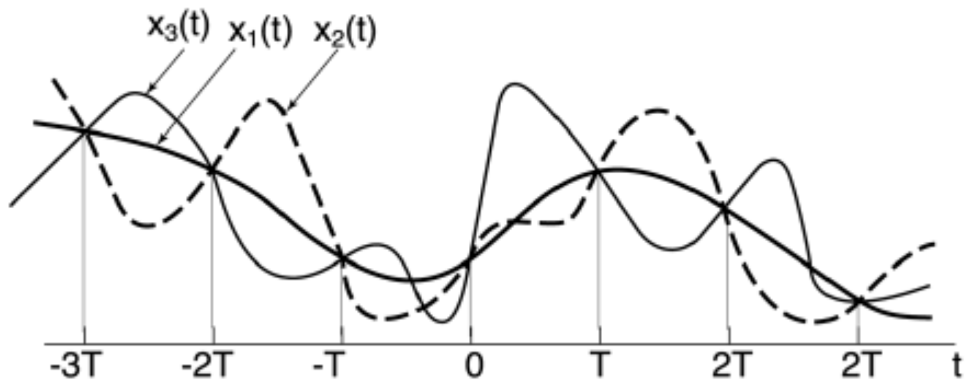
Sampling- Problem statement

- Most of the signals that we encounter in the real world are CT signals, e.g. $x(t)$.
- For lots of applications (data transmission, storage, processing) it is convenient to transform them into DT signals.
- How do we convert them into DT signals $x[n]$?
  - By periodic sampling, i.e. taking snapshots of $x(t)$ every T seconds.

When Would a Set of Samples Be Adequate?

- Lots of signals can have the same samples:



- Sampling usually occurs with loss of information (all values of x (t) between 2 successive sampling points are lost)

- Key question for sampling: Under what conditions can we reconstruct the original CT signal $x(t)$ from its samples?

