

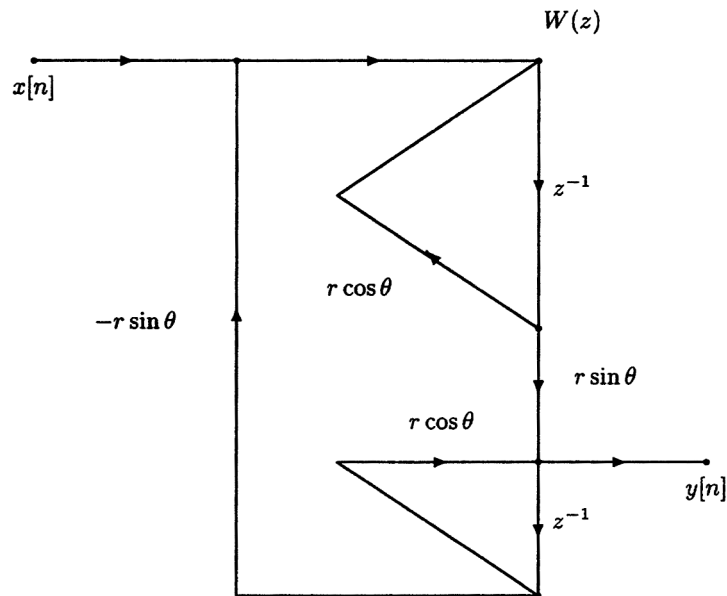
6.1. We proceed by obtaining the transfer functions for each of the networks. For network 1,

$$Y(z) = 2r \cos \theta z^{-1} Y(z) - r^2 z^{-2} Y(z) + X(z)$$

or

$$H_1(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 2r \cos \theta z^{-1} + r^2 z^{-2}}$$

For network 2, define $W(z)$ as in the figure below:



then

$$W(z) = X(z) - r \sin \theta z^{-1} Y(z) + r \cos \theta z^{-1} W(z)$$

and

$$Y(z) = r \sin \theta z^{-1} W(z) + r \cos \theta z^{-1} Y(z)$$

Eliminate $W(z)$ to get

$$H_2(z) = \frac{Y(z)}{X(z)} = \frac{r \sin \theta z^{-1}}{1 - 2r \cos \theta z^{-1} + r^2 z^{-2}}$$

Hence the two networks have the same poles.

6.3.

$$H(z) = \frac{2 + \frac{1}{4}z^{-1}}{1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}}$$

System (d) is recognizable as a transposed direct form II implementation of $H(z)$.

6.4. (a) From the flow graph, we have:

$$Y(z) = 2X(z) + \left(\frac{1}{4}X(z) - \frac{1}{4}Y(z) + \frac{3}{8}Y(z)z^{-1}\right)z^{-1}$$

That is:

$$Y(z)\left(1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}\right) = X(z)\left(2 + \frac{1}{4}z^{-1}\right).$$

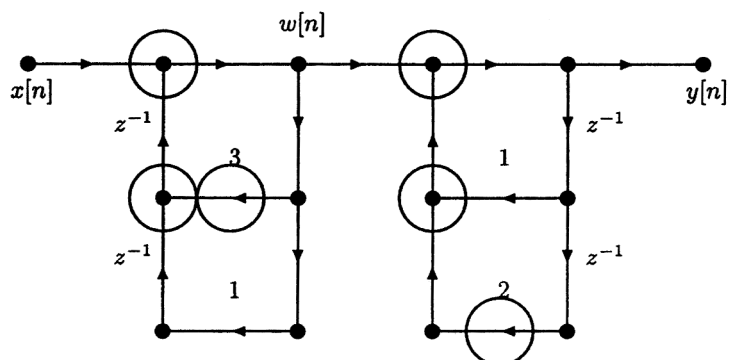
The system function is thus given by:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2 + \frac{1}{4}z^{-1}}{1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}}.$$

