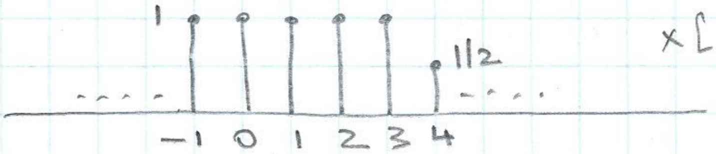
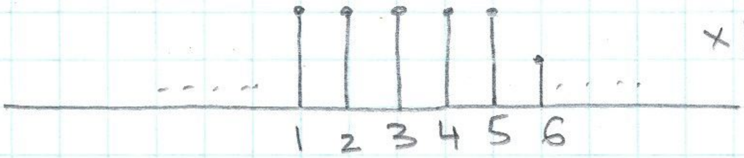
Question 1



$x[n]$.

a) $x[n-2]$ is obtained from $x[n]$ by delaying it with 2 units.

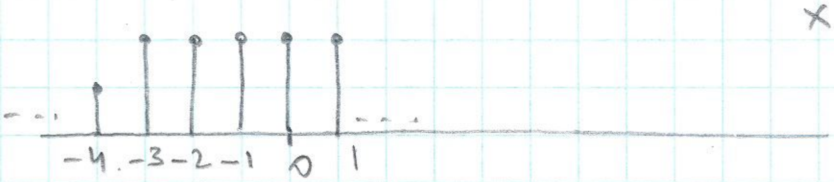


$x[n-2]$.

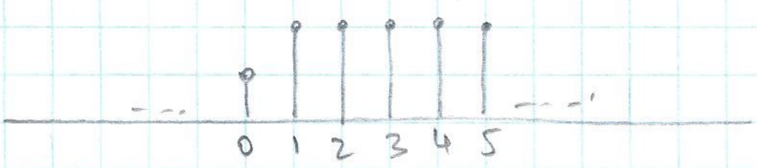
b). $x[4-n]$. - time reversal followed by time shift.

$$x[n] \xrightarrow[\text{reversal}]{} x[-n] \xrightarrow[\text{delay by 4}]{} x[-(n-4)].$$

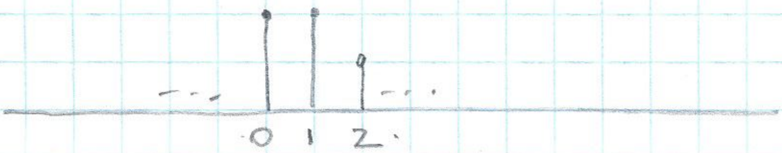
$x[-n]$.



$x[-(n-4)]$.



c) $x[2n]$ - time compression ; only samples at even values of n are extracted.

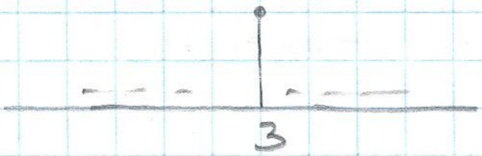


Question 1 (cont'd).

(d) $x[n]\ u[2-n]$.

$$u[2-n] = \begin{cases} 0 & \text{for } n > 2 \\ 1 & \text{for } n \leq 2. \end{cases}$$

(e) $x[n-1]\ \delta[n-3] = x[2]\ \delta[n-3]$.



Question 2.

2.23 b

$$T(x[n]) = x[n^2].$$

This transformation "samples" $x[n]$ at locations which can be expressed as k^2.

- The system is stable. If the input $x[n]$ is bounded then the output $x[n^2]$ is also bounded.

- The system is not causal. For example

$$T(x[4]) = x[16].$$

- The system is linear.

- let $y_1[n] = T(x_1[n]) = x_1[n^2]$.

$$y_2[n] = T(x_2[n]) = x_2[n^2].$$

$$T(a x_1[n] + b x_2[n]) = a x_1[n^2] + b x_2[n^2] =$$

$$= a y_1[n] + b y_2[n].$$

- The system is not time-invariant.

If $y[n] = T(x[n]) = x[n^2]$. $\quad y[n-1] = x[(n-1)^2]$.

$$T(x[n-1]) = x(n^2-1) \neq y[n-1].$$

Question 2 cont'd.

