ELEC 310. Lecture 5
LTI systems

Textbook: sections 2.3

Today

- Linear Time-Invariant Systems
- The impulse response of an LTI system
- The convolution sum (three methods)

LTI systems in a nutshell

- Linearity

$$ax_1[n] + bx_2[n] \rightarrow ay_1[n] + by_2[n]$$

0 input$\rightarrow$0 output

- Time invariance

$$\begin{array}{ll} \text{If} & x[n] \rightarrow y[n] \\ \text{then} & x[n - n_0] \rightarrow y[n - n_0] \end{array}$$

- If causal :

$$x[n] = 0 \; for \;\; t \leq 0 \rightarrow y[n] = 0 \; for \;\; t \leq 0$$

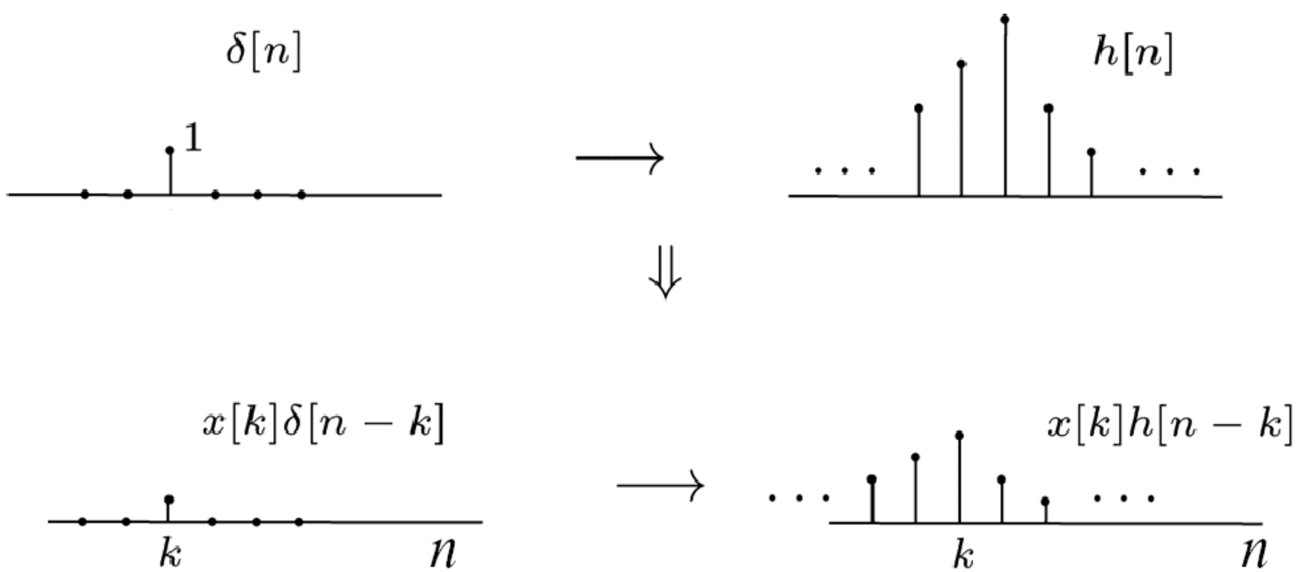
Two fundamental questions

1. What information do we need to have about an LTI system in order to be able to compute its response to any given input signal?
2. How will we actually use this information to compute the response of the LTI system to any given input signal?

Convolution sum representation of response of LTI systems

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

- Interpretation



We sum up responses over all k

Performing DT convolution- 1st method

- Scale, shift, stack, and add
  (also called shift, multiply, and sum)

Suppose we want to compute the convolution of two signals $x_1[n]$ and $x_2[n]$

The two signals will play different roles:

- Choose which signal to stack (for instance, $x_1[n]$)
- The scaling factors will consist in the other signal' (i.e. $\alpha_k = x_1[k]$)
- create scaled and shifted version $\alpha_k x_2[n-k]$)
- Stack them up, then add them up
- Numerical example…

Performing DT convolution-2nd method

- Flip, shift, multiply and add
- This is the discrete-time version of the CT method of convolution
- Computes the area of overlap between one signal and the flipped and shifted version of the other
- Step 1: Draw $x_1[k]$, $x_2[-k]$, and an axis for $y[n]$
- Step 2: Slide $x_2[n-k]$ across $x_1[k]$ and compute $y[n]$ as the 'area' of overlap.
- Numerical example…

Performing DT convolution - 3rd method

- Analytical solving of the convolution sum

$$x_1[n] * x_2[n] = \sum_{k=-\infty}^{+\infty} x_1[k] x_2[n-k]$$

- Numerical example…

Discussion

- Each computation method is suitable for specific classes of signals
  - The first two methods (graphical) are good for signals of finite length
  - The analytical method works best for causal signals of infinite length