(b) To get the difference equation, we just inverse Z -transform the equation in *a*. We get:

$$y[n] + \frac{1}{4}y[n-1] - \frac{3}{8}y[n-2] = 2x[n] + \frac{1}{4}x[n-1].$$

6.5. The flow graph for this system is drawn below.



(a)

$$w[n] = x[n] + 3w[n-1] + w[n-2]$$

$$y[n] = w[n] + y[n-1] + 2y[n-2]$$

(b)

$$W(z) = X(z) + 3z^{-1}W(z) + z^{-2}W(z)$$

$$Y(z) = W(z) + z^{-1}Y(z) + 2z^{-2}Y(z)$$

So

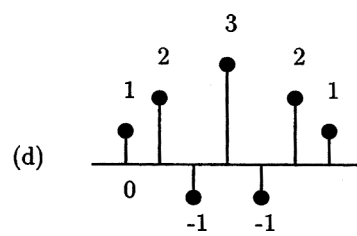
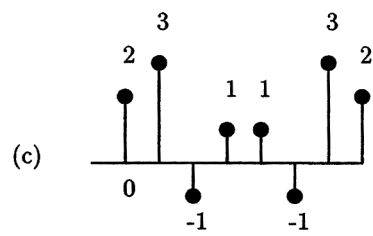
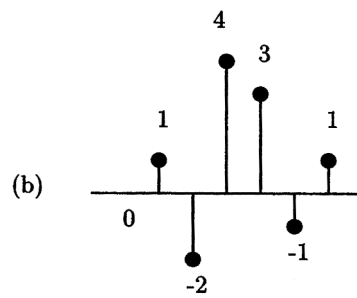
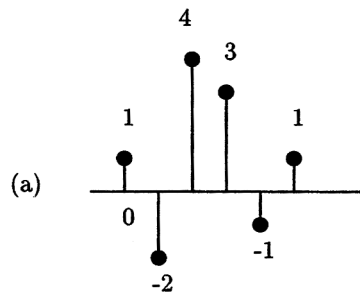
$$\begin{aligned} \frac{Y(z)}{X(z)} &= H(z) \\ &= \frac{1}{(1 - z^{-1} - 2z^{-2})(1 - 3z^{-1} - z^{-2})} \\ &= \frac{1}{1 - 4z^{-1} + 7z^{-3} + 2z^{-4}} \end{aligned}$$

(c) Adds and multiplies are circled above: 4 real adds and 2 real multiplies per output point.

(d) It is not possible to reduce the number of storage registers. Note that implementing $H(z)$ above in the canonical direct form II (minimum storage registers) also requires 4 registers.

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6.6. The impulse responses of each system are shown below.

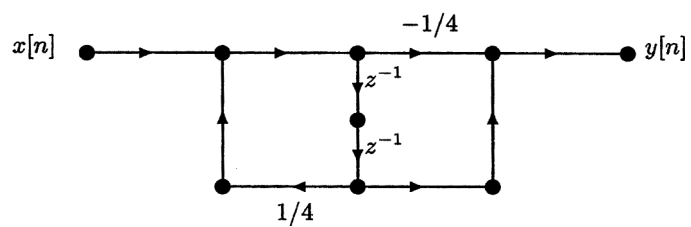


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6.7. We have

$$H(z) = \frac{-\frac{1}{4} + z^{-2}}{1 - \frac{1}{4}z^{-2}}.$$

Therefore the direct form II is given by:

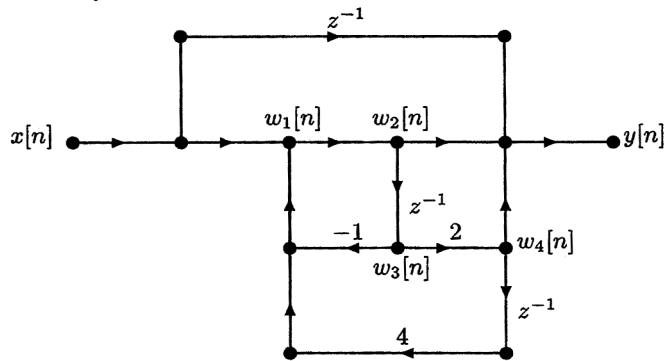


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6.8. By looking at the graph, we get:

$$y[n] = 2y[n-2] + 3x[n-1] + x[n-2].$$

6.9. The signal flow graph for the system is:



(a) First we need to determine the transfer function. We have

$$\begin{aligned} w_1[n] &= x[n] - w_3[n] + 4w_4[n-1] \\ w_2[n] &= w_1[n] \\ w_3[n] &= w_2[n-1] \\ w_4[n] &= 2w_3[n] \\ y[n] &= w_2[n] + x[n-1] + w_4[n]. \end{aligned}$$

