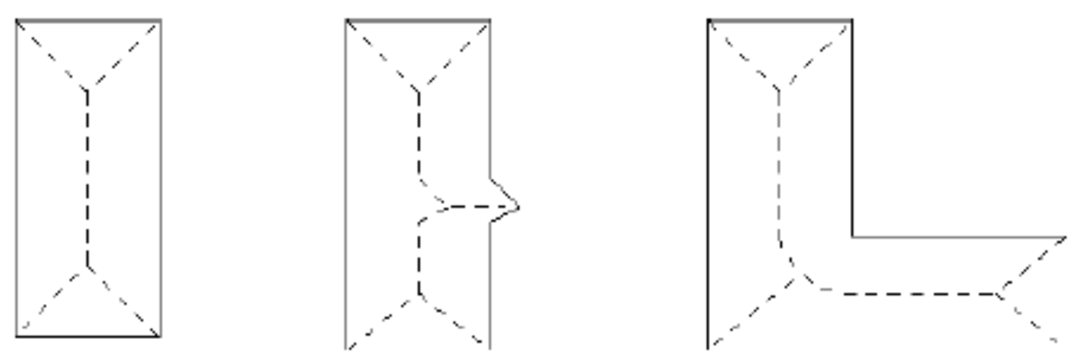


Figure 3 Effect of the hit-and-miss based right angle convex corner detector on a simple binary image. Note that the `detector' is rather sensitive.

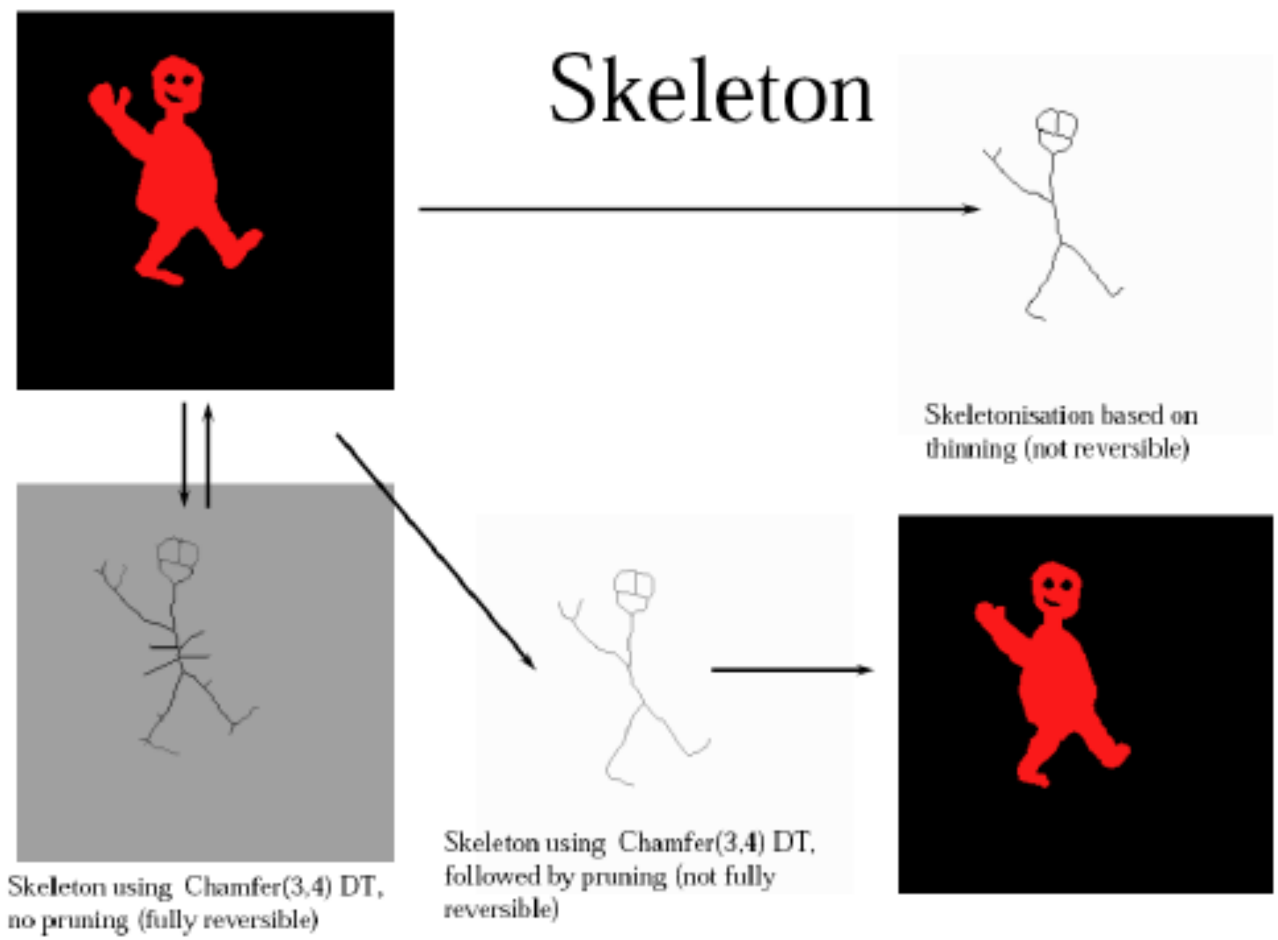
BW2 = bwhitmiss(BW1,SE1,SE2)

