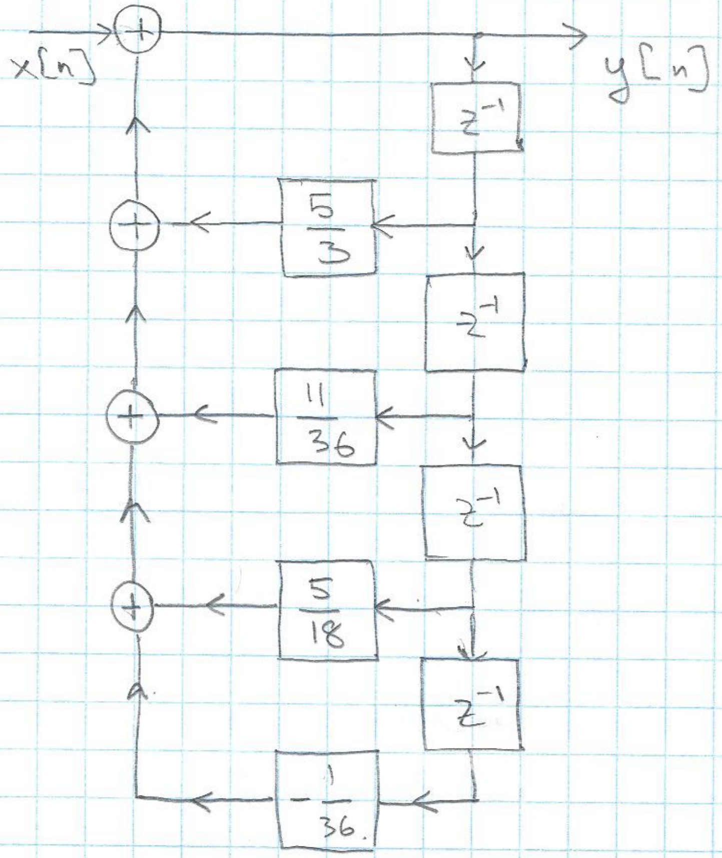
ASSIGNMENT 5 SOLUTION.

Question 1.

(a) We multiply the 2^{nd} order polynomials of each denominator. We obtain:

$$H_1(z) = \frac{1}{\left(1 - z^{-1} + \frac{1}{4}z^{-2}\right)\left(1 - \frac{2}{3}z^{-1} + \frac{1}{9}z^{-2}\right)}$$

$$= \frac{1}{1 - \frac{5}{3}z^{-1} - \frac{11}{36}z^{-2} - \frac{5}{18}z^{-3} + \frac{1}{36}z^{-4}}$$



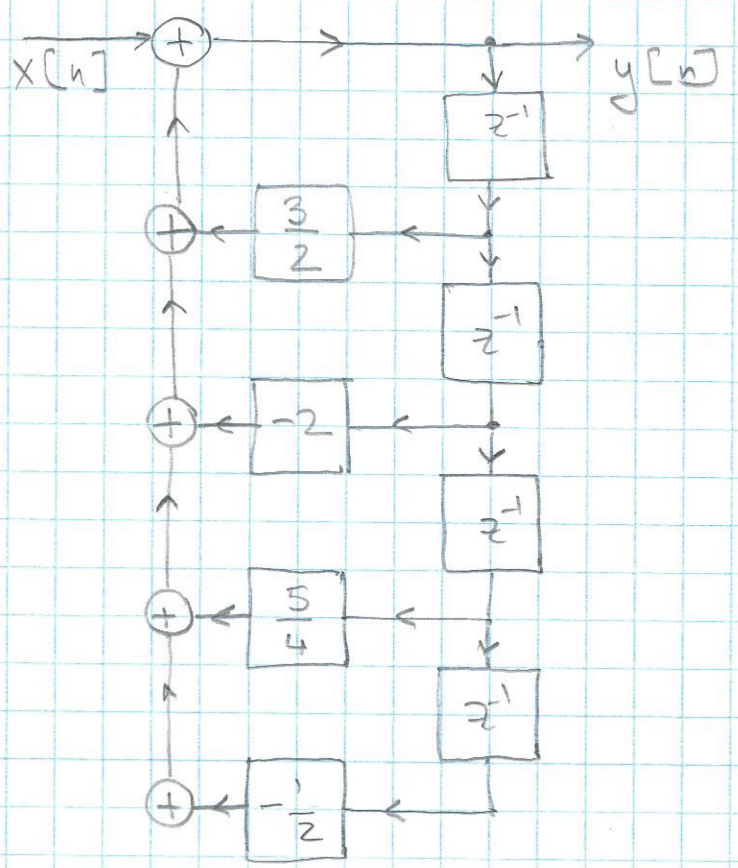
DIRECT FORM II

FOR $H_1(z)$

(2) ASSIGNMENT 5 SOLUTION.

$$H_2(z) = \frac{1}{\left(1 - z^{2} + \frac{1}{2} z^{-2}\right)\left(1 - \frac{1}{2} z^{-1} + z^{-2}\right)}$$

$$= \frac{1}{1 - \frac{3}{2} z^{-1} + 2 z^{-2} - \frac{5}{4} z^{-3} + \frac{1}{2} z^{-4}}.$$



DIRECT FORM II

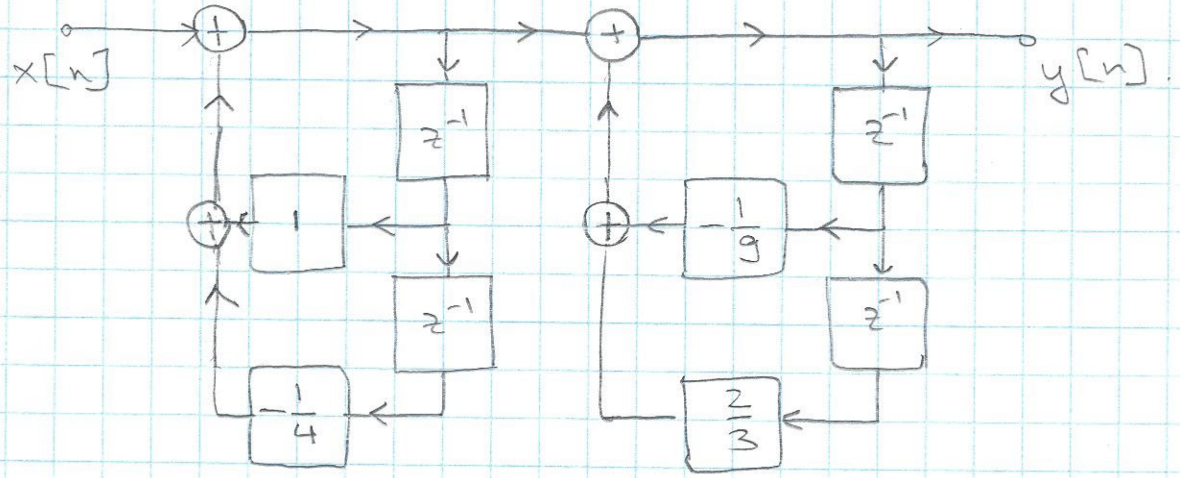
FOR $H_2(z)$.

③ ASSIGNMENT 5 SOLUTION.

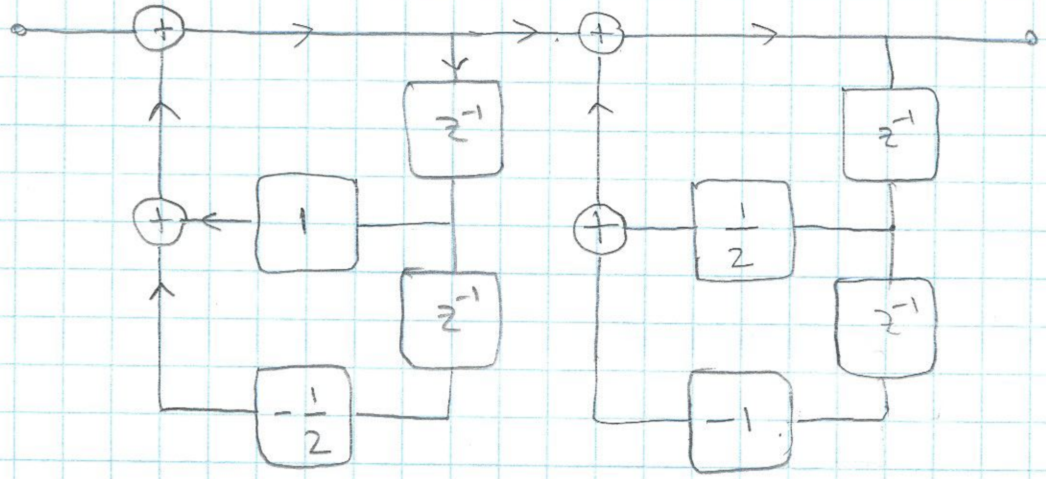
Question 1b.

The cascade connection of two second-order block diagrams can be drawn using the initial expression of $H_1(z)$ and $H_2(z)$.

For $H_1(z)$



For $H_2(z)$



Note: for both systems, the cascade connection can be done in any order.

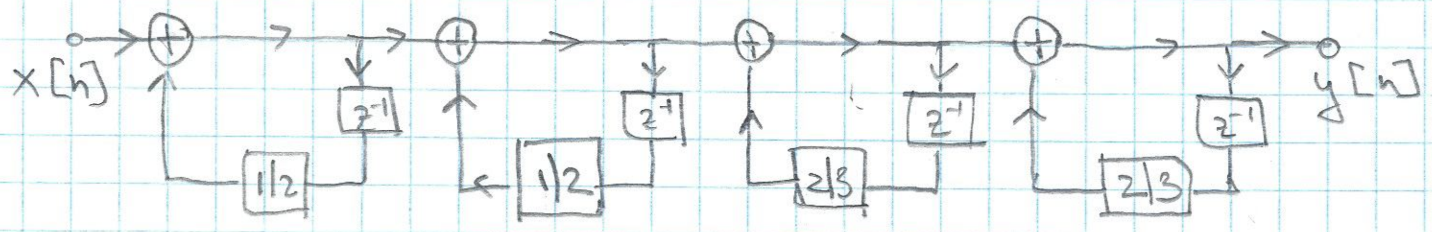
④ ASSIGNMENT 5 SOLUTION.