Periodic (impulse-train) sampling

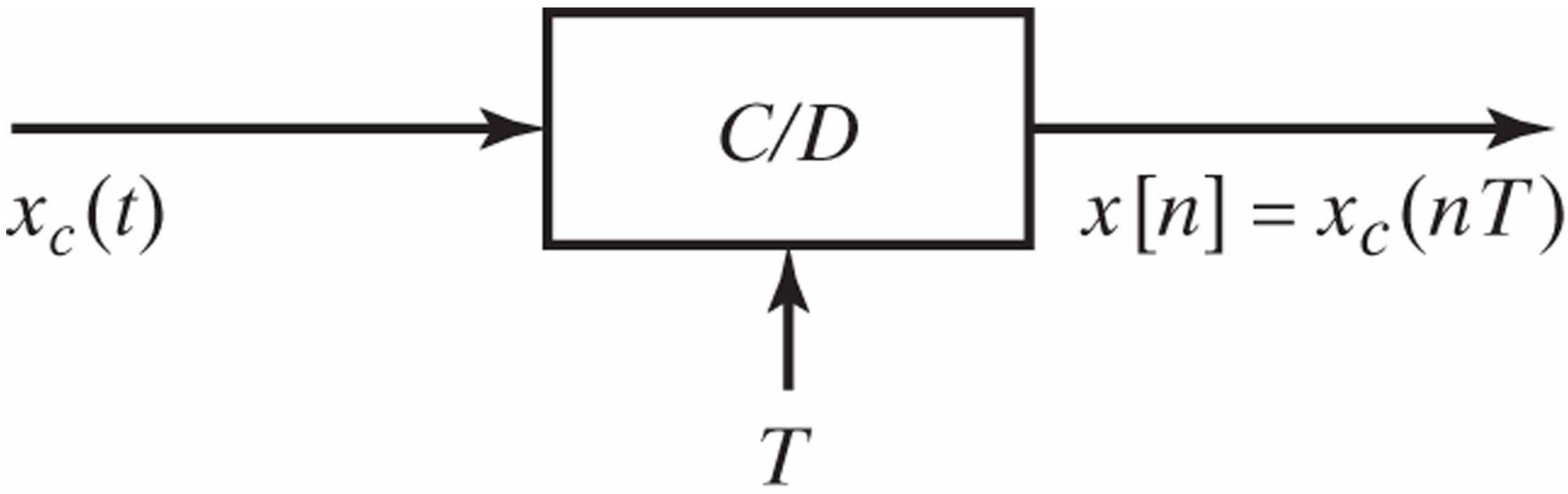