Detection of the skeleton of the object

- A skeleton is an object representation that is:
- Single-pixel thin
- Lies in the "middle" of the object
- Preserves the topology of the object



Shape representation via skeleton



Morphological derivation of the skeleton

- Design hit-and-miss operators that find boundary points that can safely be removed without breaking the object in two
- Apply recursively to strip layer by layer from the exterior of the shape until convergence
- Structuring elements (plus other four rotations)

0	0	0
x	1	x
1	1	1

x	0	0
1	1	0
x	1	x

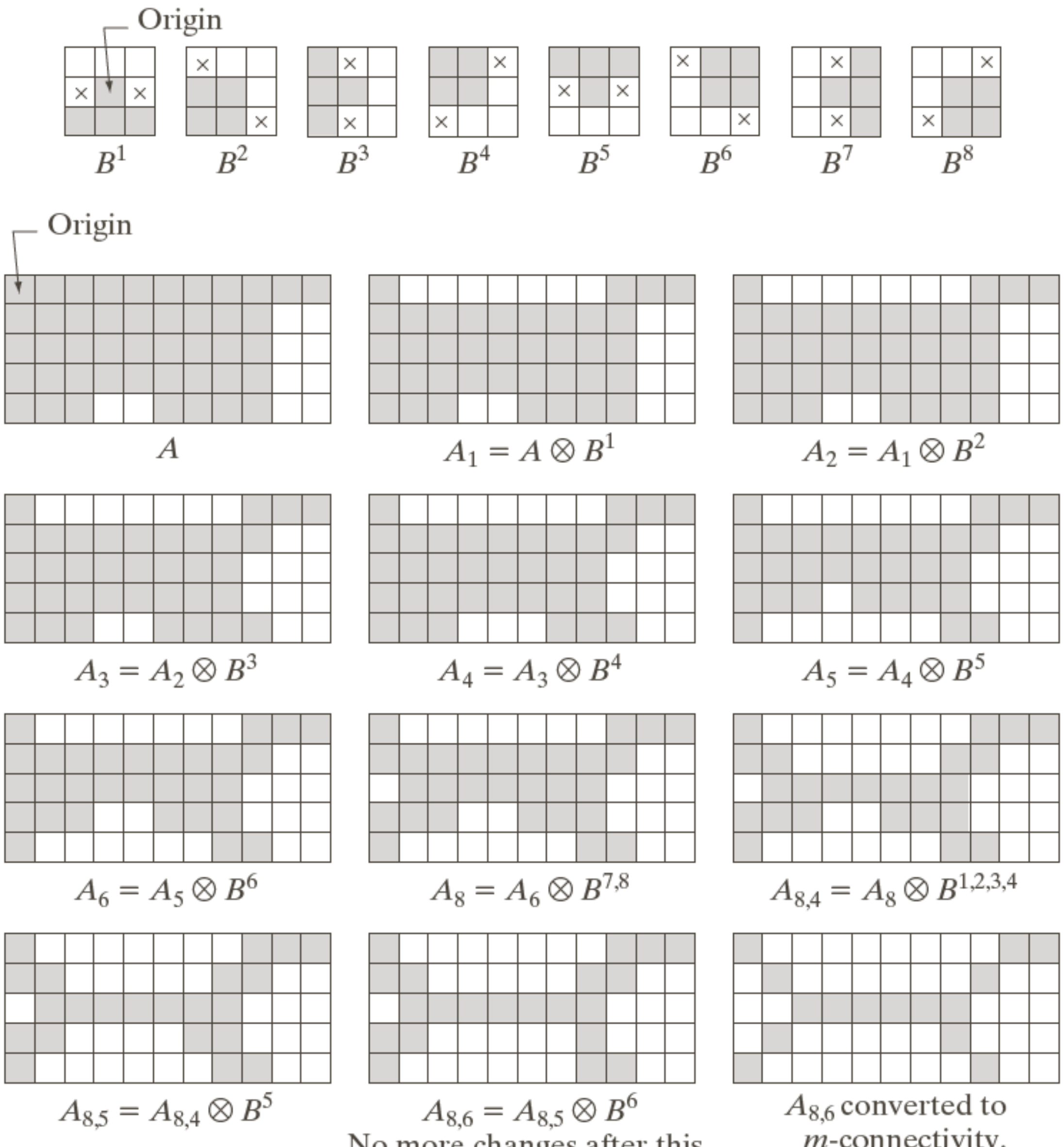
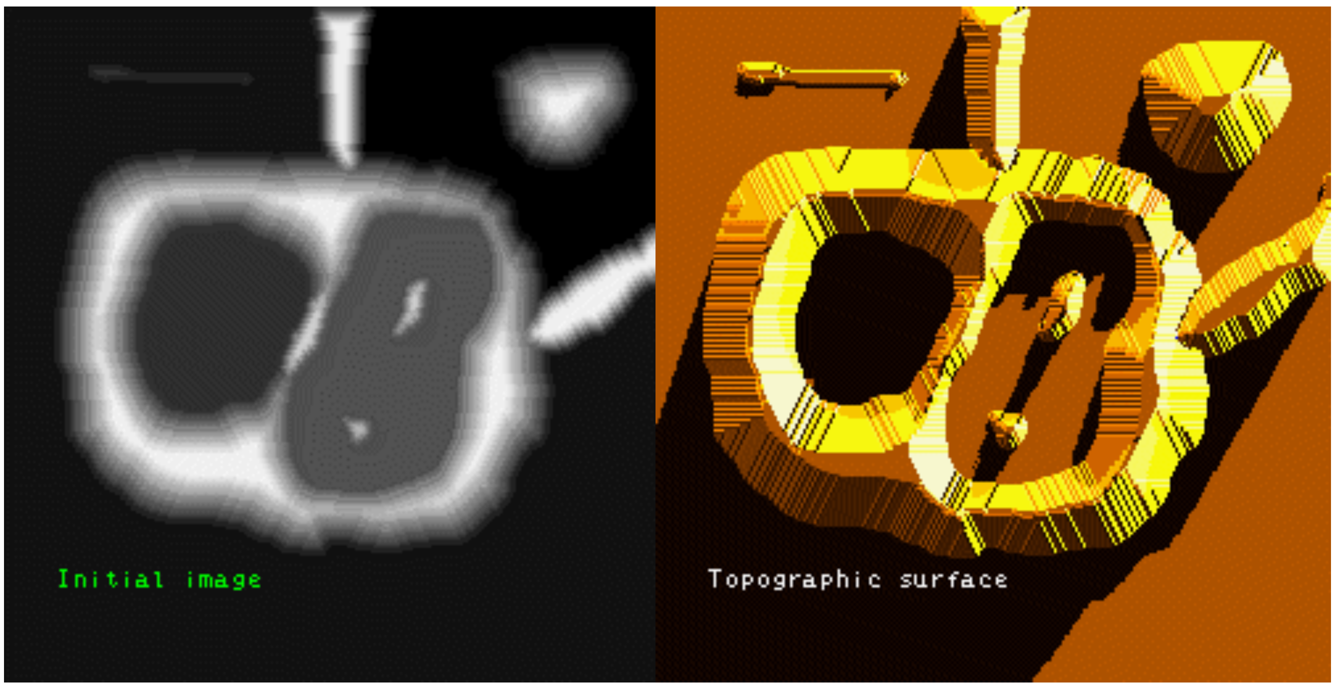


FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to m-connectivity.

