The Z transform (4)

Today

Properties of the Z-transform (example)

Analysis of LTI systems using the Z Transform

- Section 3.5
- Examples 3.20, 3.21

! We will not discuss the unilateral Z-Transform (section 3.6 not required)

Example

- 10.31. We are given the following five facts about a DT signal x[n] with Z-transform X(z)

- 1. x[n] is real and right-sided
- 2. X(z) has exactly two poles
- 3. X(z) has exactly two zeros at the origin
- 4. X(z) has a pole at $z = \frac{1}{2} e^{j\pi/3}$
- 5. X(1)=8/3

Determine X(z) and its ROC.

ZT property related to time reversal

$$x[-n] \xleftrightarrow{ZT} X(z^{-1})$$

Inverted R (i.e. the set of points z^{-1} where z is in R)

System functions

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n] = x[n] * h[n]$$

Y(z)=H(z)X(z)

ROC is at least the intersection of the ROCs of H(z) and X(z)

$$H(z) = \sum_{n=-\infty}^{+\infty} h[n] z^{-n} \text{ is called the system function}$$

H(z) + ROC tell us everything about the system

In particular, the ROC is useful for evaluating the causality and stability of the system

CAUSALITY

- *h*[*n*] right-sided: ROC is the exterior of a circle *possibly* including *z*=∞:

$$H(z) = \sum_{n=N_1}^{\infty} h[n] z^{-n}$$

- If $N_1<0$, then the term $h[N1]z^{-N1} \rightarrow \infty$ for $z \rightarrow \infty$
- ROC is the outside of a circle, but does not include ∞
- h[n] causal $\leftrightarrow N_1 \geq 0$: there are no z^m terms with $m<0$, thus $z=\infty$ is in the ROC

A DT LTI system with system function H(z) is causal ↔ the ROC of H(z) is the exterior of a circle including z=∞

Causality for systems with rational system functions

$$H(z) = \frac{b_M z^M + b_{M-1} z^{M-1} + \cdots + b_1 z + b_0}{a_N z^N + a_{N-1} z^{N-1} + \cdots + a_1 z + a_0}$$

1) no poles at ∞ : this implies $M \leq N$

2) the ROC is the exterior of a circle outside the outermost pole

Stability

- An LTI system is stable iff $\sum_{n=-\infty}^{+\infty} |h[n]| < \infty$

- The unit circle is in the ROC

Stability and causality

- A causal LTI system with a rational system function is stable if and only if all its poles are inside the unit circle (magnitudes <1)

Example 1.

- Find the impulse response h[n] of the system with the following system function and verify that the system is non-causal:

$$H(z) = \frac{z^2}{z-1}, |z| > 1.$$

Example 2.

- Consider a DT LTI system given by the following difference equation:

$$y[n-1]+\frac{10}{3}y[n]+y[n+1]=x[n]$$

Determine the unit impulse response of the system given that the system is stable.