ELEC 310
Digital Signal Processing

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Today

Complex numbers

- i versus j
- Rectangular form
- Polar form
- Relationships between the two forms
- Graphical representations of complex numbers
- Complex arithmetic

Representation of sinusoidal DT signals via complex numbers

- Periodicity

NOTE: most of today's lecture will use the blackboard.

Exercise

- Let z_0 be a complex number with polar coordinates (r_0, θ_0) and Cartesian coordinates (x_0, y_0).
- Determine expressions for the Cartesian coordinates of the following complex numbers in terms of x_0 and y_0.
- Plot these points in the complex plane knowing that $r_0=2$ and $\theta_0=\pi/4$

Periodicity properties of DT

$$x(t) = e^{j\omega_0 t}$$

1) The larger the magnitude of ω_0, the higher is the rate of oscillation in the signal

$$x[n] = e^{j\omega_0 n}$$

$$e^{j(\omega_0 + 2\pi)n} = e^{j\omega_0 n}$$

The signal with frequency ω_0 is **identical** to signals with frequencies $\omega_0 \pm 2k\pi$, k integer

We need to choose an interval of length 2π , either $[-\pi, \pi)$ or $[0, 2\pi)$.

How does the rate of oscillation vary from 0 to 2π?

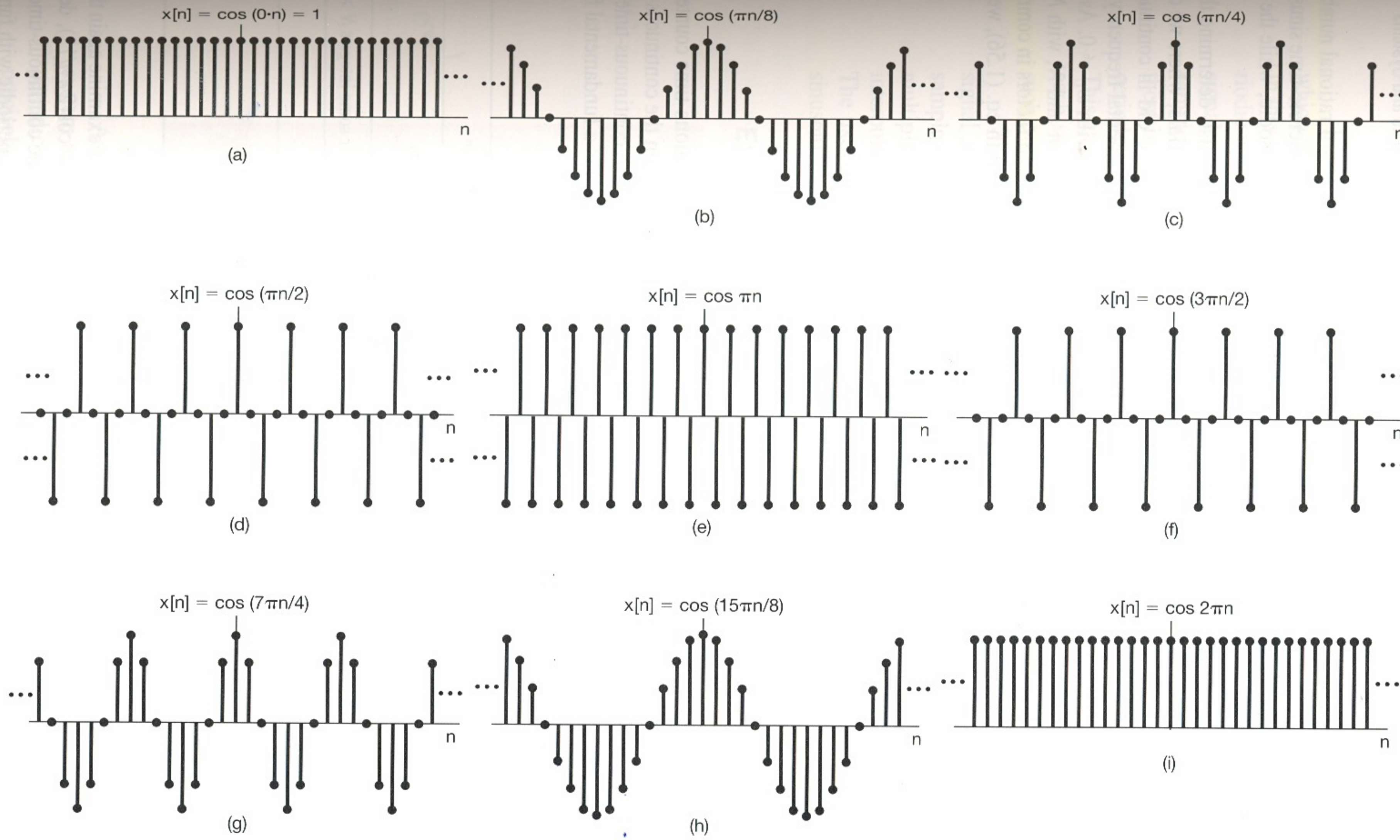

Figure 1.27 Discrete-time sinusoidal sequences for several different frequencies.

Periodicity properties of DT

- CT

$$x(t) = e^{j\omega_0 t}$$

$$x[n] = e^{j\omega_0 n}$$

1) The larger the magnitude of ω_0, the higher is the rate of oscillation in the signal
2) This signal is periodic for any non-zero value of ω_0

The DT signal x[n] is periodic only if $\omega_0/2\pi$ is a rational number.

Fundamental frequency $\omega_0 = 2\pi \frac{m}{N}$ where m and N have no factors in common.

Example

- Determine the fundamental period of

$$x[n] = 2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{8}n\right) - 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right)$$