The Watershed Transform

- Main idea: consider a gray-level image as a topographic surface



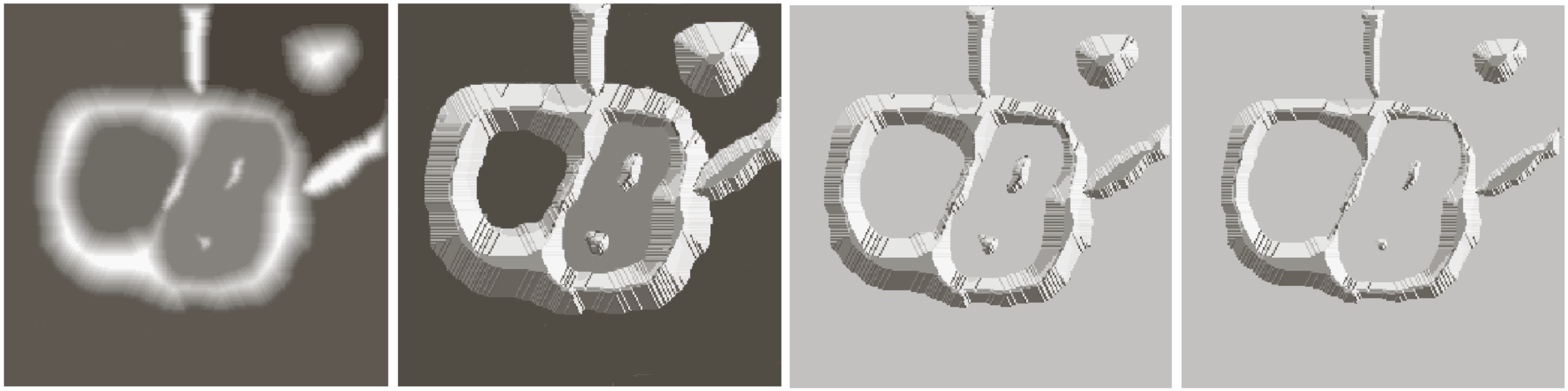
From Gonzalez and Woods, Digital Image Processing, third edition

Watershed transform (cont'd)

In the topographic interpretation, we consider three types of points:

- a) points belonging to a regional minimum
- b) points at which a drop of water would fall with certainty towards a single minimum (**catchment basin**)
- c) points at which water would be equally likely to fall to more than one minimum (**watershed lines**)

Our goal: to find the watershed lines



a b
c d

FIGURE 10.54 (a) Original image. (b) Topographic view. (c)–(d) Two stages of flooding.

e f
g h

FIGURE 10.54 *(Continued)* (e) Result of further flooding. (f) Beginning of merging of water from two catchment basins (a short dam was built between them). (g) Longer dams. (h) Final watershed (segmentation) lines. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