Taking the Z -transform of the above equations, rearranging and substituting terms, we get:

$$H(z) = \frac{1 + 3z^{-1} + z^{-2} - 8z^{-3}}{1 + z^{-1} - 8z^{-2}}.$$

The difference equation is thus given by:

$$y[n] + y[n-1] - 8y[n-2] = x[n] + 3x[n-1] + x[n-2] - 8x[n-3].$$

The impulse response is the response to an impulse, therefore:

$$h[n] + h[n-1] - 8h[n-2] = \delta[n] + 3\delta[n-1] + \delta[n-2] - 8\delta[n-3].$$

From the above equation, we have:

$$\begin{aligned} h[0] &= 1 \\ h[1] &= 3 - h[0] = 2. \end{aligned}$$

(b) From part (a) we have:

$$y[n] + y[n-1] - 8y[n-2] = x[n] + 3x[n-1] + x[n-2] - 8x[n-3].$$

6.10. (a)

$$\begin{aligned}w[n] &= \frac{1}{2}y[n] + x[n] \\v[n] &= \frac{1}{2}y[n] + 2x[n] + w[n-1] \\y[n] &= v[n-1] + x[n].\end{aligned}$$

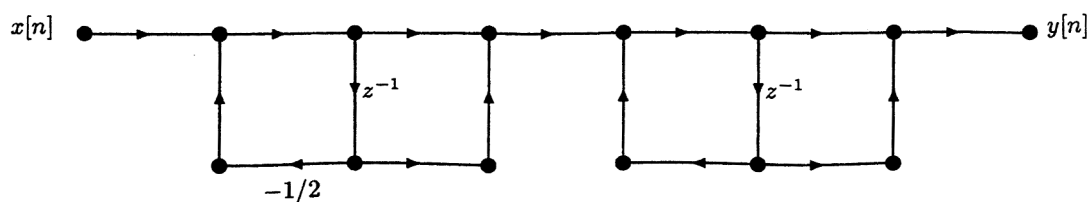
(b) Using the Z -transform of the difference equations in part (a), we get the transfer function:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 2z^{-1} + z^{-2}}{1 - \frac{1}{2}z^{-1} - \frac{1}{2}z^{-2}}.$$

We can rewrite it as :

$$H(z) = \frac{(1 + z^{-1})(1 + z^{-1})}{(1 + \frac{1}{2}z^{-1})(1 - z^{-1})}.$$

We thus get the following cascade form:

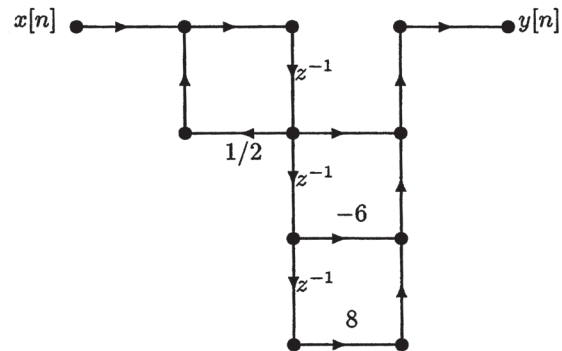


(c) The system function has poles at $z = -\frac{1}{2}$ and $z = 1$. Since the second pole is on the unit circle, the system is not stable.

