Structures for LTI systems (2)

Outline

Block diagrams continued

- Example 1 – 1st order difference equations
- 2nd order difference equations
- Classification of block diagrams
- examples

Example 1

- A causal, digital filter structure is given by a block diagram (sketched on the board).
- a) Find H(z) for this causal filter. Plot the zero-pole pattern and indicate the region of convergence.

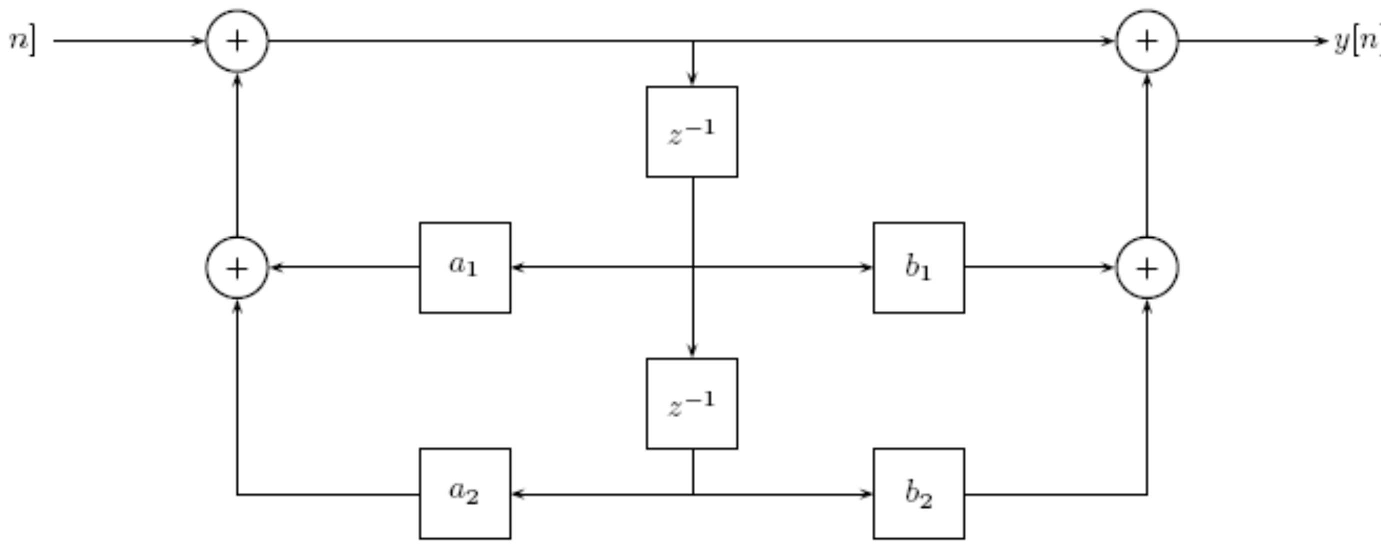
- b) For what values of K is the system stable?

- c) Determine y[n] if k=1 and $x[n] = \left(\frac{2}{3}\right)^n$ for all n

Block diagram representations for second order systems

$$y[n]-a_1y[n-1]-a_2y[n-2]=x[n]+b_1x[n-1]+b_2x[n-2]$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1+b_1z^{-1}+b_2z^{-2}}{1-a_1z^{-1}-a_2z^{-2}}$$



Classification of block diagram representations

- Direct form:
  - draw using H(z) as expressed directly for the difference equation

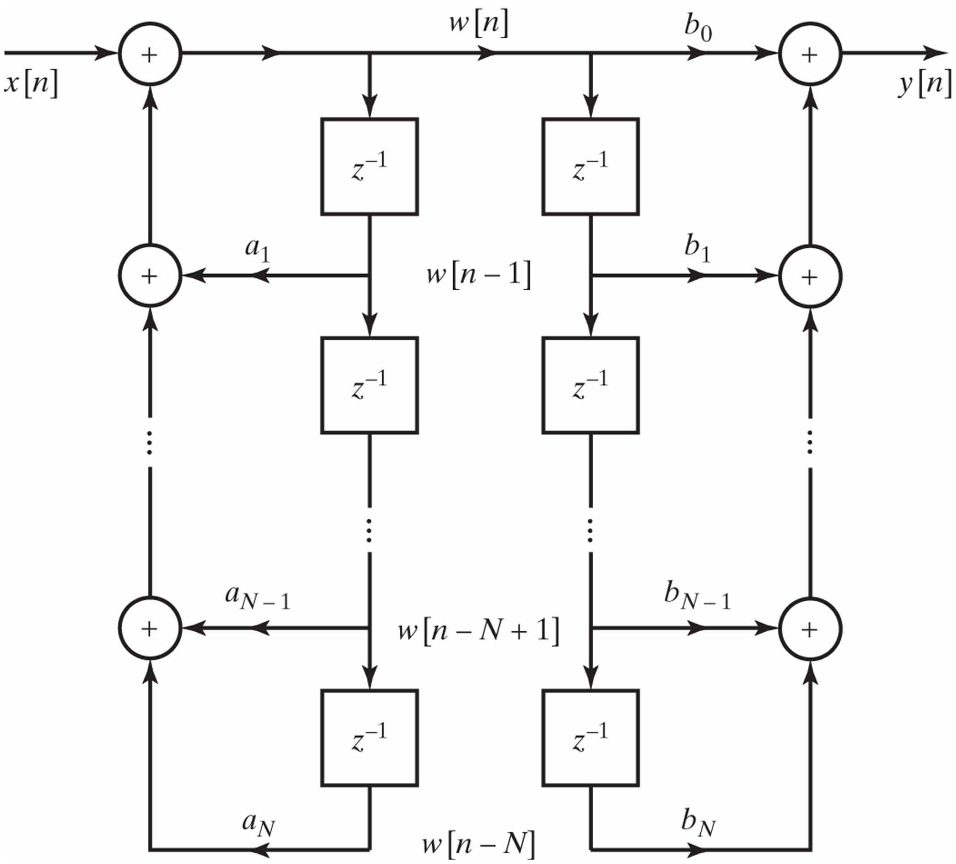
  Direct Form 1

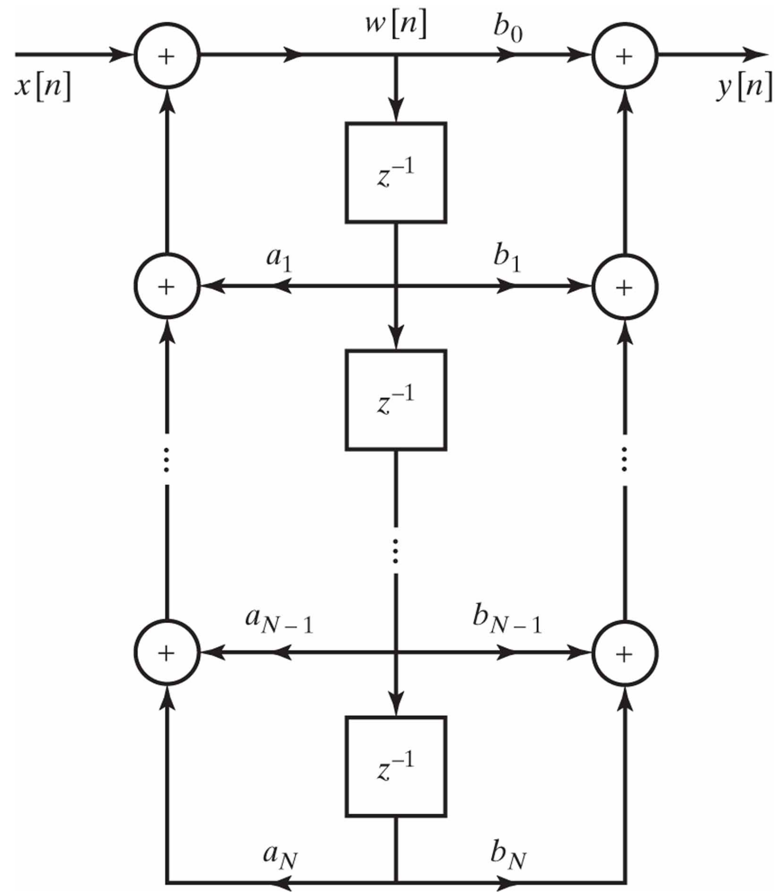
  Direct Form 2
- Cascade form
  - Whenever we express H(z) as a **product** of simpler blocs $H_1(z)$, $H_2(z)$, $H_3(z)$ etc.
- Parallel form
  - Whenever we express H(z) as a **sum** of simpler blocs $H_1(z)$, $H_2(z)$, $H_3(z)$ etc.
  - These blocs can come from partial fraction expansion

Observations

- all forms are equivalent
- Cascade and parallel forms are **not** unique

Direct Forms I and II for nth order equations





Example 2

- Consider the system function $H(z)=\dfrac{1-\frac{7}{4}z^{-1}-\frac{1}{2}z^{-2}}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}$

- a) Draw its direct form II representation
- b) Draw its cascade form representation based on the fact that

$$H(z)=\left(\frac{1+\frac{1}{4}z^{-1}}{1+\frac{1}{2}z^{-1}}\right)\left(\frac{1-2z^{-1}}{1-\frac{1}{4}z^{-1}}\right)$$

- c) Draw its parallel form representation based on the fact that

$$H(z)=4+\frac{5/3}{1+\frac{1}{2}z^{-1}}-\frac{14/3}{1-\frac{1}{4}z^{-1}}$$