Dam construction

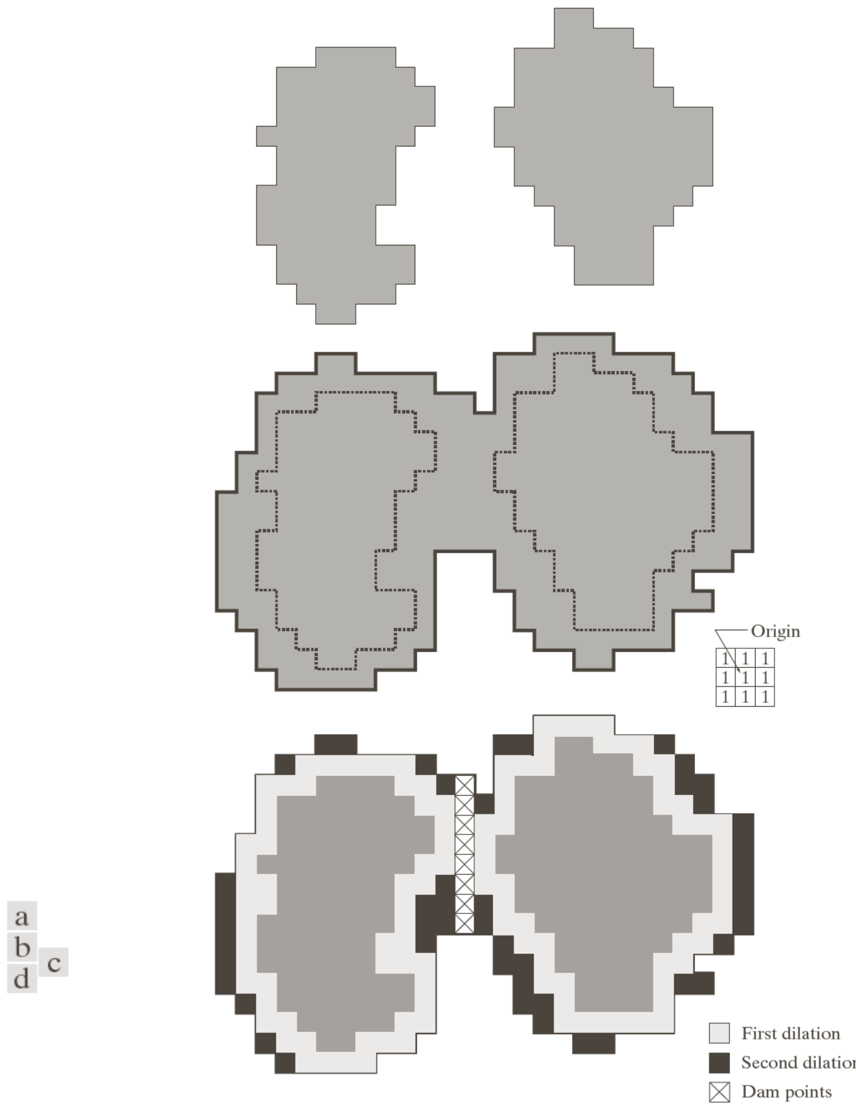
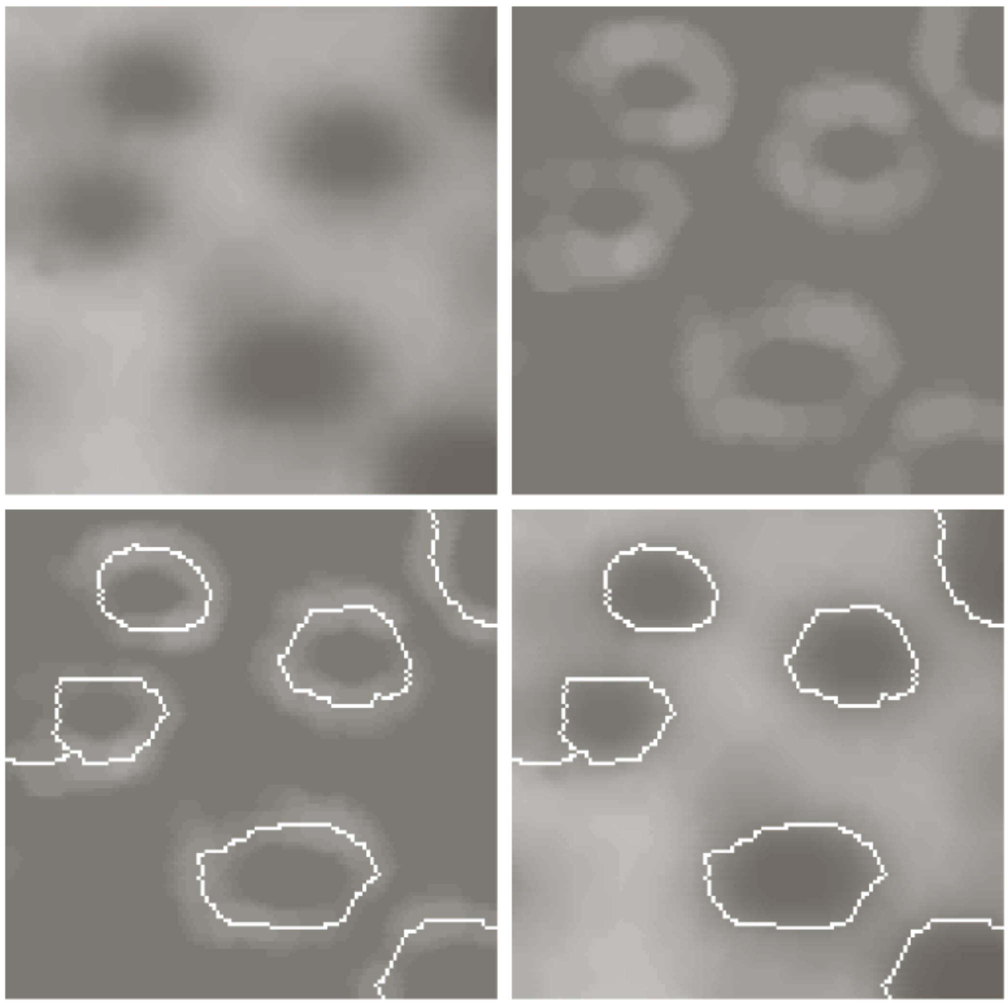