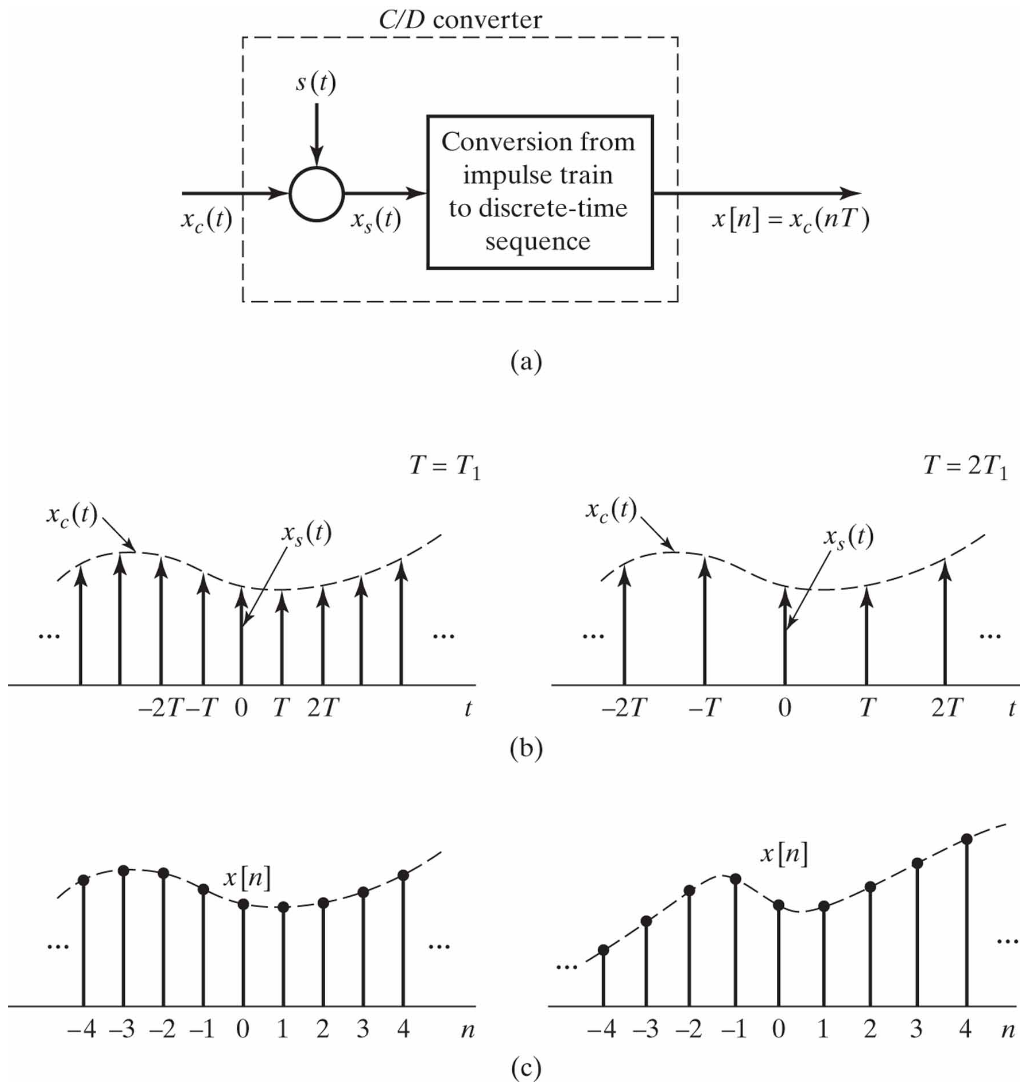
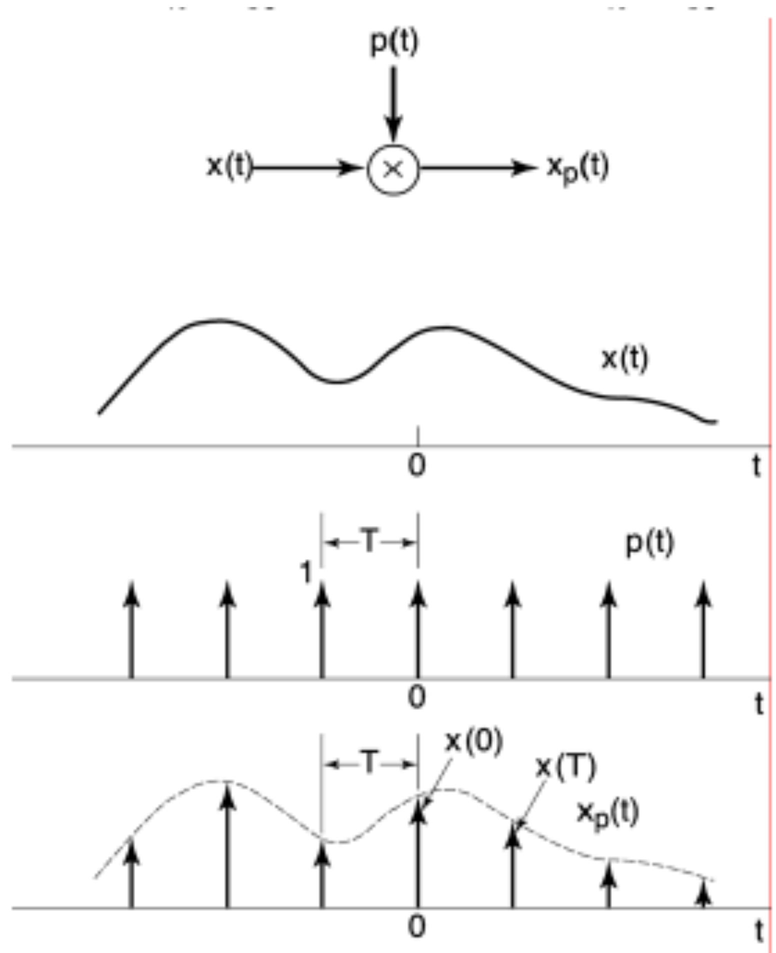


