ELEC 310. Lecture 12. Frequency domain representation of discrete-time signals and systems

Textbook: section 2.6

Eigenfunctions of LTI systems

- Last time we discussed about periodic exponentials as eignefunctions of LTI systems
- In fact, ANY complex exponential is an eigenfunction of LTI systems.
- Here are the main types of complex exponentials

$x[n] = z^n$ with $z = \alpha \cdot e^{j\omega}$

1) $\omega = 0 \Rightarrow x[n]$ is a real exponential (non periodic)

2) α is real $\Rightarrow x[n]$ is periodic if $\dfrac{\omega}{2\pi}$ is rational

3) α is complex $\Rightarrow x[n]$ is a complex exponential (most general case) and not periodic

- Let's prove this eigenfunction property

Why are eigenfunctions important?

- If we can decompose a general signal into a sum of eigenfunctions, then we can use:
  - the eigenfunction property
  - superposition
- for computing the system's response to the general signal
- Informally: if the input to an LTI system is represented as a linear combination of complex exponentials, then the output can also be represented as a linear combination of the same complex exponential signals.

The frequency response of LTI system

- The frequency response of an LTI system is the eigenvalue $H(e^{j\omega})$ that corresponds to the eigenfunction $x[n] = e^{j\omega n}$

- We can compute the frequency response of LTI systems using the eigenfunction property

$$for\ x[n] = e^{j\omega n}$$

$$H(e^{j\omega}) = \sum_{k=-\infty}^{+\infty} h[k] e^{-j\omega k}$$

Differences between the CT case and the DT case

$H(e^{j\omega})$ is <u>always</u> periodic with 2π.

Let's prove this

Important consequences:

- We only need to specify $H(e^{j\omega})$ over an interval of length 2π.
- Most of the times we will use the $[-\pi, \pi]$ interval
- Low frequencies: frequencies close to 0
- High frequencies: frequencies close to $\pm\pi$.

Figure 2.17 Ideal lowpass filter showing (a) periodicity of the frequency response and (b) one period of the periodic frequency response.