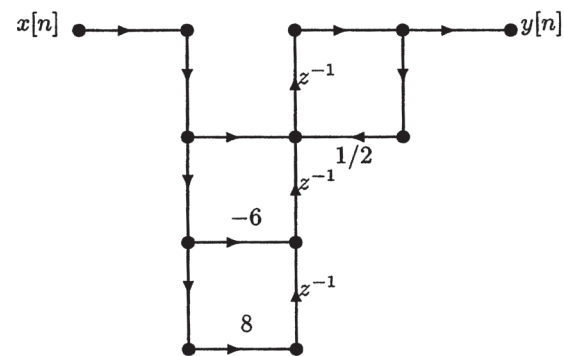
6.11. (a) $H(z)$ can be rewritten as:

$$H(z) = \frac{z^{-1} - 6z^{-2} + 8z^{-3}}{1 - \frac{1}{2}z^{-1}}.$$

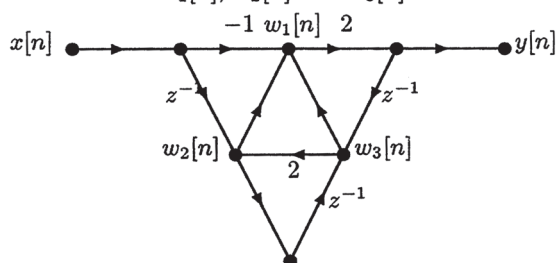
We thus get the following direct form II flow graph :



(b) To get the transposed form, we just reverse the arrows and exchange the input and the output. The graph can then be redrawn as:



6.12. We define the intermediate variables $w_1[n]$, $w_2[n]$ and $w_3[n]$ as follows:



We thus have the following relationships:

$$\begin{aligned} w_1[n] &= -x[n] + w_2[n] + w_3[n] \\ w_2[n] &= x[n-1] + 2w_3[n] \\ w_3[n] &= w_2[n-1] + y[n-1] \\ y[n] &= 2w_1[n]. \end{aligned}$$

Z -transforming the above equations and rearranging and grouping terms, we get:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{-2 + 6z^{-1} + 2z^{-2}}{1 - 8z^{-1}}.$$

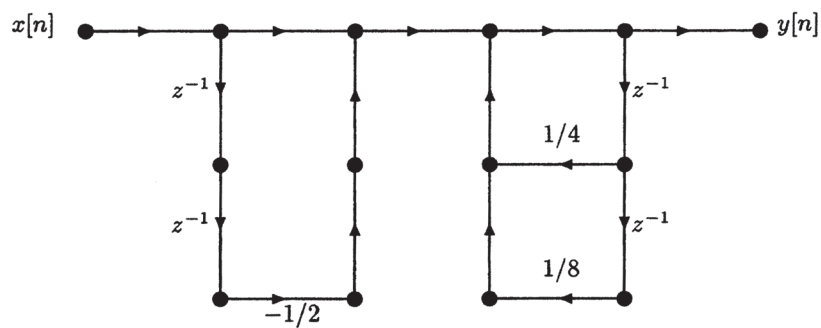
Taking the inverse Z -transform, we get the following difference equation:

$$y[n] - 8y[n-1] = -2x[n] + 6x[n-1] + 2x[n-2].$$

6.13.

$$H(z) = \frac{1 - \frac{1}{2}z^{-2}}{1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}.$$

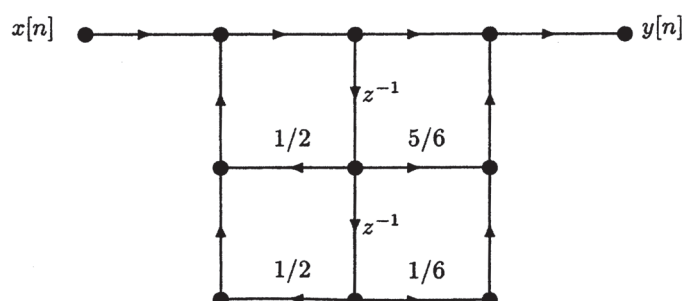
The direct form I implementation is:



6.14.

$$H(z) = \frac{1 + \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}}{1 - \frac{1}{2}z^{-1} - \frac{1}{2}z^{-2}}.$$

