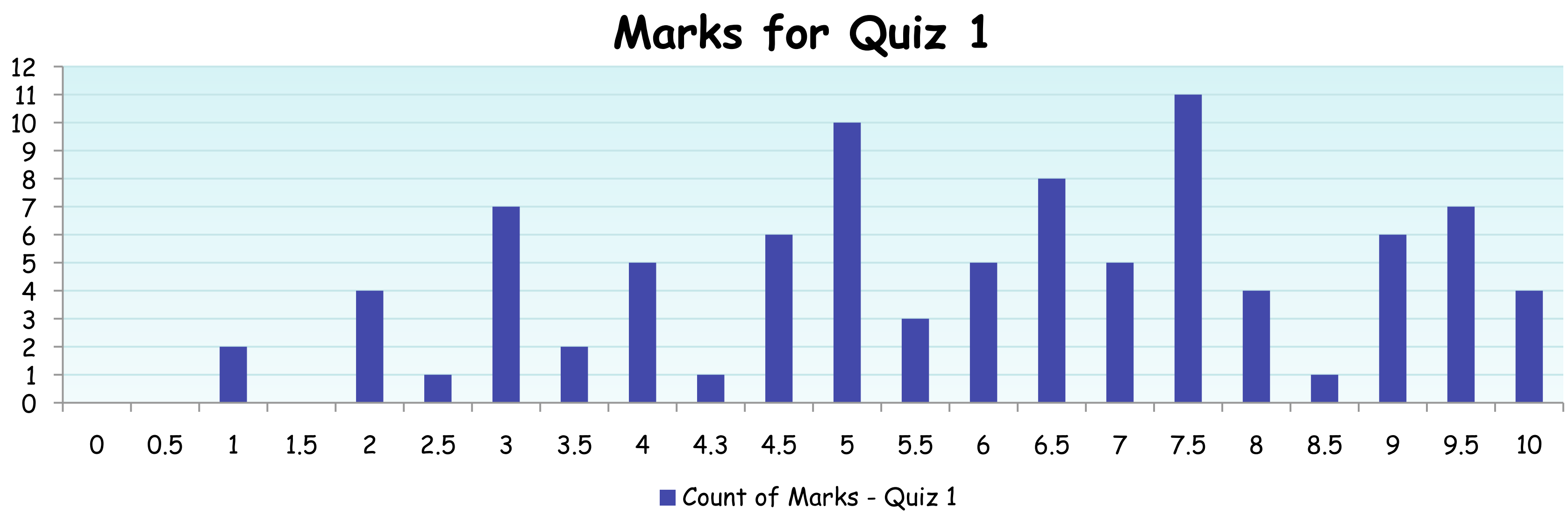
Today

- Discussion of Quiz 1
- The Z transform. Introduction of the concept

Distribution of marks



Common errors

Question 1.

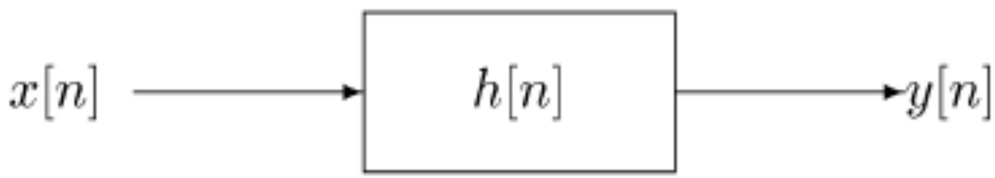
- Variable substitution affects summation limits
  - Some final results had x[n] defined for values of k
- Formula for the geometric series unknown:
  - the ZT makes extensive use of this
- Convolution with δ unknown.

Question 2.

- The fundamental period of a discrete signal is always a positive integer (not 8/3)
- 8/3 is a rational number

The z-transform: derivation

- Analogous to the Laplace transform in CT: it can be applied to a broader class of signals than the DT Fourier Transform



$$x[n] = z^n \Rightarrow y[n] = H(z)z^n$$

where $H(z) = \sum_{n=-\infty}^{+\infty} h[n]z^{-n}$ assuming that the summation is convergent

- For $z=e^{j\omega}$ H(z) is the Fourier Transform of the signal h[n]
- **In the more general case ($|z| \neq 1$) H(z) is referred to as the Z-transform of h[n].**
- definition

