Outline

Sampling (Chapter 4)

Last lecture:

Sampling with a periodic train of impulses

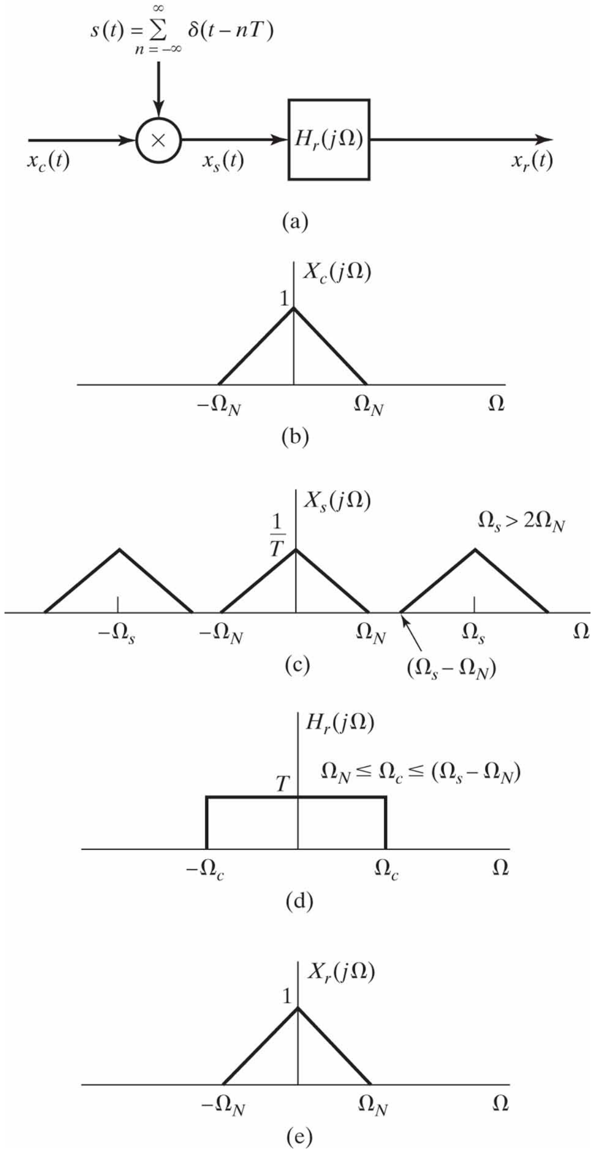
Sampling in the frequency domain

The Sampling Theorem —the Nyquist Rate

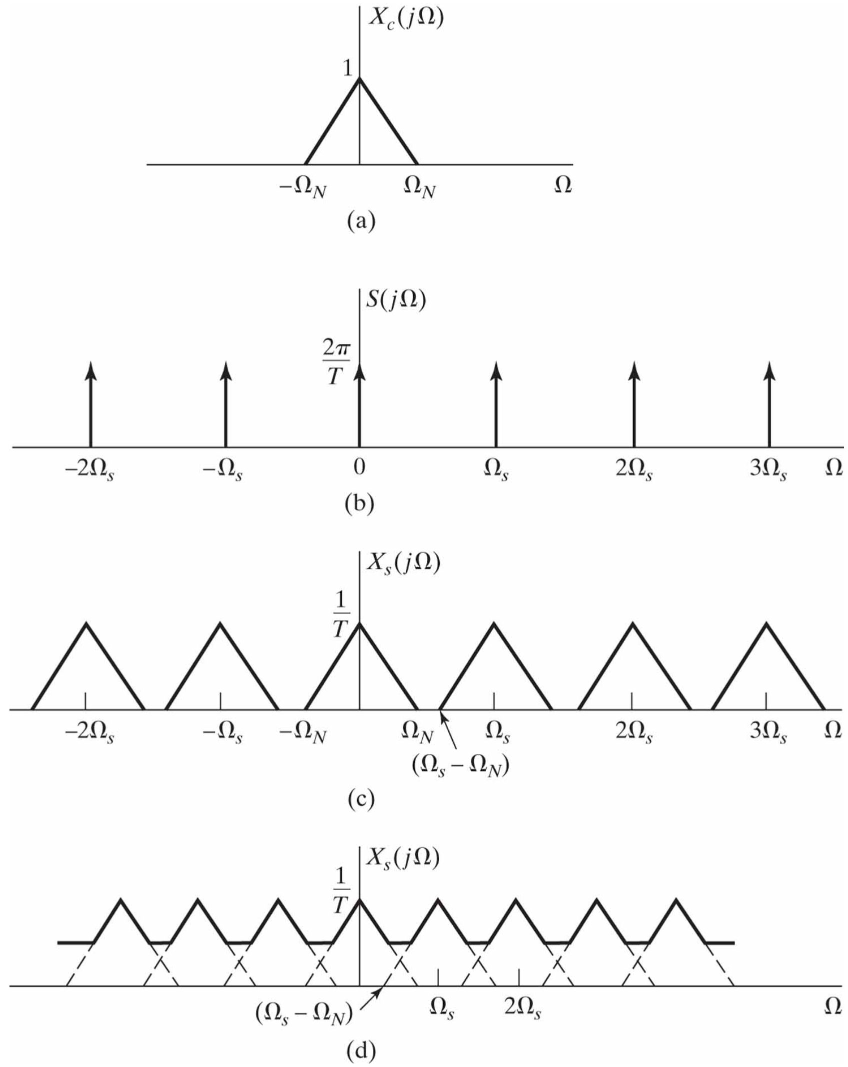
Today

- We will learn how to reconstruct the original signal from its samples (section 4.3)
- example

Review: Analysis of the impulse-train sampling in the frequency domain



Review: Undersampling (aliasing)



a b c

FIGURE 4.17 Illustration of aliasing on resampled images. (a) A digital image with negligible visual aliasing. (b) Result of resizing the image to 50% of its