

The Discrete Time Fourier Transform (2)

Outline

Examples of direct DTFT computations

From these examples, we will derive some important DTFT transform pairs

DTFT equations

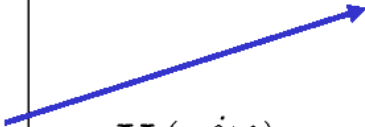
$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

Synthesis equation

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

Analysis equation

Any 2π
interval in ω



Examples

Last lecture we found the DTFT of:

- $x_1[n] = \delta[n]$
- $x_2[n] = \delta[n - n_0]$
- $x_3[n] = \delta[n] + \delta[n - 1]$

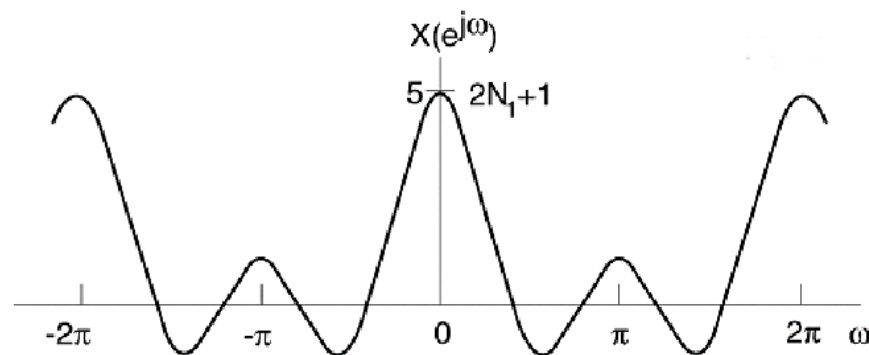
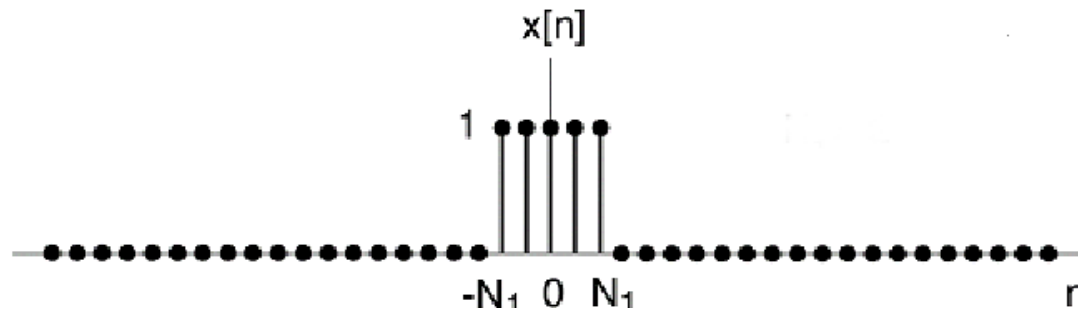
Examples (cont'd)

- Determine the inverse Fourier Transform for

$$X(e^{j\omega}) = 1 + 3e^{-j\omega} + 2e^{-j2\omega} - 4e^{-j3\omega} + e^{-j10\omega}$$

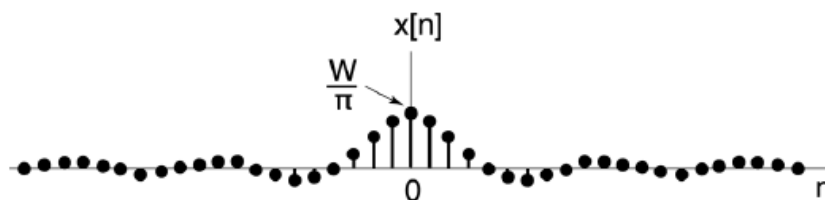
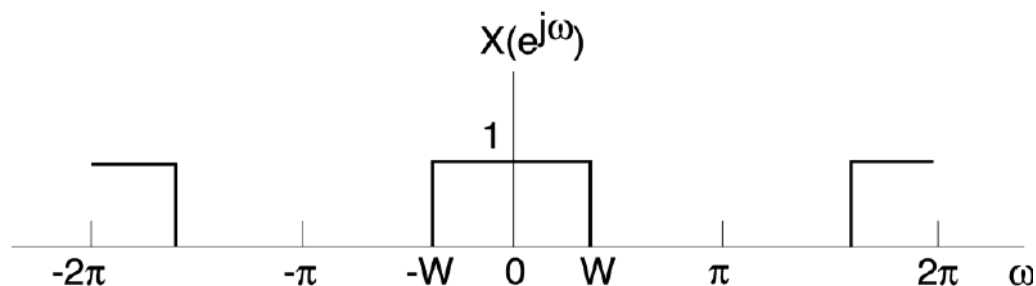
Examples (cont'd)

- Find the DTFT for the DT rectangular pulse (drawn for $N_1=2$)



Rectangular pulse in frequency

Find the Inverse Fourier transform for $X(e^{j\omega})$ shown below



We have derived the following transform pairs

Signal	Fourier transform
$a^n u[n], \quad a < 1$	$\frac{1}{1 - ae^{-j\omega}}$
$x[n] \begin{cases} 1, & n \leq N_1 \\ 0, & n > N_1 \end{cases}$	$\frac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$
$\frac{\sin Wn}{\pi n} = \frac{W}{\pi} \text{sinc} \left(\frac{Wn}{\pi} \right)$ $0 < W < \pi$	$X(e^{j\omega}) = \begin{cases} 1, & 0 \leq \omega \leq W \\ 0, & W < \omega \leq \pi \end{cases}$ $X(e^{j\omega})$ periodic with period 2π
$\delta[n]$	1