Relationship between the Z-transform and DTFT

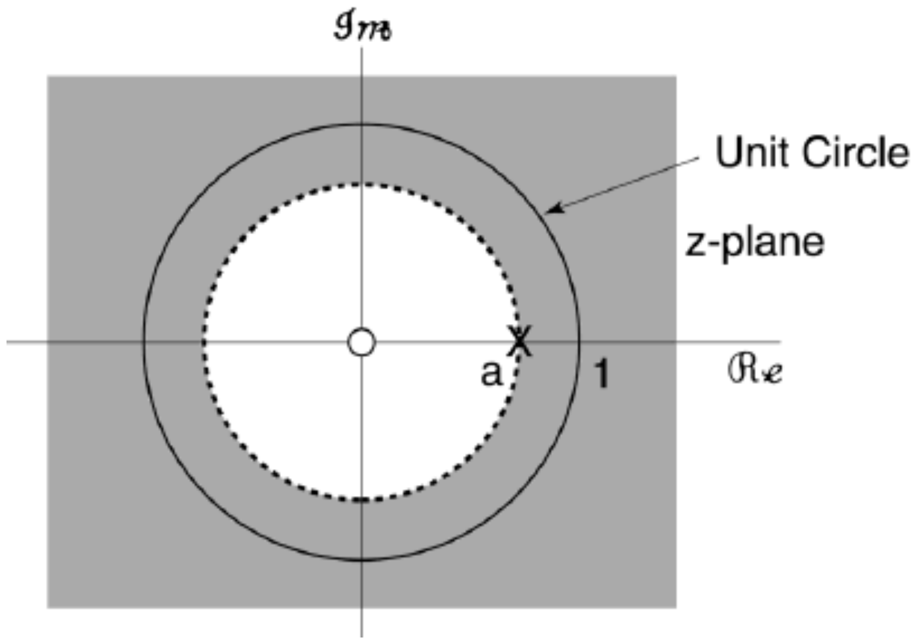
- We express z in the polar form $z=re^{j\omega}$
- We derive the relationship between ZT and FT

Consequences

- The Z transform reduces to the Fourier transform when the magnitude of z is unity.
  - i.e. on the contour of the **unit circle** in the complex plane
- Region of convergence considerations

Example 1

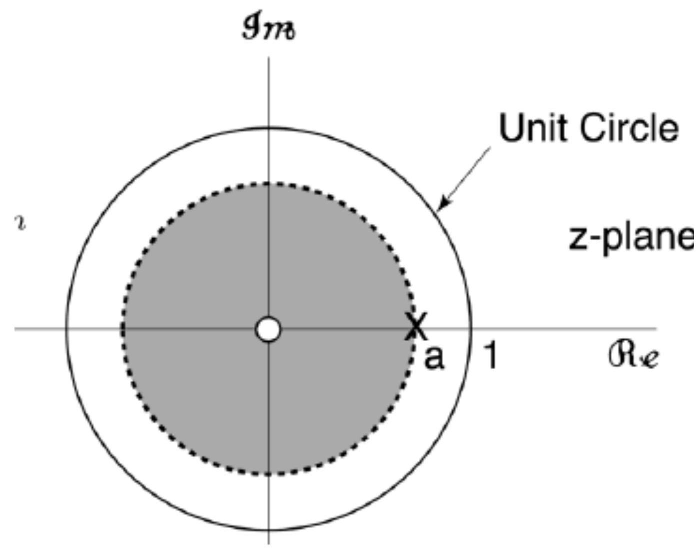
- $x[n]=a^n u[n]$



Pole-zero plot and region of convergence for 0<a<1.

Example 2

- $x[n]=-a^n u[-n-1]$



Pole-zero plot and region of convergence for 0<a<1

Observations

- Specification of the Z transform requires both algebraic expression and region of convergence
- Rational Z-transforms are obtained if x[n]=linear combination of exponentials for $n>0$ and $n<0$

$$X(z) = \frac{N(z)}{D(z)}$$

Polynomials in z

- Rational Z-transforms are completely characterized by their poles and zeros (except for the gain)

Useful formulas

The direct Z transform:

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n] z^{-n}$$

NOTE: We have not defined the inverse Z Transform yet

Relationships between Z and Fourier Transforms:

A. $X(z) = Fourier\{x[n] r^{-n}\}$

Convergence: The Z transform is convergent iff

the sequence $x[n] r^{-n}$ is absolutely summable.

All these values of z form the Region of Convergence (ROC)

B. $X(z)\Big|_{z=e^{j\omega}} = Fourier\{x[n]\}$

The Z Transform reduces to Fourier when $|z|=1$ (the unit circle)