Sampling of Discrete-Time signals

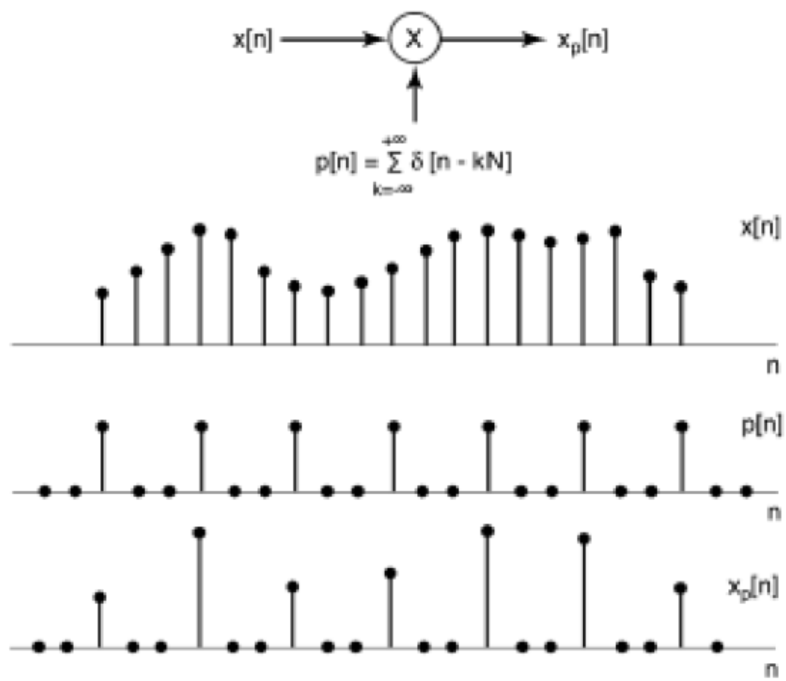
Outline

Impulse-train sampling of DT signals
DT Decimation (Downsampling)
DT Interpolation (Upsampling)

- Reading: section 7.5 from Oppenheim Signals and Systems (scan posted on the course site)

DT Sampling

- Motivation: Reducing the number of data points to be stored or transmitted, e.g. in CD music recording.

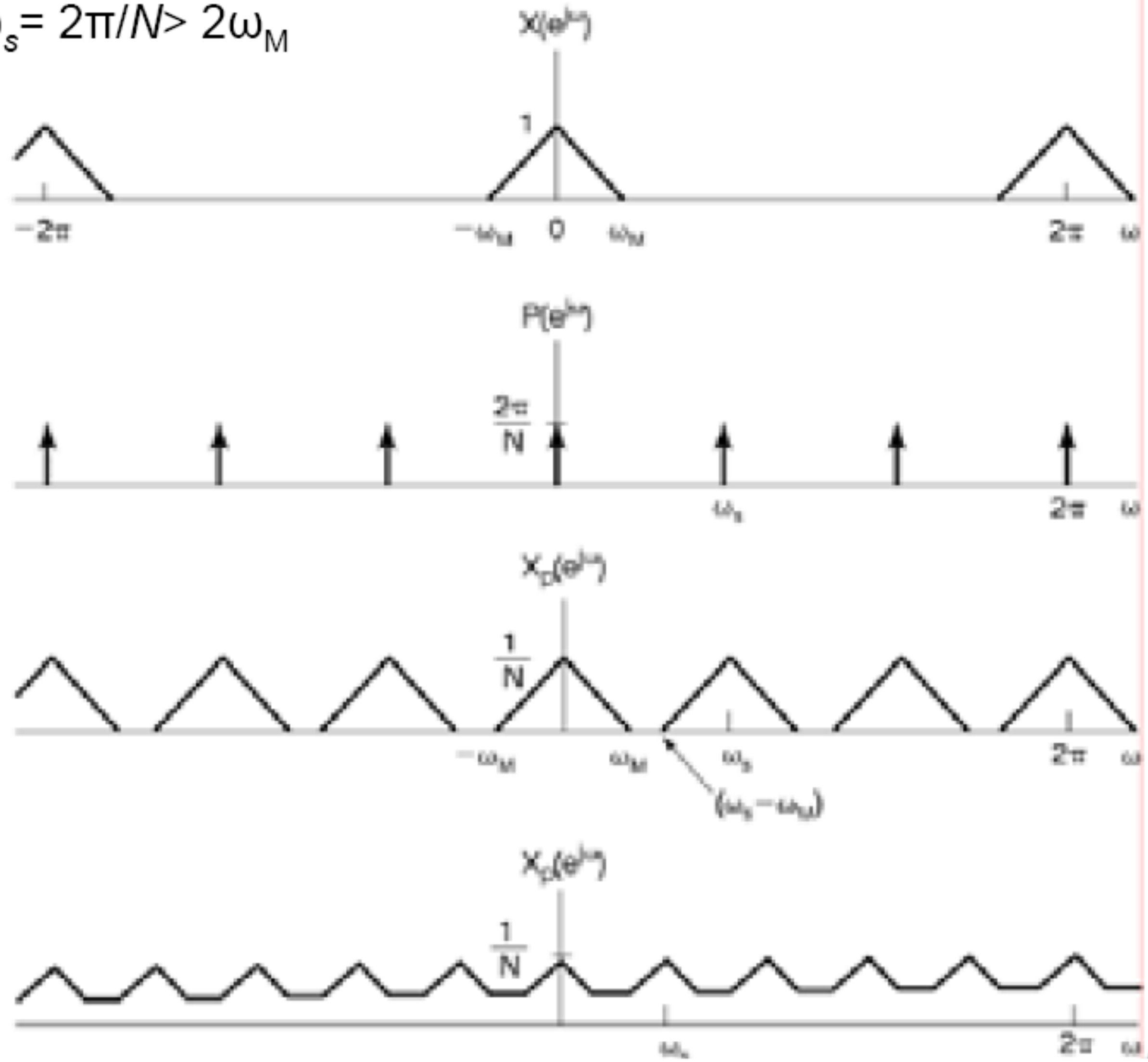


Analysis of DT sampling in the frequency domain

- We aim to find the relationship between

  $X_p(e^{j\omega})$ and $X(e^{j\omega})$

We can reconstruct $x[n]$ if $\omega_s = 2\pi/N > 2\omega_M$



Nyquist rate is met

$\omega_M < \omega_s/2 = \pi/N$

Aliasing!

Decimation-downsampling

- Downsampling is the process of extracting every Nth value from the original signal x[n]
- Downsampling is also called decimation for historical reasons (N=10)
- Useful to think of decimation by N as DT sampling with N followed by discarding the zero values
- $x_p[n]$ has (n-1) zero values between nonzero values. Why keep them around?

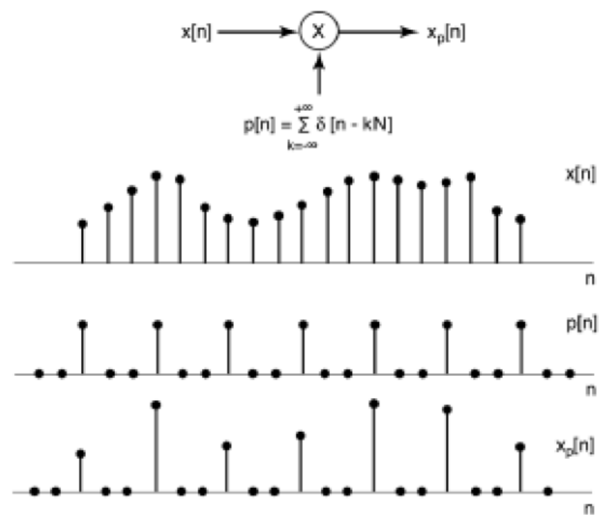
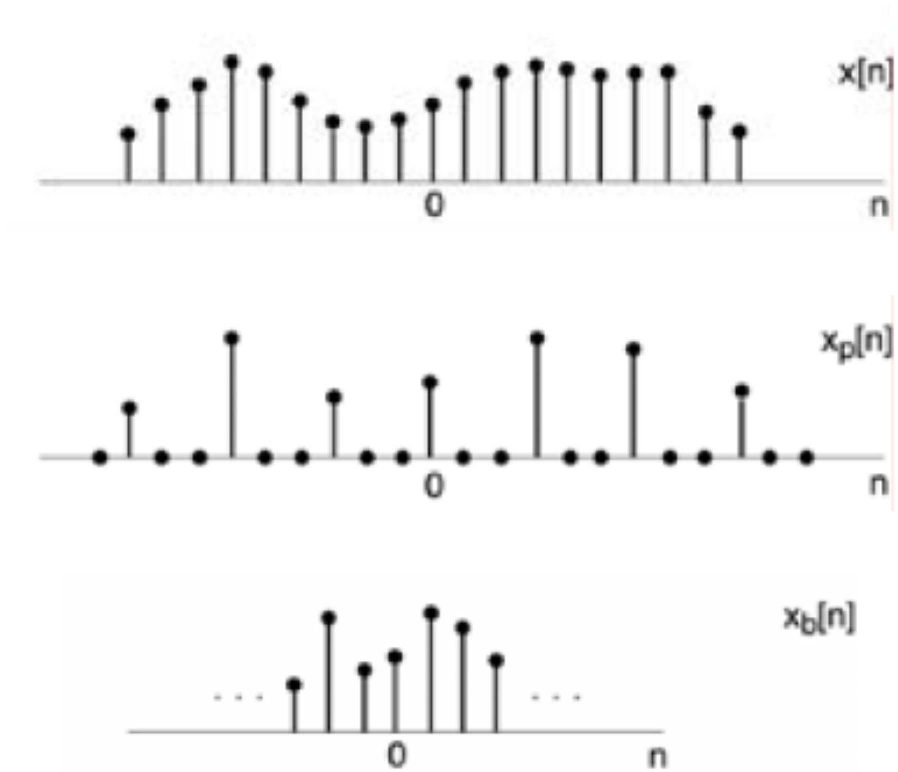


Illustration of decimation in the time-domain



Decimation in the frequency domain

- We compute the relationship between

$X_b(e^{j\omega})$ and $X_p(e^{j\omega})$

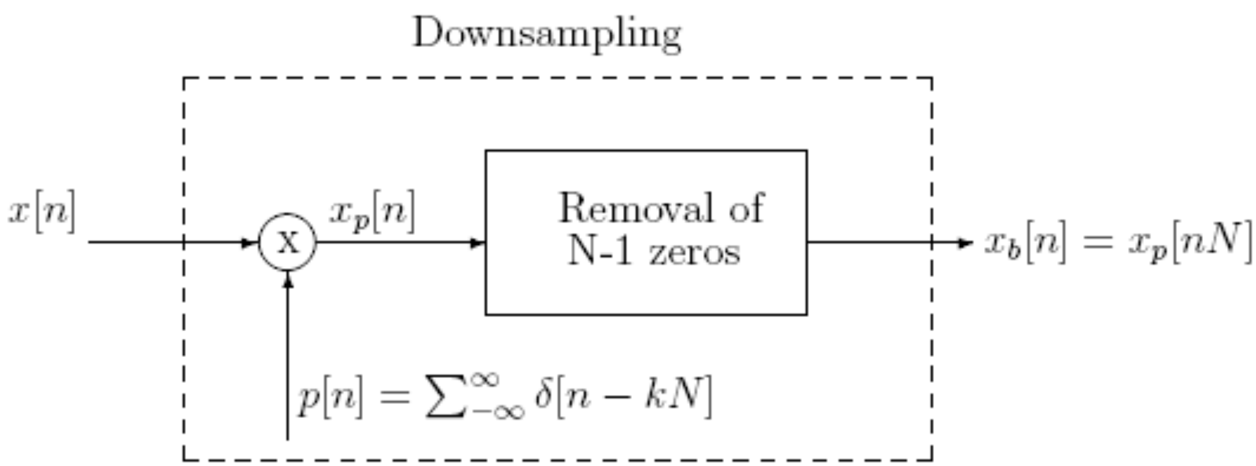
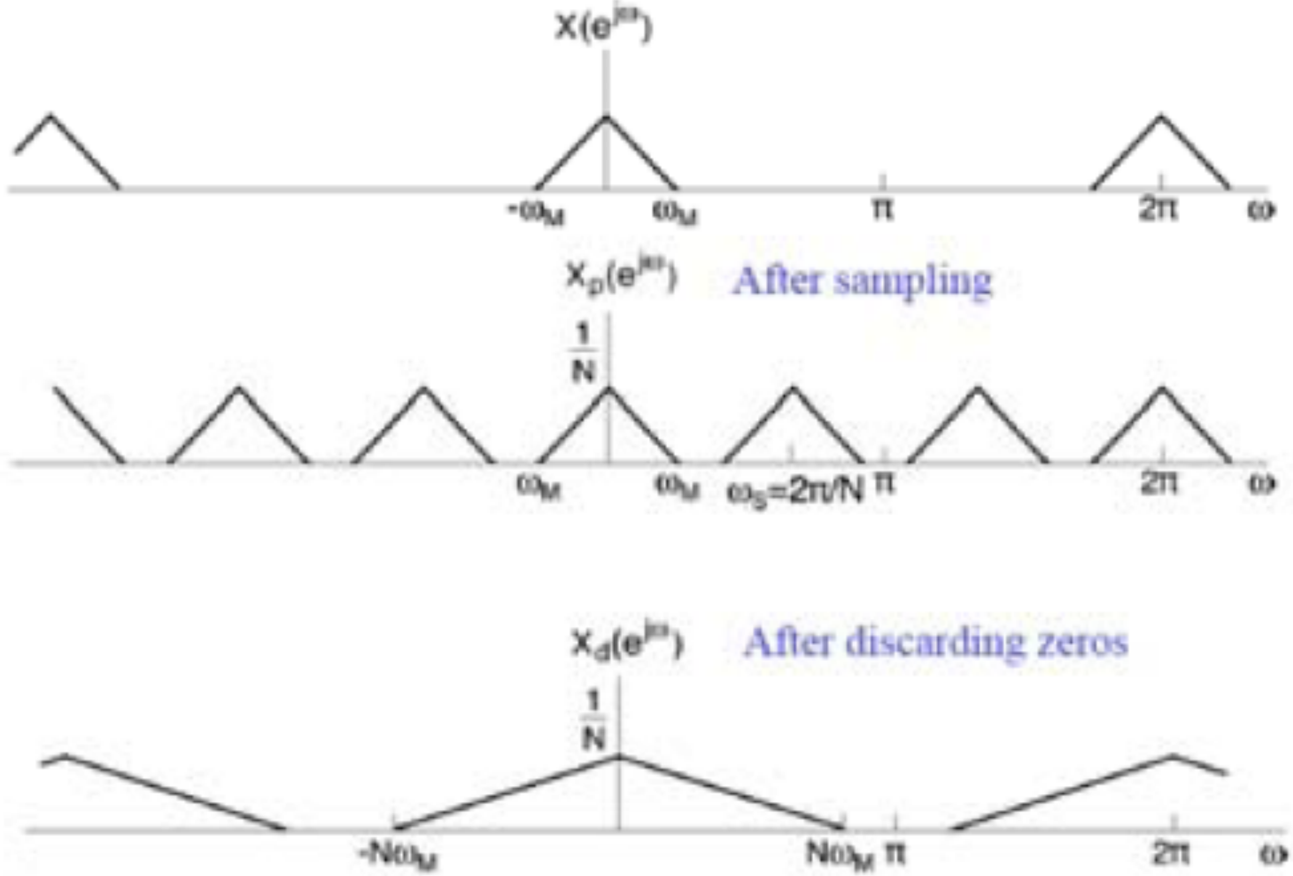
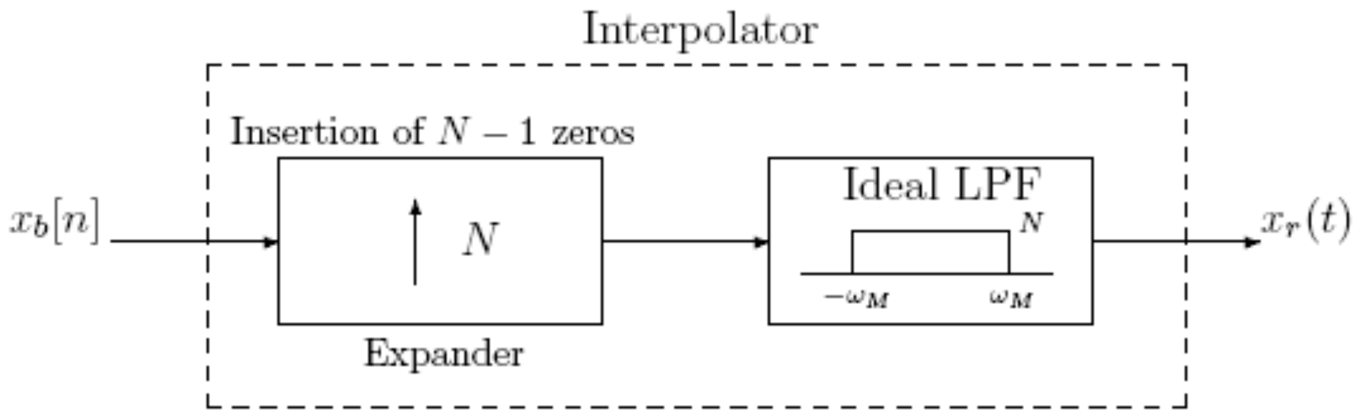
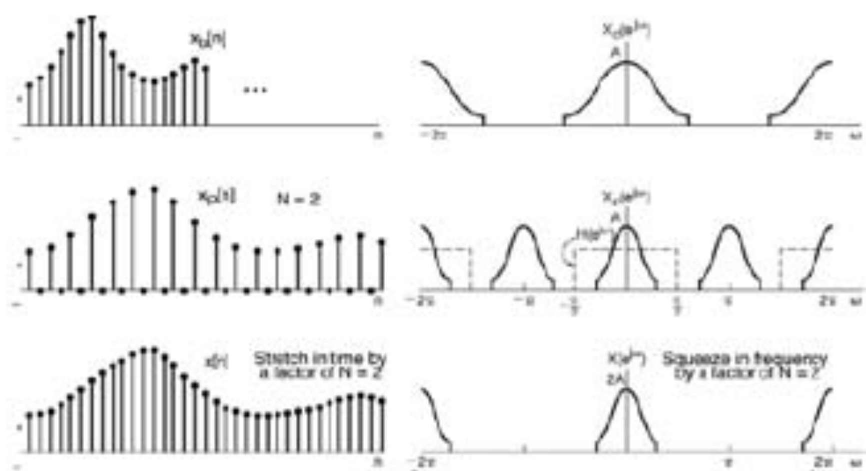


Illustration of decimation in the frequency domain



The inverse operation: upsampling





Combining upsampling and downsampling

- see example 7.5 from scanned chapter

Example

- A signal x[n] has a Fourier Transform that is zero for $(\pi/4) \le |\omega| \le \pi$.
- Another signal

$$g[n] = x[n] \cdot \sum_{k=-\infty}^{\infty} \delta[n-1-4k]$$

is generated by sampling x[n]. Specify the frequency response of a low-pass filter that produces x[n] as output when g[n] is its input.