Figure 4.2 Sampling with a periodic impulse train, followed by conversion to a discrete-time sequence. (a) Overall system. (b) $x_s(t)$ for two sampling rates. (c) The output sequence for the two different sampling rates.



Periodic (impulse-train) sampling: time domain

- We need a convenient way to express the periodic sampling of a CT signal
- We use the sampling property of the unit impulse in order to construct **a sampling function.**



- Textbook notation of the sampling function: s(t) instead of p(t)

Analysis of sampling in the frequency domain

$$x_s(t) = x(t) \cdot s(t)$$

We use the multiplication property of the Fourier transform

The analysis is performed on the blackboard

Figure 4.3 Frequency-domain representation of sampling in the time domain. (a) Spectrum of the original signal. (b) Fourier transform of the sampling function. (c) Fourier transform of the sampled signal with $\Omega_s > 2\Omega_N$. (d) Fourier transform of the sampled signal with $\Omega_s < 2\Omega_N$.

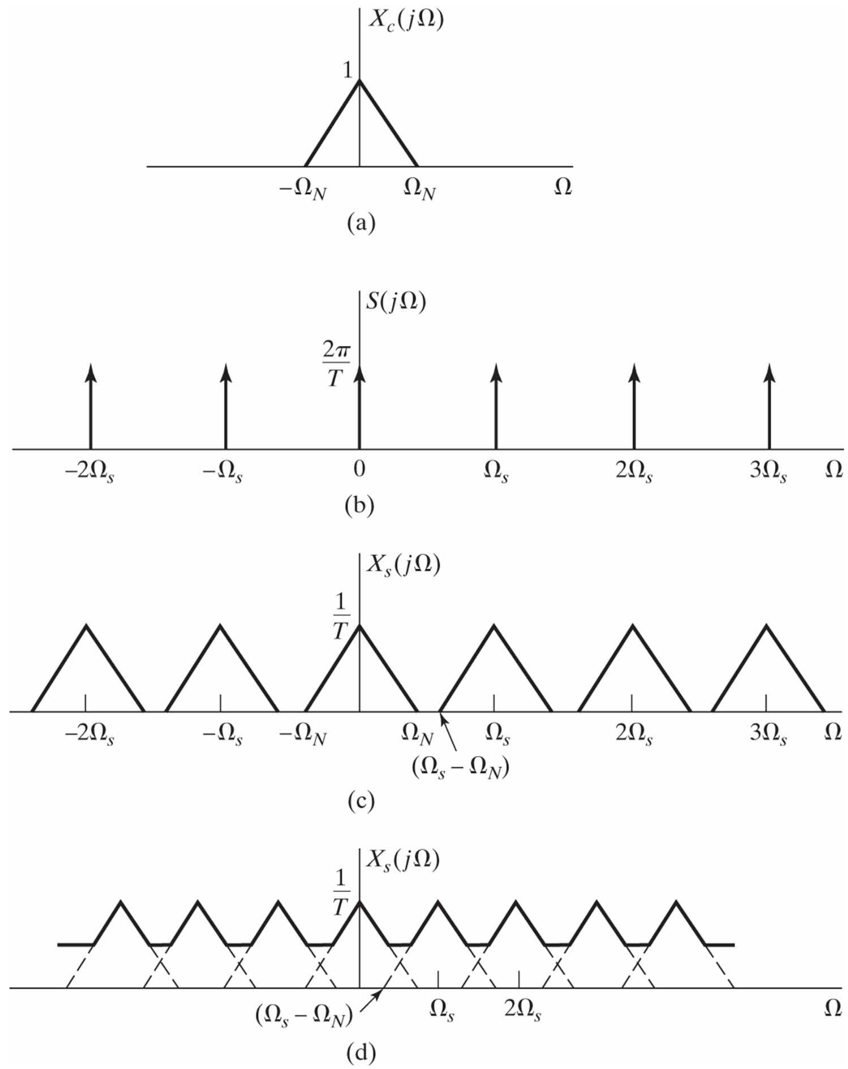
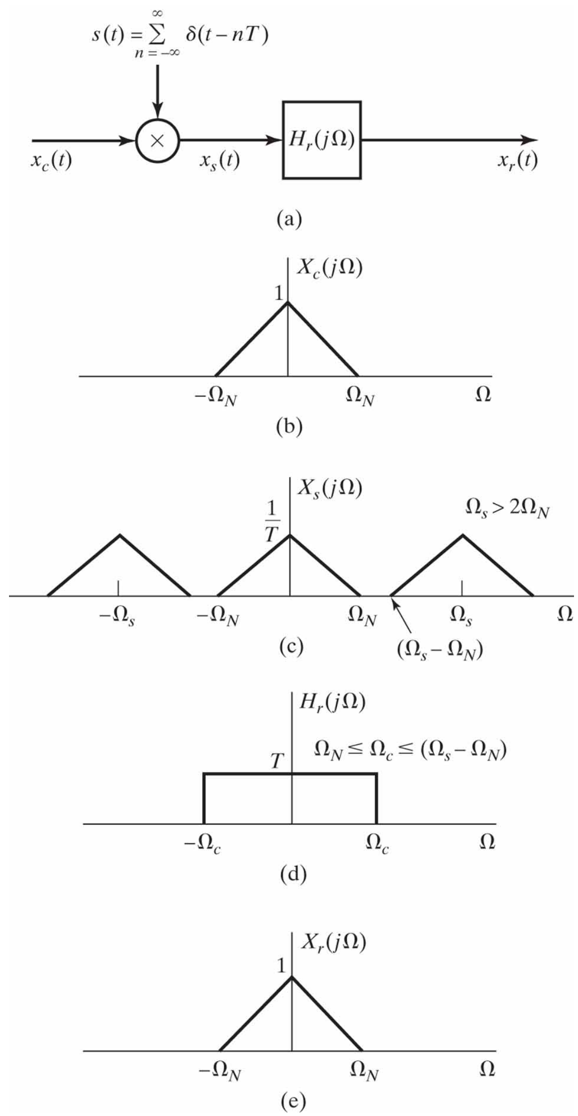