original size by pixel deletion. Aliasing is clearly visible. (c) Result of blurring the image in (a) with a 3×3 averaging filter prior to resizing. The image is slightly more blurred than (b), but aliasing is not longer objectionable. (Original image courtesy of the Signal Compression Laboratory, University of California, Santa Barbara.)

Review: The sampling theorem

- Let x(t) be a band-limited signal, so that

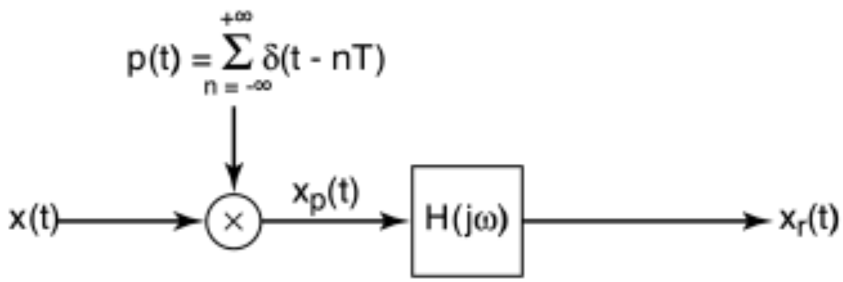
$$X(j\omega) = 0 \quad \text{for} \quad |\omega| > \omega_M$$

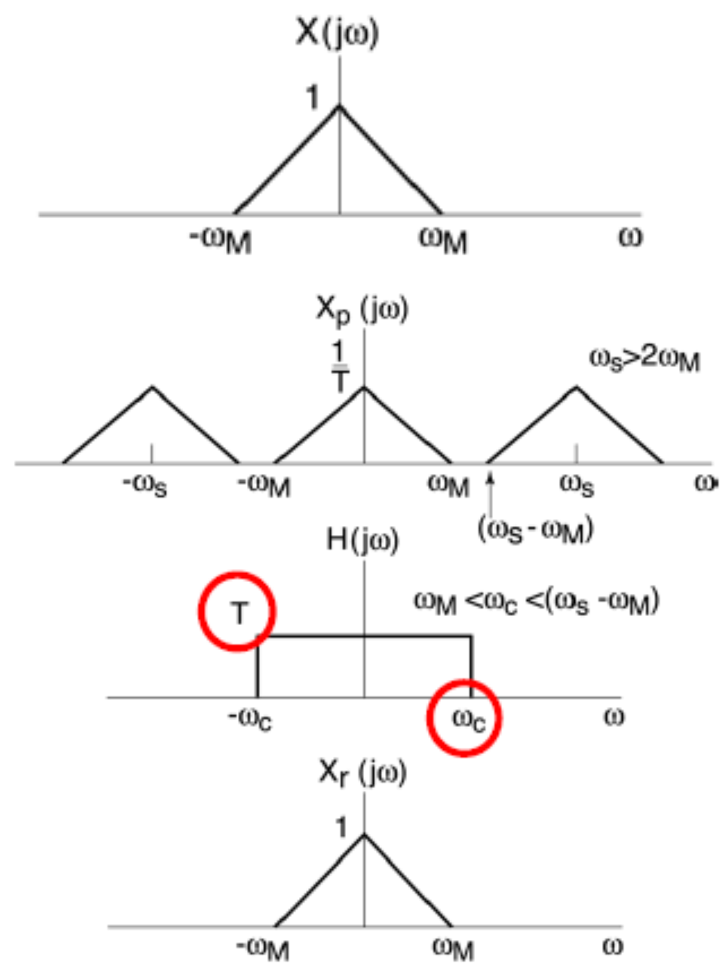
- Then x(t) is completely determined by its samples {x(nT)} if