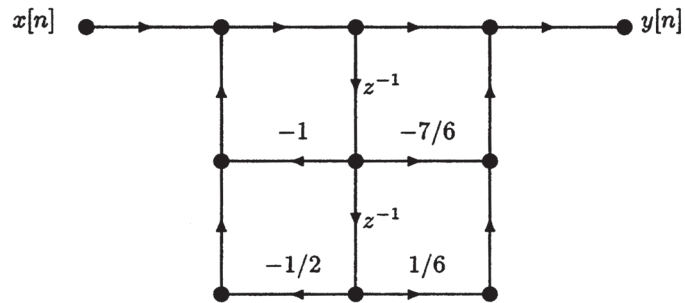
The direct form II implementation is:



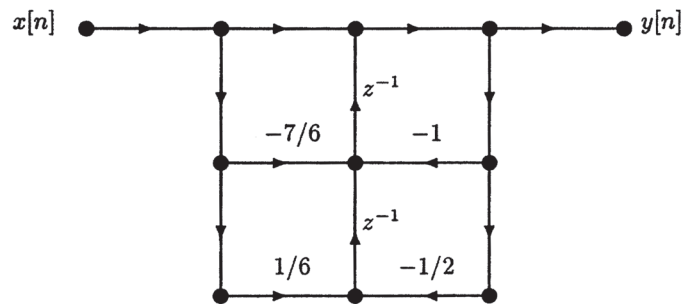
6.15.

$$H(z) = \frac{1 - \frac{7}{6}z^{-1} + \frac{1}{6}z^{-2}}{1 + z^{-1} + \frac{1}{2}z^{-2}}.$$

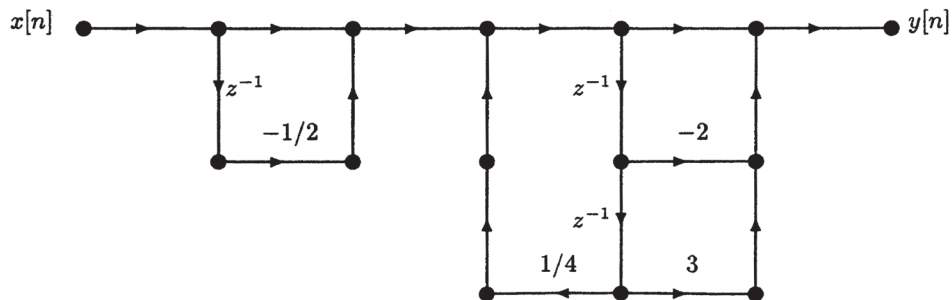
To get the transposed direct form II implementation, we first get the direct form II:



Now, we reverse the arrows and exchange the role of the input and the output to get the transposed direct form II:



6.16. (a) We just reverse the arrows and reverse the role of the input and the output, we get:



(b) The original system is the cascade of two transposed direct form II structures, therefore the system function is given by:

$$H(z) = \left(\frac{1 - 2z^{-1} + 3z^{-2}}{1 - \frac{1}{4}z^{-2}} \right) \left(1 - \frac{1}{2}z^{-1} \right).$$

The transposed graph, on the other hand, is the cascade of two direct form II structures, therefore the system function is given by:

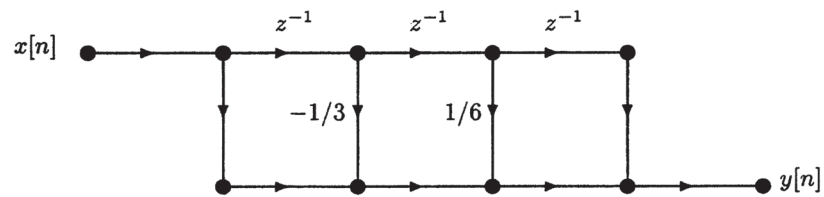
$$H(z) = \left(1 - \frac{1}{2}z^{-1} \right) \left(\frac{1 - 2z^{-1} + 3z^{-2}}{1 - \frac{1}{4}z^{-2}} \right).$$

This confirms that both graphs have the same system function $H(z)$.