Figure 4.4 Exact recovery of a continuous-time signal from its samples using an ideal lowpass filter.



The sampling theorem

- Let x(t) be a band-limited signal, so that

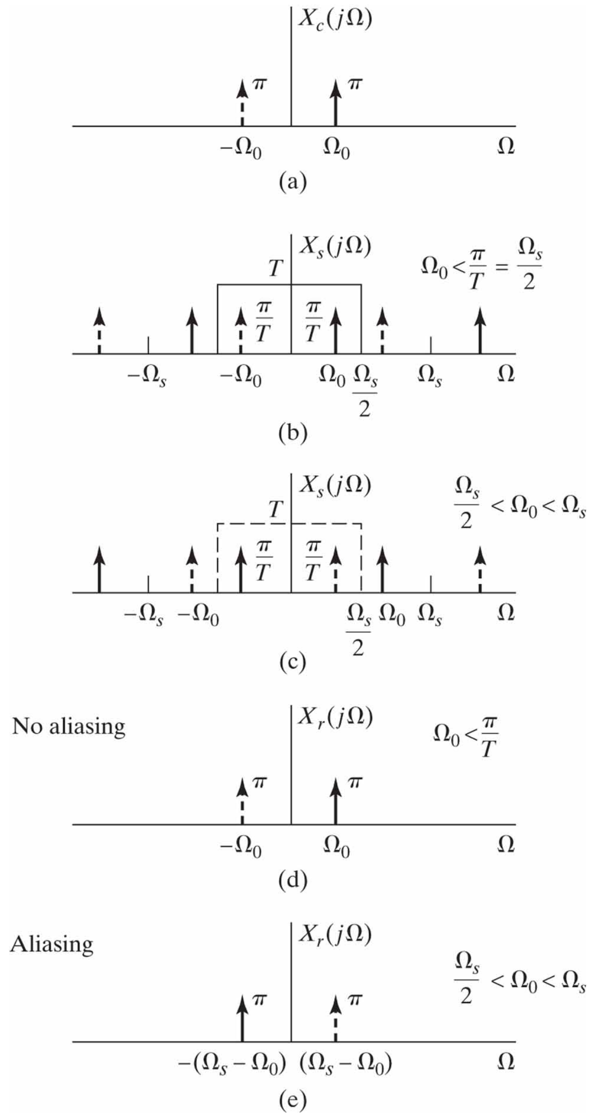
$$X(j\omega) = 0 \quad \text{for} \quad |\omega| > \omega_M$$

- Then x(t) is completely determined by its samples {x(nT)} if

$$\omega_s > 2\omega_M = \text{The Nyquist rate}$$

- where $\omega_s = 2\pi/T$

Figure 4.5   The effect of aliasing in the sampling of a cosine signal.



