The Discrete Time Fourier Transform. Linear constant coefficient difference equations

Today

- Example using the convolution property
- The frequency response of LTI systems defined by a linear constant coefficient difference equation
- Example
- Wrap-up of the DTFT

- Assignment 1 posted. Due date: Feb 24.

Example (2) for convolution property

- Consider an LTI filter with

$$h[n] = \frac{\sin\frac{\pi n}{16}}{\pi n} - \frac{\sin\frac{\pi n}{12}}{\pi n}$$

- Determine the response y[n] of the filter to the signal

$$x[n] = \sin\frac{\pi n}{8} - 2\cos\frac{\pi n}{4}$$

Systems characterized by linear-constant coefficient difference equations

- Our goal: to determine the frequency response of the LTI system $H(e^{j\omega})$ from the implicit input-output relationship

$$\sum_{k=1}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

- 1st method: use the convolution property, as well as linearity and time shifting properties of the DTFT
- 2nd method: determine h[n] first. see example 2.25.

Example

- Consider a system consisting of a cascade of two LTI systems with the following frequency responses

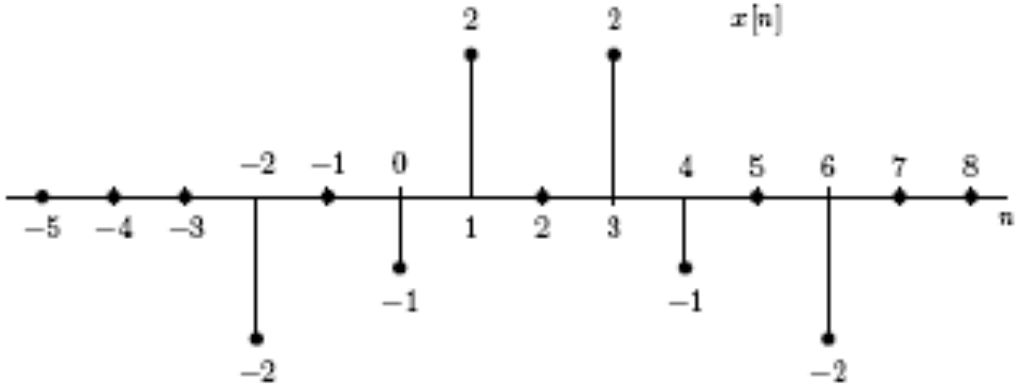
$$H_1\left(e^{j\omega}\right)=\frac{2-e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}}$$

$$H_2\left(e^{j\omega}\right)=\frac{1}{1-\frac{1}{2}e^{-j\omega}+\frac{1}{4}e^{-j2\omega}}$$

- Find the difference equation describing the overall system

Wrap-up example for DTFT properties and equations

- Let $X(e^{j\omega})$ denote the Fourier Transform of the signal x[n] sketched below.



- a) Find X(1)
- b) Find α such that $e^{j\omega\alpha} X(e^{j\omega})$ is real
- c) Determine and sketch the signal whose Fourier Transform is $\mathrm{Re}\{X(e^{j\omega})\}$
- d) Evaluate the following integral

$$\int_{-\pi}^{\pi} X\left(e^{j\omega}\right) d\omega$$

What you need to know about the DT Fourier Transform

- How to compute the direct and inverse FT using:
  - The analysis and synthesis equations
  - Convergence issues
  - The properties of the FT
  - Fourier Transform Pairs
  - See also examples 2.22 and 2.23 from your textbook

- How to determine a signal x[n] from an incomplete description in time and frequency domains
  - Properties of FT, decomposition of signals into even and odd parts
  - See example from L16.

- How to analyze LTI systems in the frequency domain
  - Convolution property
  - See also examples 2.24, 2.25