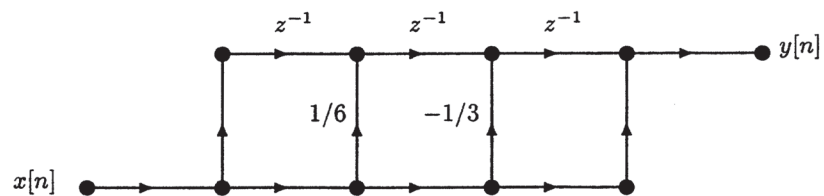
6.17.

$$H(z) = 1 - \frac{1}{3}z^{-1} + \frac{1}{6}z^{-2} + z^{-3}.$$

(a) Direct form implementation of this system:



(b) Transposed direct form implementation of the system:



6.18. The flow graph is just a cascade of two transposed direct form II structures, the system function is thus given by:

$$H(z) = \left(\frac{1 + \frac{4}{3}z^{-1} - \frac{4}{3}z^{-2}}{1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}} \right) \left(\frac{1}{1 - az^{-1}} \right).$$

Which can be rewritten as:

$$H(z) = \frac{(1 + 2z^{-1})(1 - \frac{2}{3}z^{-1})}{(1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2})(1 - az^{-1})}.$$

In order to implement this system function with a second-order direct form II signal flow graph, a pole-zero cancellation has to occur, this happens if $a = \frac{2}{3}$, $a = -2$ or $a = 0$. If $a = \frac{2}{3}$, the overall system function is:

$$H(z) = \frac{1 + 2z^{-1}}{1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}}.$$

If $a = -2$, the overall system function is:

$$H(z) = \frac{1 - \frac{2}{3}z^{-1}}{1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}}.$$

And finally if $a = 0$, the overall system function is:

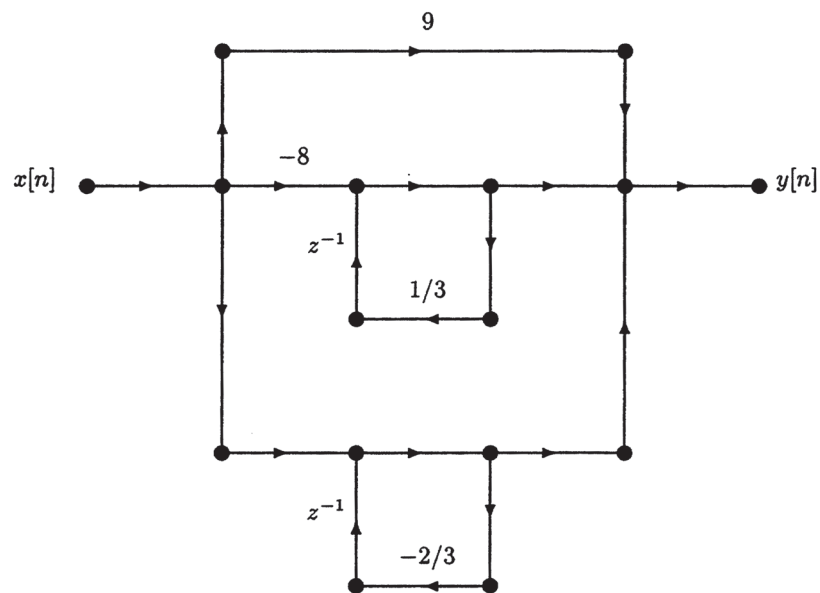
$$H(z) = \frac{(1 + 2z^{-1})(1 - \frac{2}{3}z^{-1})}{1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}}.$$

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6.19. Using partial fraction expansion, the system function can be rewritten as:

$$H(z) = \frac{-8}{1 - \frac{1}{3}z^{-1}} + \frac{1}{1 + \frac{2}{3}z^{-1}} + 9.$$

Now we can draw the flow graph that implements this system as a parallel combination of first-order transposed direct form II sections:



6.20. The transfer function can be rewritten as:

$$H(z) = \frac{(1 + 2z^{-1} + \frac{5}{4}z^{-2})}{(1 + \frac{1}{4}z^{-2})(1 - \frac{5}{2}z^{-1} + z^{-2})}$$

which can be implemented as the following cascade of second-order transposed direct form II sections:

