ELEC 310. Lecture 3
Basic concepts in DT signals

Textbook: section 2.1
! Additional readings posted on the course site

Today

- DT Signals: mathematical description, representation and manipulation
  - Periodicity (continued from last lecture)
  - Energy and power
  - Transformations of the independent variable

Example

- Determine the fundamental period of

$$x[n] = 2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{8}n\right) - 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right)$$

Signal energy and power

- Notions that were initially applicable to signals produced by physical systems only (i.e. v(t), i(t) across a resistor of resistance R)

- **Generalization: The energy and power characterize any type of signals (not only electrical)**

- The energy of a DT signal x[n] over [n_1, n_2] is computed as

$$E_x = \sum_{-\infty}^{\infty} |x[n]|^2$$

- Example: find the energy contained in $x[n] = (\frac{1}{2})^n u[n]$

Signal energy and power (cont'd)

- A signal which has a finite energy is called "an energy signal"
- Many DT signals don't have a finite energy. In this case we consider their average power (we average over time).
- Formulas for CT and DT signals:

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt \qquad P_x = \lim_{N\to\infty} \frac{1}{2N} \sum_{-N}^{N-1} |x[n]|^2$$

Signals having infinite energy and finite power are called power signals.

For periodic signals, their average power is:

$$P_x = \frac{1}{N} \sum_{n=k}^{k+N-1} |x[n]|^2 = \frac{1}{N} \sum_{n=<N>} |x[n]|^2$$

Classes of signals (energy viewpoint)

- In terms of total energy and average power, we can identify three important classes of signals:
- **1) Finite total energy**

  $E_\infty < \infty \rightarrow P_\infty = \lim_{T\to\infty} \frac{E_\infty}{2T} = 0$

- **2) Finite average power**

  $0 < P_\infty < \infty \rightarrow E_\infty = \infty$

- **3) Infinite P_∞, E_∞**

Transformations of a signal

- Operations on the independent variable:
  - time shift,
  - time scale,
  - time reversal

Importance of transformations of independent variable

- Naturally occur in physical systems
- Are useful for identifying specific categories of signals (and systems)
- Will be further used in specifying important properties of the Fourier transform and Z transform

- Example on combining transformations posted on the course site.