$$\omega_s > 2\omega_M = \text{The Nyquist rate}$$

- where $\omega_s = 2\pi/T$

Reconstruction of x(t) from sampled signals





If there is no overlap between shifted spectra, a low-pass filter can extract $X(j\omega)$ from $X_p(j\omega)$

Reconstruction of the original signal from its sampled version

- This reconstruction process is called interpolation
- Interpolation=fitting a continuous signal to a set of sample values
- Examples
  - Band-limited (ideal) interpolation: uses the impulse response of an ideal low-pass filter: formula 4.25

  Practical interpolation schemes:
  - Zero-hold: obtains a piecewise constant signal
  - Linear interpolation: adjacent sample points are connected by a straight line

Figure 4.7 (a) Block diagram of an ideal bandlimited signal reconstruction system. (b) Frequency response of an ideal reconstruction filter. (c) Impulse response of an ideal reconstruction filter.

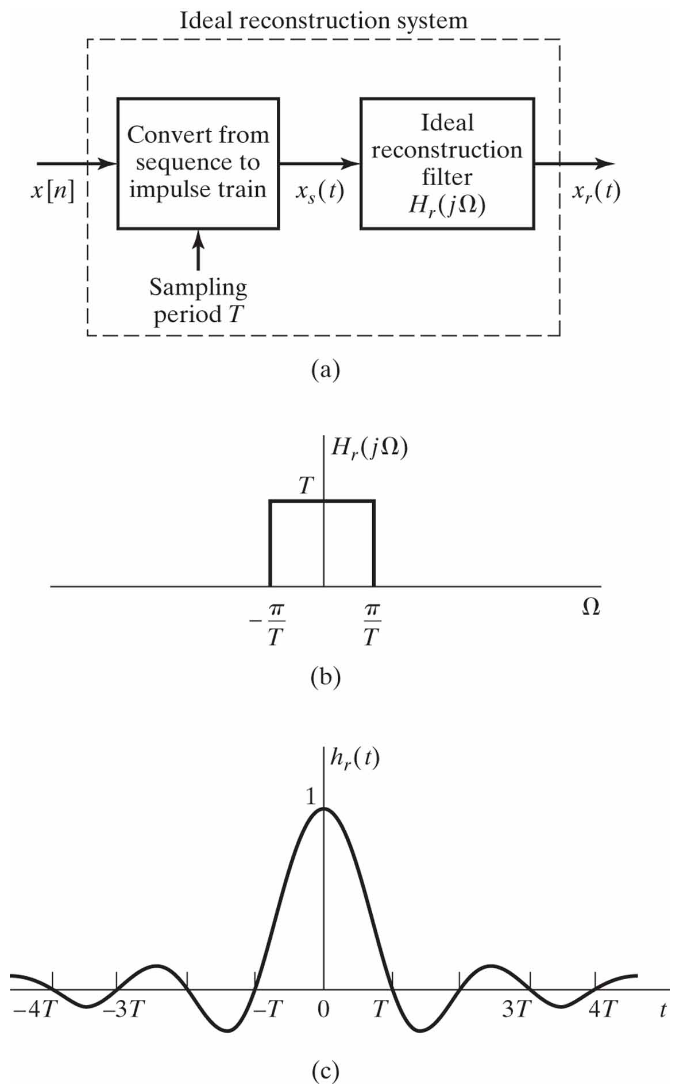
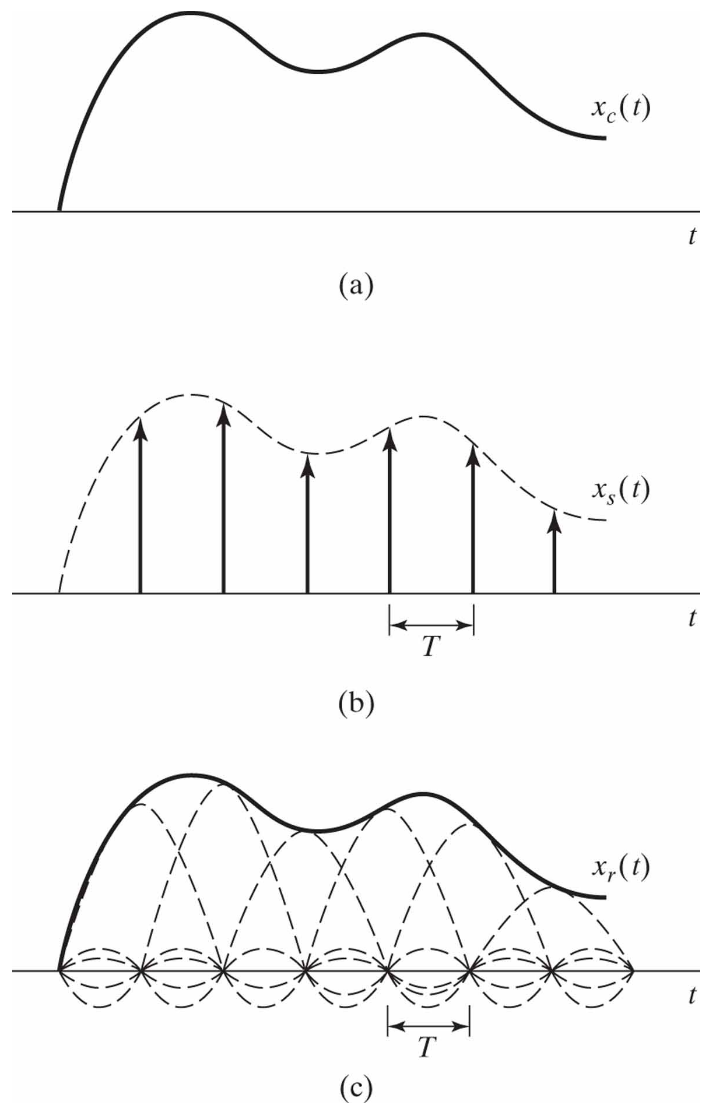
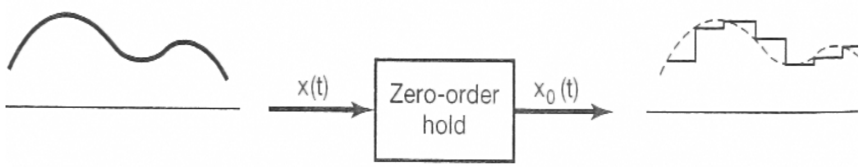


