L19 Example 2a)

$$x[n] = 3\delta[n-1] - \delta[n-2] + 2\delta[n-3].$$

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n]\, z^{-n}$$

$$= x[1]\, z^{-1} - x[2]\, z^{-2} + x[3]\, z^{-3}$$

$$= 3z^{-1} - z^{-2} + 2z^{-3}$$

$$= \frac{3z^{2} - z + 2}{z^{3}}$$

$X(0) = \infty \Rightarrow X(z)$ has a pole in 0.

$$ROC = \{|z| > 0\}.$$

the unit circle is in the ROC

$\downarrow$

The Fourier transform exists.

$$X(e^{j\omega}) = \frac{3e^{j2\omega} - e^{j\omega} + 2}{z e^{j3\omega}}.$$

L19. Example 2b.

$$x[n] = \begin{cases} \left(\frac{1}{3}\right)^n \cos\left(\frac{\pi}{4} n\right) & n \leq 0 \\ 0 & n > 0 \end{cases}$$

$$X(z) = \sum_{n=-\infty}^{0} x[n] \, z^{-n}$$

$$= \sum_{n=-\infty}^{0} \left(\frac{1}{3}\right)^n \cos\left(\frac{\pi}{4} n\right) z^{-n}$$

$$= \frac{1}{2} \sum_{n=-\infty}^{0} \left(\frac{1}{3}\right)^n e^{j\frac{\pi}{4} n} z^{-n} + \frac{1}{2} \sum_{n=-\infty}^{0} \left(\frac{1}{3}\right)^n e^{-j\frac{\pi}{4} n} z^{-n}$$

We substitute $m = -n$

$$= \frac{1}{2} \sum_{m=0}^{\infty} \left(\frac{1}{3} e^{j\frac{\pi}{4}} z^{-1}\right)^{-m} + \frac{1}{2} \sum_{m=0}^{\infty} \left(\frac{1}{3} e^{-j\frac{\pi}{4}} z^{-1}\right)^{-m}$$

$$= \frac{1}{2} \underbrace{\sum_{m=0}^{\infty} \left(3 e^{-j\frac{\pi}{4}} z\right)^m}_{= \frac{1}{1 - 3e^{-j\frac{\pi}{4}} z}} + \frac{1}{2} \underbrace{\sum_{m=0}^{\infty} \left(3 e^{j\frac{\pi}{4}} z\right)^m}_{= \frac{1}{1 - 3e^{j\frac{\pi}{4}} z}}$$

for $\left|3 e^{-j\frac{\pi}{4}} z\right| < 1$, $\boxed{|z| < \frac{1}{3}}$ for $\boxed{|z| < \frac{1}{3}}$

$$X(z) = \frac{1}{2} \cdot \frac{1}{1 - 3e^{j\frac{\pi}{4}} z^{-1}} + \frac{1}{2} \frac{1}{1 - 3e^{-j\frac{\pi}{4}} z^{-1}}$$

ROC : $|z| < \frac{1}{3}$. does not contain the unit circle

$\Downarrow$

the Fourier Transform does not exist