Structures for LTI systems

Outline

Midterm discussion
DT sampling example (upsampling and downsampling)
Block diagrams (textbook section 6.1)

Midterm A

QUESTION 1 (8 points)

Consider an LTI system with unit impulse response

$$h[n] = \frac{\sin(\pi n / 3)}{\pi n}$$

- (1p) Provide a graphical representation for the frequency response of the system .
- (4p) Determine the output of the system y[n] if the input is .
- (3p) Give an example of a non-zero input signal for which the output of the system will be 0 for all n.

Midterm A

QUESTION 2. (7 points)

Consider the following Fourier Transform:

$$X(e^{j\omega}) = \frac{1 - \frac{1}{3}e^{-j\omega}}{1 - \frac{1}{4}e^{-j\omega} - \frac{1}{8}e^{-2j\omega}}$$

Determine the signal x[n] that corresponds to this Fourier Transform.

Midterm B

QUESTION 1 (8 points)

Consider a stable LTI system whose system function H(z) is given by

$$H(z) = \frac{3}{1 + \frac{1}{3} z^{-1}}$$

a) (2 p) Find the region of convergence of the system function
b) (6p) Suppose the input to the system is the unit step sequence: x[n]=u[n]. Find the output y[n] that corresponds to this input by computing the inverse Z transform of Y(z).

Midterm B

QUESTION 2. (7 points)

A causal LTI system is described by the following difference equation:

$$y[n]-0.25y[n-2]=x[n]-x[n-1]$$

a) (3p) Find the system function H(z) for this system and its region of convergence.

b) (1p) Determine whether this system is stable or not.

c) (3p) Determine the Z transform X(z) of the input signal x[n] so that the corresponding output of the above system is :

$$y[n]=\delta[n]-\delta[n-1].$$

d) (1 point bonus) Determine the signal x[n] that corresponds to X(z) found in c) by computing the inverse Z Transform.

Example

- A real valued discrete time signal x[n] has a band-limited Fourier Transform

$$X(e^{j\omega}) = 0 \text{ for } \frac{3}{14} \le |\omega| \le \pi$$

- The non-zero portion of the Fourier Transform can be made to occupy the whole available spectrum $|\omega| < \pi$ by first performing upsampling by a factor of L and then downsampling by a factor of M. Find the values of L and M.

Rationale for the study of structures

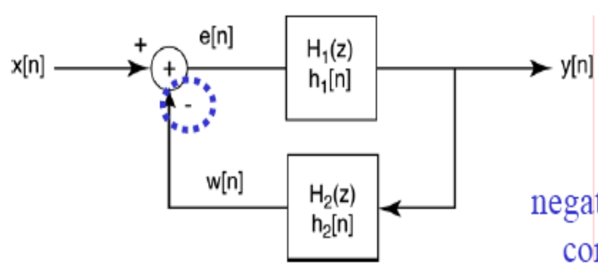
- We know that an LTI system with a rational system function has the property that its input and output satisfy a linear constant -coefficient difference equation (LCCDE)
- Structures are about the implementation of such LTI systems
- Causal systems described by LCCDE can be represented by structures consisting of an interconnection of basic operations (addition, multiplication by a constant, delay)
- The physical implementation of these basic elements depends on the technology to be used

System functions and block diagrams

- We have seen that the ZT is useful for replacing time-domain operations such as convolution and time-shifting with algebraic operations
- The ZT is also helpful for:
  - analyzing interconnections of LTI systems
  - representing systems as interconnections of basic building blocks

The Z transform and system interconnections

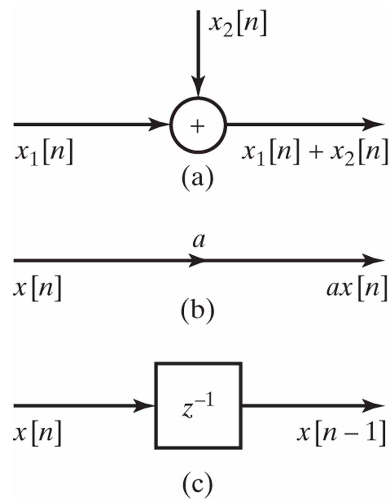
- Consider two systems given by their system functions $H_1(z)$ and $H_2(z)$
- We want to characterize all the possible system interconnections via system functions:
  - Series : $H(z)=H_1(z)\cdot H_2(z)$
  - Parallel: $H(z)= H_1(z)+H_2(z)$
  - Feedback



negative feedback configuration

Block diagram representations

- For causal LTI systems only!
- Block diagrams involve three basic operations:
  - Addition
  - Multiplication by a coefficient
  - Unit delay



- Derivation of block diagrams for systems described by first-order difference (on chalkboard)