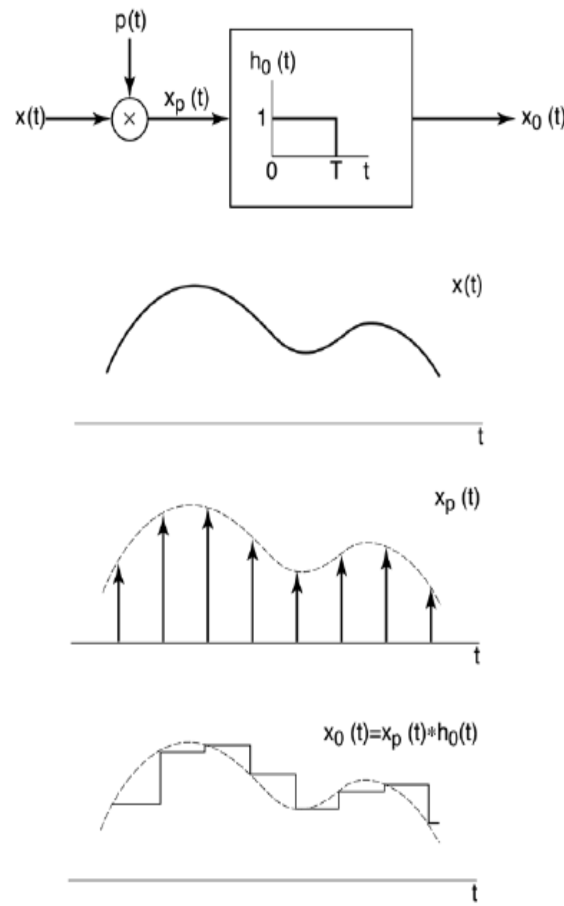
Figure 4.8 Ideal bandlimited interpolation.