2.23. c

$$T(x[n]) = x[n] \sum_{k=0}^{\infty} \delta[n-k]$$

We notice that $\sum_{k=0}^{\infty} \delta[n-k] = u[n]$.

$$T(x[n]) = x[n]\, u[n].$$

It is easy to prove that this transformation is stable, causal, linear.

The transformation is not time-invariant. We can prove this with a counterexample.

$$T(\delta[n]) = \delta[n].$$

but

$T(\delta[n+1]) = 0$. which is different from $\delta[n+1]$

2.23. d

$$T(x[n]) = \sum_{k=n-1}^{\infty} x[k].$$

- not stable. Let $x[n] = 1$ for all n.

$$T(x[n]) = \sum_{k=n-1}^{\infty} x[n] = \infty. \quad \text{for all } n.$$

- not causal because it sums forward in time.
- linear. $\sum_{k=n-1}^{\infty} a x_1[k] + b x_2[k] = a \sum_{k=n-1}^{\infty} x_1[k] + b \sum_{k=n-1}^{\infty} x_2[k]$.
- it is T.I. If $y[n] = T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$. then $y[n-n_0] = \sum_{k=n-n_0-1}^{\infty} x[k]$

thus $y(n-n_0) = T(x[n-n_0])$

QUESTION 3.

We know that.

$$v[n] = u[n] * \{a^n u[-n-1]\} = \begin{cases} \dfrac{a^n}{1-1/a} & n \leq -1 \\ \dfrac{1/a}{1-1/a} & n > -1. \end{cases}$$

Let us define $v[n] = a^n u[-n-1]$.

(c) $x[n] = u[n]$

$h[n] = (0.5) \cdot 2^n u[-n]$.

$h[n] = 2^{n-1} u[-(n-1)-1]$

Let us define $v[n] = 2^n u[-n-1]$. , for $a=2$.

We can express $h[n]$ in terms of $v[n]$ as follows.

$$\boxed{h[n] = v[n-1].}$$

$y[n] = x[n] * h[n] = u[n] * v[n-1] = w[n-1]$.

$$y[n] = \begin{cases} \dfrac{2^{n-1}}{1-1/2} & n-1 \leq -1 \\ \dfrac{1/2}{1-1/2.} & n-1 > -1. \end{cases}$$

$$y[n] = \begin{cases} 2^n & n \leq 0 \\ 1 & n > 0. \end{cases}$$

QUESTION 3 cont'd.

(d) We use again $v[n]$ and $w[n]$.

$$y[n] = x[n] * h[n]$$
$$= (u[n] - u[n-10]) * v[n]$$
$$= w[n] - w[n-10].$$

$$w[n] = \begin{cases} \frac{2^{n}}{1-1/2} & n \leq -1. \\ 1 & n > -1. \end{cases}$$

$$\boxed{w[n] = \begin{cases} 2^{n+1} & n \leq -1 \\ 1 & n > -1. \end{cases}}$$

$$w[n-10] = \begin{cases} 2^{n-10+1} & n-10 \leq -1. \\ 1 & n-10 > -1. \end{cases}$$

$$\boxed{w[n-10] = \begin{cases} 2^{n-9} & n \leq 9. \\ 1 & n > 9. \end{cases}}$$

We need to examine the following cases for n.

$$n \leq -1, \quad -1 < n \leq 9, \quad n > 9.$$

$$y[n] = w[n] - w[n-10] = \begin{cases} 2^{n+1} - 2^{n-9} & n \leq -1. \\ 1 - 2^{n-9} & -1 < n \leq 9. \\ 0 & n > 9. \end{cases}$$

QUESTION 4.

$$y[n] = x[n] * h[n]$$
$$= \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k].$$
$$= \sum_{k=-\infty}^{+\infty} x[k]\, u[n-k]. = \sum_{k=-\infty}^{n} x[k].$$

The convolution may be broken into five regions over the range of n:

$\boxed{n<0}$ $$y[n] = \sum_{k=-\infty}^{n} \underbrace{x[k]}_{0 \text{ for all ks.}} = 0.$$