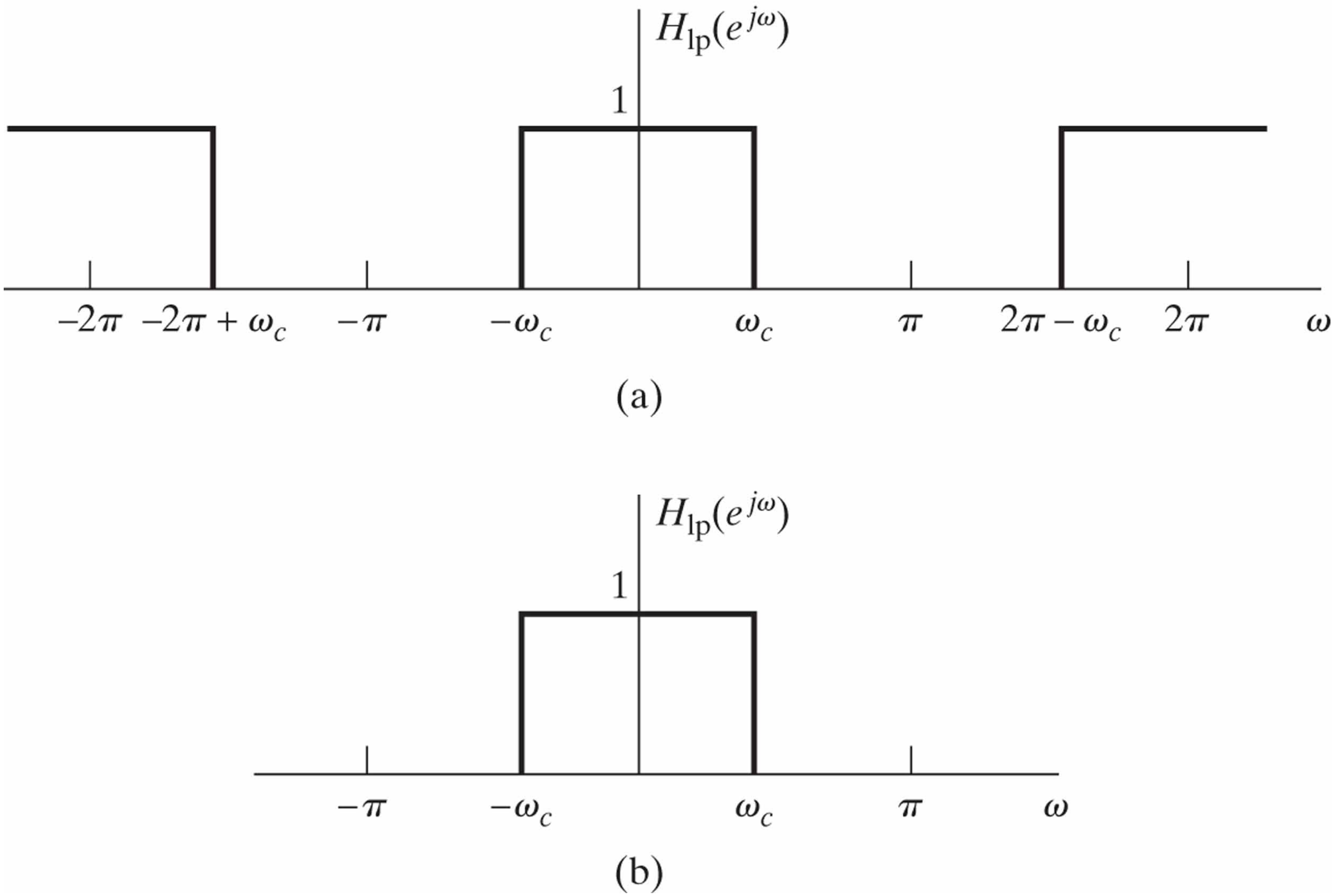
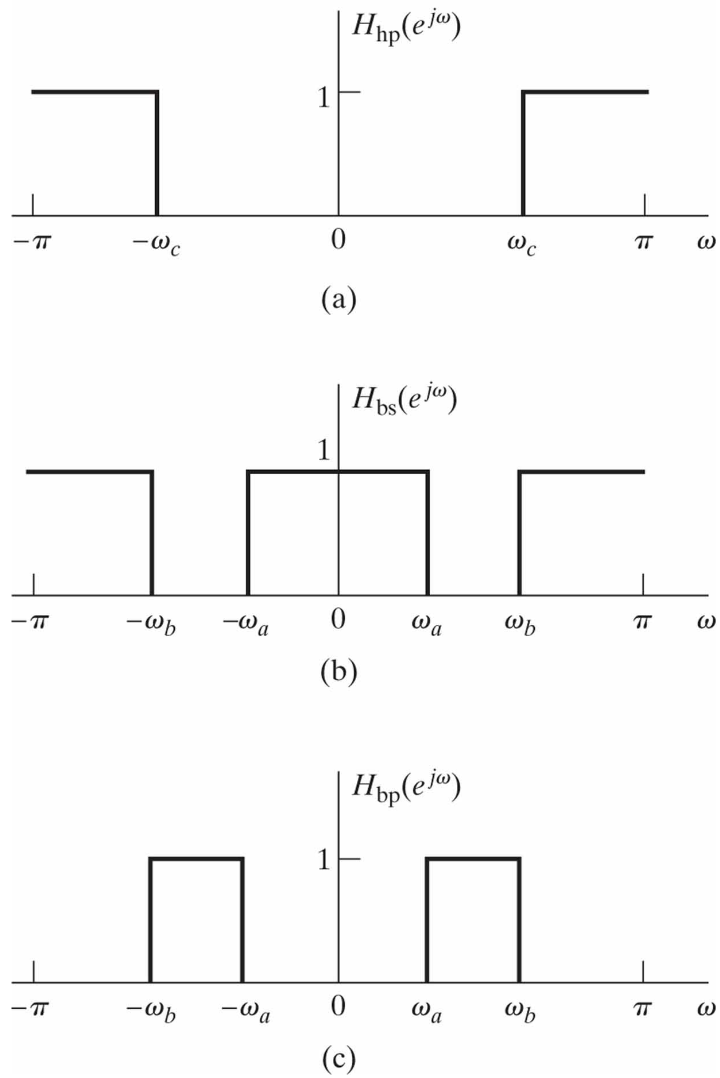


Figure 2.18 Ideal frequency-selective filters. (a) Highpass filter. (b) Bandstop filter. (c) Bandpass filter. In each case, the frequency response is periodic with period 2π. Only one period is shown.