Reconstruction by zero-order hold

- In practice sampling is not done using trains of impulses
- A practical example: the zero-order hold

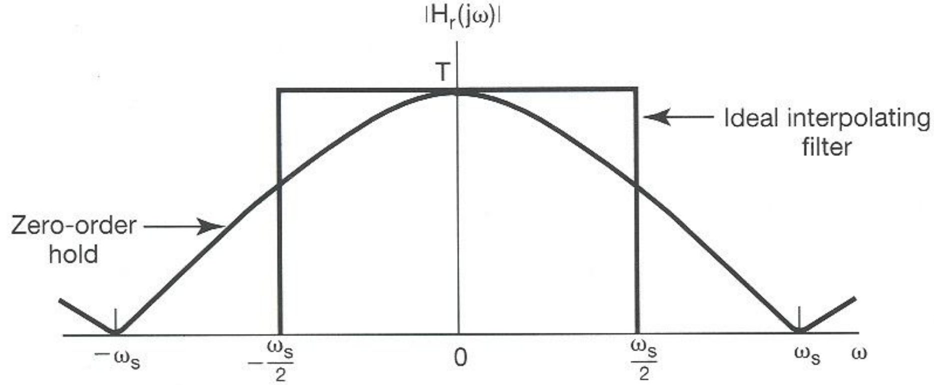


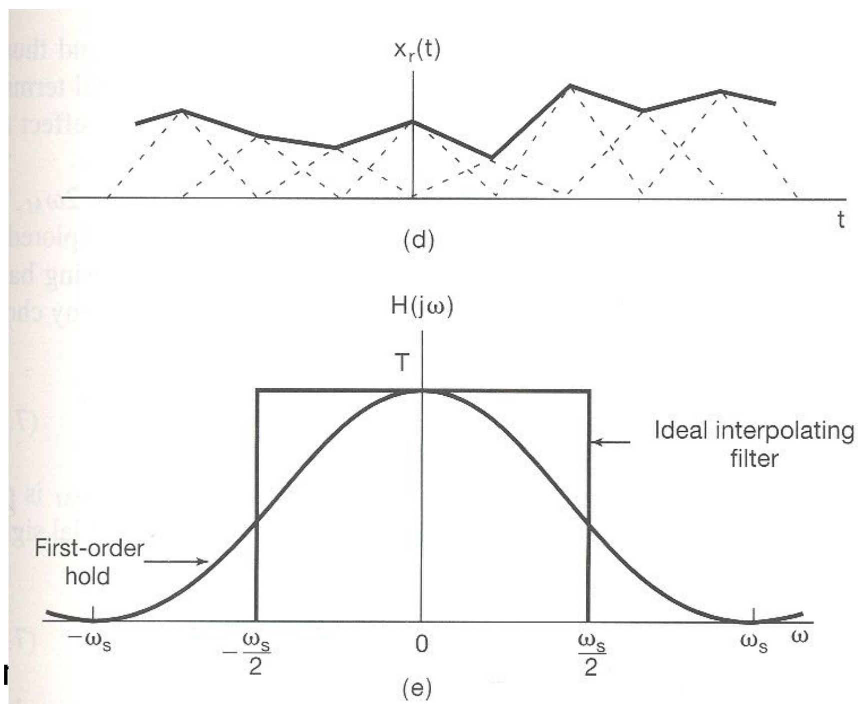


The zero-order hold system samples x (t) at a given instant and holds that value until the next sampling instant

In theory, the zero-order hold can be generated by impulse-train sampling followed by an LTI with rectangular impulse response

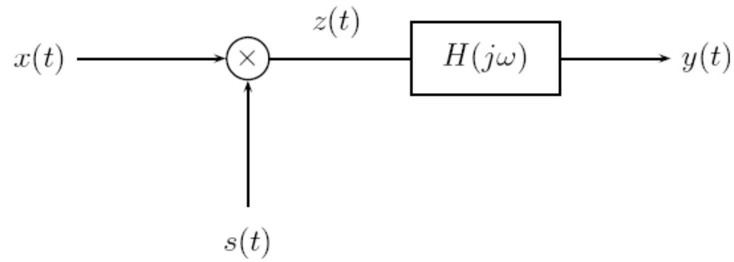
A comparison between the frequency responses of the most common interpolating filters





EXAMPLE

- A sinusoidal input signal, x(t) = cos(10t) is sampled and filtered as shown below.



- The frequency response of the filter is

$$|H(j\omega)| = \begin{cases} 1, & 90 < |\omega| < 180 \\ 0, & \text{otherwise} \end{cases}$$

$$\angle H(j\omega) = -\frac{\pi\omega}{200},$$

- Suppose the sampling function is $s(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT) \text{where } T = \frac{2\pi}{90}$

1. Provide a labeled sketch of Z(jω) the Fourier transform of z(t)
2. Find the output y(t) that corresponds to the input x(t)