Question 1c.

We can write

$$H_1(z) = \left[\frac{1}{1-\frac{1}{2}z^{-1}}\right]\left[\frac{1}{1-\frac{1}{2}z^{-1}}\right]\left[\frac{1}{1-\frac{2}{3}z^{-1}}\right]\left[\frac{1}{1-\frac{2}{3}z^{-1}}\right].$$

Therefore, $H_1(z)$ can be drawn as a cascade of 4 1st order systems for which the coefficient multipliers are all real.



$$H_2(z) = \frac{1}{1-\frac{(1+j)}{2}z^{-1}} \cdot \frac{1}{1-\frac{1-j}{2}z^{-1}} \cdot \frac{1}{1+\frac{1+j\sqrt{15}}{4}z^{-1}} \cdot \frac{1}{1-\frac{1-j\sqrt{15}}{4}z^{-1}}$$

All poles of $H_2(z)$ are complex

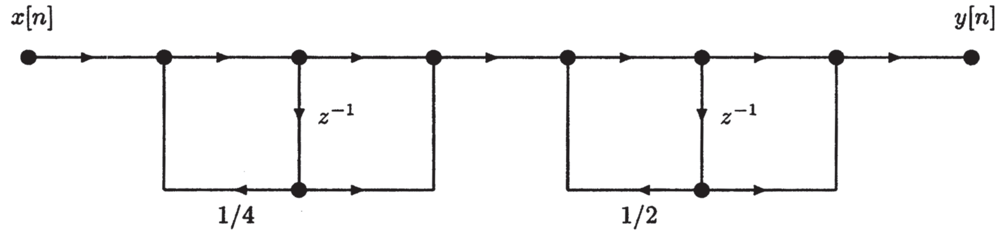
$\Downarrow$

$H_2(z)$ can not be drawn as a cascade of four systems for which the coefficient multipliers are real.

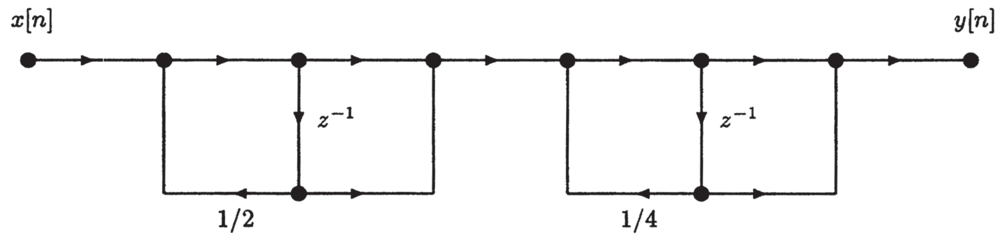
6.22.

$$H(z) = \frac{(1+z^{-1})^2}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1})}.$$

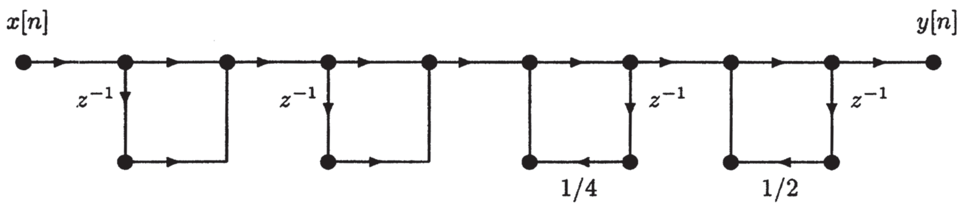
$$H(z) = \left(\frac{1+z^{-1}}{1-\frac{1}{4}z^{-1}}\right)\left(\frac{1+z^{-1}}{1-\frac{1}{2}z^{-1}}\right).$$



$$H(z) = \left(\frac{1+z^{-1}}{1-\frac{1}{2}z^{-1}}\right)\left(\frac{1+z^{-1}}{1-\frac{1}{4}z^{-1}}\right).$$



Plus 12 systems of this form:



with the three types of 1st-order systems taken in various orders.

6.25. A.

$$H(z) = \frac{1+0.81z^{-2}}{1-0.3z^{-1}-0.4z^{-2}} \times \frac{1+2z^{-1}}{1+0.8z^{-1}}$$
$$= \frac{\left(1+j0.9z^{-1}\right)\left(1-j0.9z^{-1}\right)\left(1+2z^{-1}\right)}{\left(1-0.8z^{-1}\right)\left(1+0.5z^{-1}\right)\left(1+0.8z^{-1}\right)}.$$

B. Yes, the overall system is stable. All of the poles are inside the unit circle, which guarantees stability for a causal system.

C. No, the system is not minimum-phase. There is a zero outside the unit circle at $z=-2$.

D.

$$H(z) = \frac{1+2z^{-1}+0.81z^{-2}+1.62z^{-3}}{1+0.5z^{-1}-0.64z^{-2}-0.32z^{-3}}.$$