FIGURE 10.55 (a) Two partially flooded catchment basins at stage $n - 1$ of flooding. (b) Flooding at stage n, showing that water has spilled between basins. (c) Structuring element used for dilation. (d) Result of dilation and dam construction.

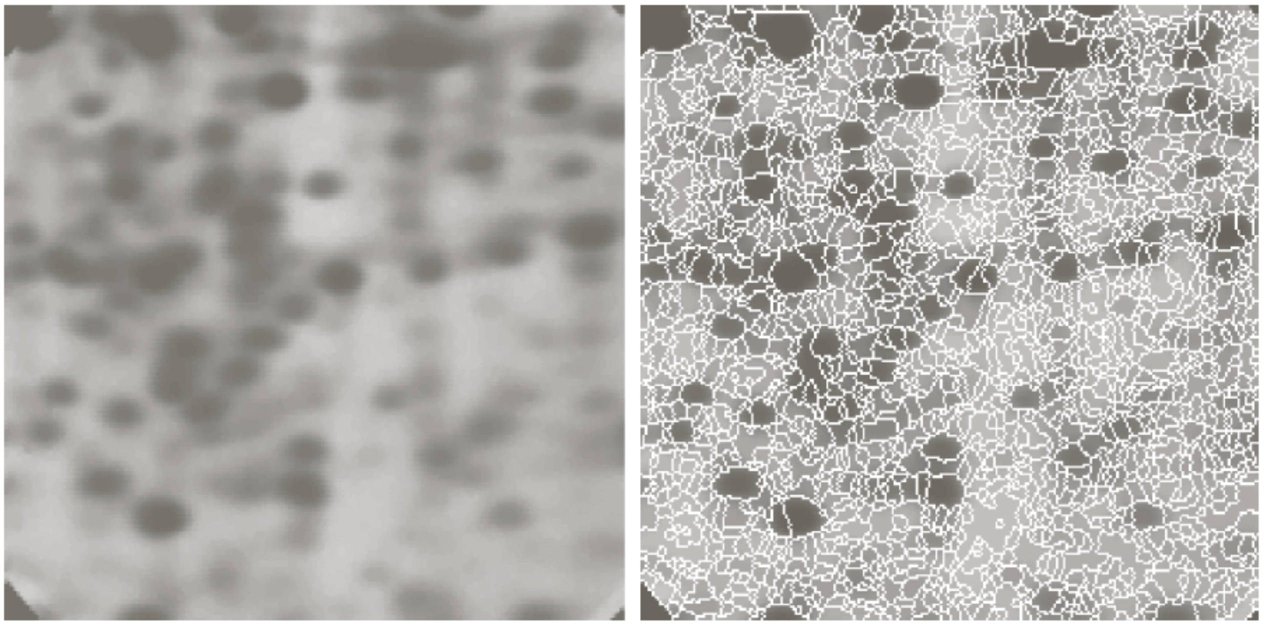
Example of watershed transform



a b
c d

FIGURE 10.56
(a) Image of blobs. (b) Image gradient. (c) Watershed lines. (d) Watershed lines superimposed on original image. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