$\boxed{0 \leq n \leq N_1}$ $$y[n] = \sum_{k=-\infty}^{n} x[k] = \sum_{k=0}^{n} a^k.$$
$$y[n] = \frac{1-a^{n+1}}{1-a}.$$

$\boxed{N_1 < n < N_2}$ $$y[n] = \sum_{k=0}^{N_1} a^k.$$
$$y[n] = \frac{1-a^{N_1+1}}{1-a}.$$

$\boxed{N_2 \leq n \leq N_2+N_1}$ $$y[n] = \sum_{k=0}^{N_1} a^k + \sum_{k=N_2}^{n} a^{k-N_2}.$$
$$y[n] = \frac{1-a^{(N_1+1)}}{1-a} + a^{-N_2}\frac{a^{N_2}-a^{n+1}}{1-a}$$
$$y[n] = \frac{2-a^{N_1+1}-a^{n-N_2+1}}{1-a}.$$

where we have used $\sum_{k=m}^{n} r^k = \frac{r^{n+1}-r^m}{r-1}$

Question 4 cont'd.

$\boxed{n > N_2 + N_1}$

$$y[n] = \sum_{k=0}^{N_1} a^k + \sum_{k=N_2}^{N_1+N_2} a^{n-N_2}.$$

$$y[n] = \frac{2 - a^{N_1+1} - a^{(N_1+N_2)-N_2+1}}{1-a}$$

$$y[n] = 2 \cdot \frac{1-a^{N_1+1}}{1-a}$$

$y[n]$ has the following expression:

$$y[n] = \begin{cases} 0 & \text{for } n < 0. \\ \dfrac{1-a^{n+1}}{1-a} & \text{for } 0 \leq n \leq N_1. \\ \dfrac{1-a^{N_1+1}}{1-a} & \text{for } N_1 < n < N_2. \\ \dfrac{2-a^{N_1+1}-a^{n+1}}{1-a} & \text{for } N_2 \leq n \leq N_1+N_2. \\ 2 \cdot \dfrac{1-a^{N_1+1}}{1-a} & \text{for } n > N_1+N_2 \end{cases}$$

QUESTION 5.

a) $x[n] = \cos\left(\frac{n}{8} - \pi\right)$.

The signal is not periodic.

Let us suppose that there is N integer so that
$x[n+N] = x[n]$ for all n.

$\cos\left(\frac{n}{8} - \pi\right) = \cos\left(\frac{n+N}{8} - \pi\right)$.

The cos function is periodic with $2k\pi$, k integer

thus $\frac{N}{8} = 2k\pi$

$N = 16k\pi \rightarrow$ there exists no k so that N integer.

$\downarrow$

$x[n]$ is not periodic.

b) $x[n] = 2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{8}N\right) - 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right)$.

$$= \left(e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n}\right) + \frac{1}{2j}\left(e^{j\frac{\pi}{8}N} - e^{-j\frac{\pi}{8}N}\right) -$$

$$- \left(e^{j\left(\frac{\pi}{2}n+\frac{\pi}{6}\right)} - e^{-j\left(\frac{\pi}{2}n+\frac{\pi}{6}\right)}\right).$$

$e^{j\frac{\pi}{4}n}$, $e^{-j\frac{\pi}{4}n}$ are periodic with $N_1 = 8$ (fundamental).

$e^{j\frac{\pi}{8}n}$, $e^{-j\frac{\pi}{8}n}$ are periodic with $N_2 = 16$ (fundamental)

$e^{j\frac{\pi}{2}n}$, $e^{-j\frac{\pi}{2}n}$ are periodic with $N_3 = 4$ (fundamental)

Question 5 (cont'd)

the period of $x[n]$ is $N = LMC(L_1, L_2, L_3)$

$$\boxed{N = 16}$$

c) $x[n] = 1 + e^{j\frac{4\pi n}{7}} - e^{j\frac{2\pi n}{5}}$

$$= 1 + e^{j\frac{2\pi}{7}\cdot 2n} - e^{j\frac{2\pi}{5}\cdot n}$$

$N_1 = 1$ $\quad N_2 = 0$ $\quad N_3 = 5$

$N = LMC(1, 7, 5) = 35.$