Aliasing

- Literally means “by a different name”
- It is used to explain the effect of undersampling a continuous signal, which causes frequencies to show up as different frequencies
- Aliased signal=signal at a different frequency
- Usually: higher frequencies are being aliased to lower frequencies

Some examples

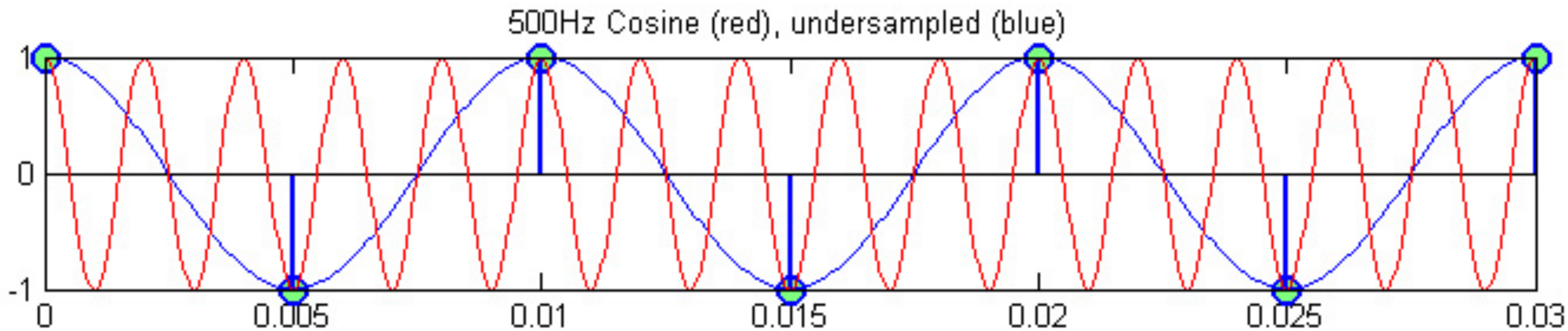
- 1D audio signals: the aliased frequency components sound lower in pitch
  - 2D images: parallel lines in pinstripe shirts aliasing into large wavy lines
- 2D time-varying signals: viewing propellers on a plane that seem to be turning slow when they are actually moving at very high speeds.

Aliasing-examples

Video demo

Moiré Patterns: http://ptolemy.eecs.berkeley.edu/eecs20/week13/moire.html

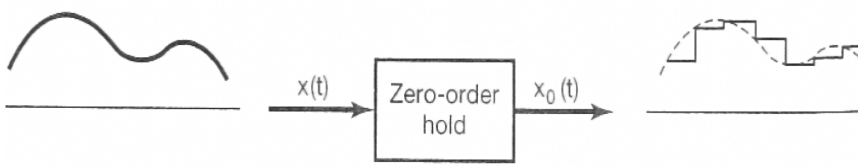
Aliasing-examples (cont'd)

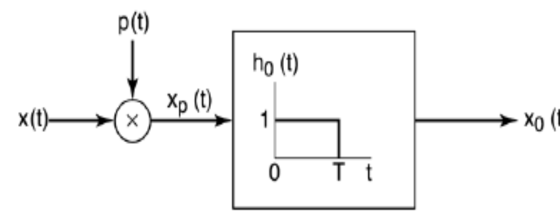


http://www.dsptutor.freeuk.com/aliasing/AliasingDemo.html

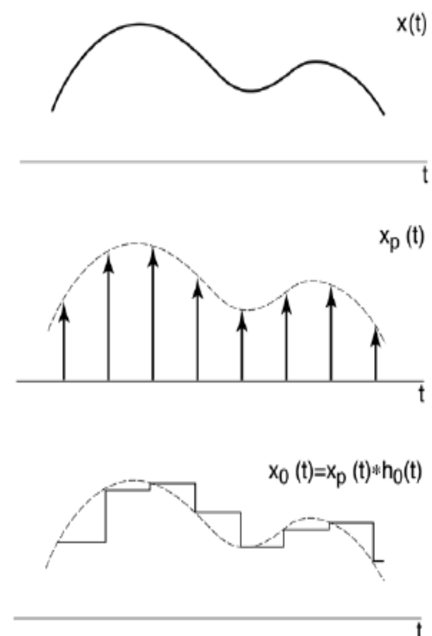
Observations on sampling

- In practice sampling is not done using trains of impulses
- A practical example: the zero-order hold





The zero-order hold system samples x (t) at a given instant and holds that value until the next sampling instant



In theory, the zero-order hold can be generated by impulse-train sampling followed by an LTI with rectangular impulse response