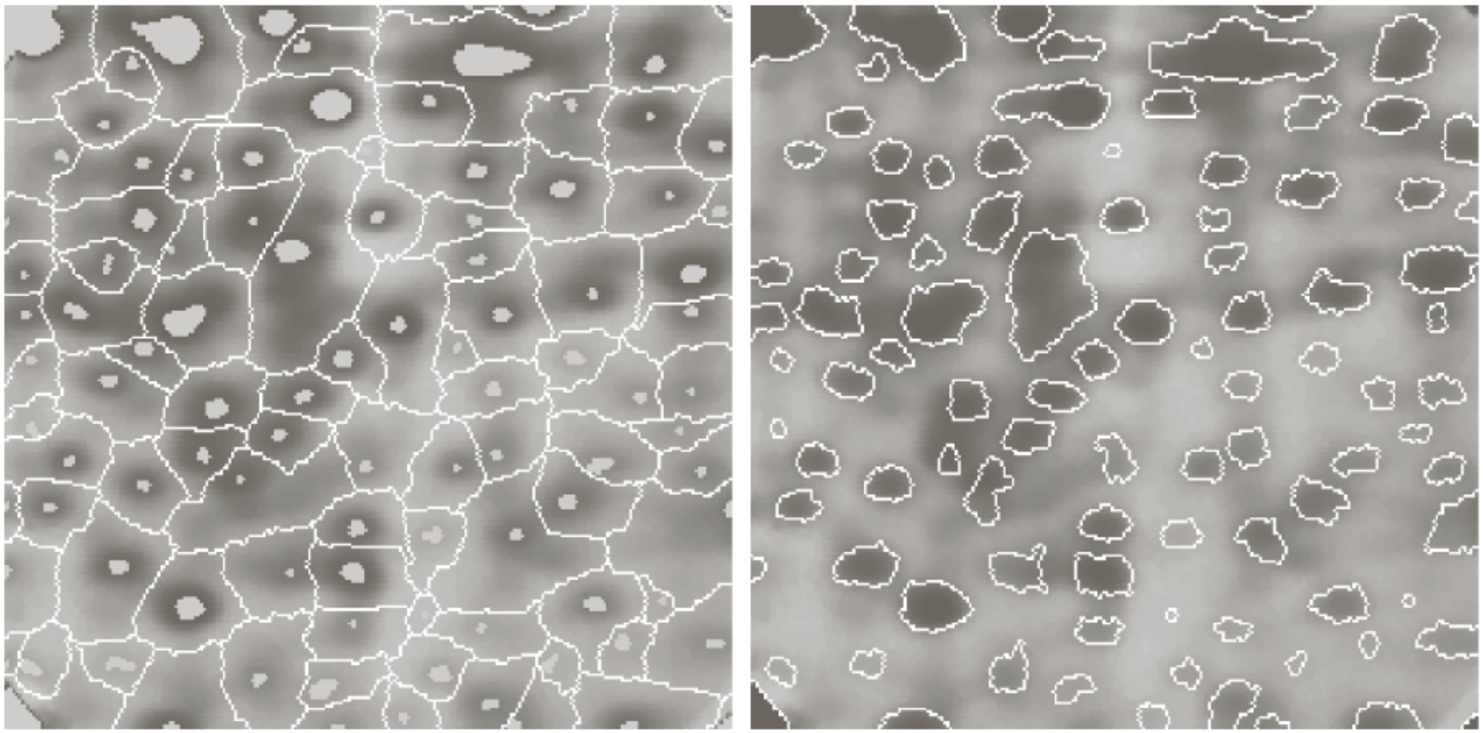
Oversegmentation



a b

FIGURE 10.57 (a) Electrophoresis image. (b) Result of applying the watershed segmentation algorithm to the gradient image. Oversegmentation is evident. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

Marker-based watershed



a b

FIGURE 10.58 (a) Image showing internal markers (light gray regions) and external markers (watershed lines). (b) Result of segmentation. Note the improvement over Fig. 10.47(b). (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)