Example: compute the frequency response of the **moving average system** (see also examples 2.3 and 2.16 from textbook)

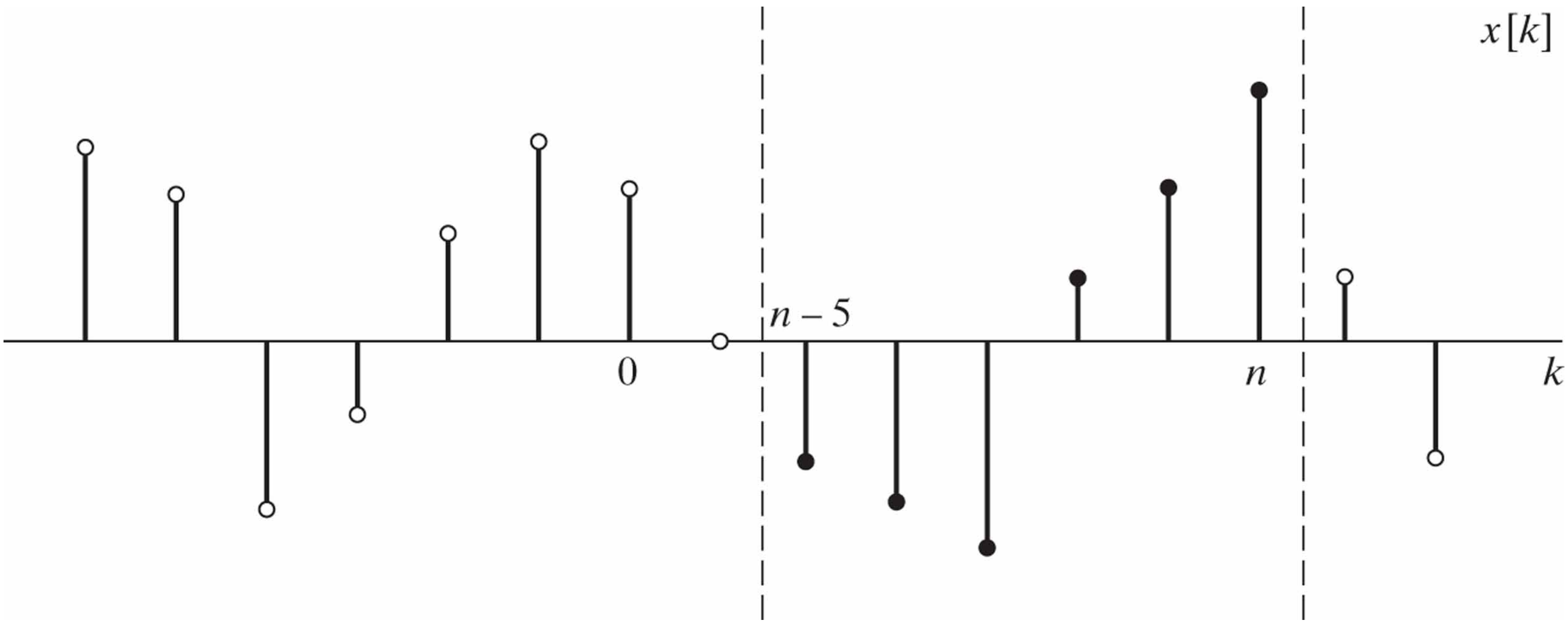


Figure 2.7 Sequence values involved in computing a moving average with $M_1 = 0$ and $M_2 = 5$.

Figure 2.19 (a) Magnitude and (b) phase of the frequency response of the moving-average system for the case $M_1 = 0$ and $M_2 = 4$.