

ELEC 310. Lecture 7

LTI systems. Properties

Textbook: section 2.4

Today

- Linear Time-Invariant Systems
 - Properties of the convolution
 - Properties of LTI systems

Reading: textbook 2.4

Assignment 1 posted on Jan 15; due by 4 pm on Jan 22.
QUIZ: Wednesday January 27.

Properties of convolution

- Convolution with $\delta[n]$
- The commutative property
- The associative property
- The distributive property
- These properties have important consequences for the computation of convolution and for interconnecting LTI systems

Convolution with $\delta[n]$

- $\delta[n]$ is the neutral element for the convolution operation
- We can express time-shifting as a convolution with a time-shifted δ .

$$x[n] * \delta[n] = x[n]$$

$$x[n] * \delta[n - n_0] = x[n - n_0]$$

Convolution is commutative

$$x[n] * h[n] = h[n] * x[n]$$

We can easily demonstrate this using variable substitution in the convolution sum.

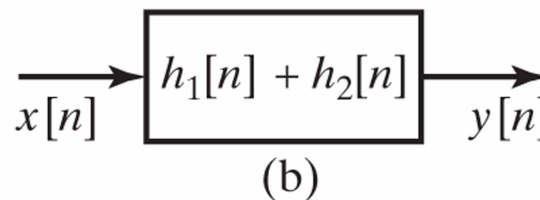
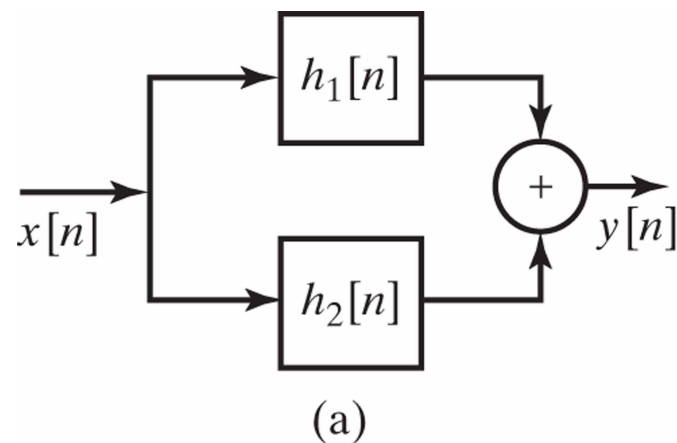
How can we interpret and use this property?

- Interchange roles of $x[n]$, $h[n]$
- Compute some convolutions easier

Convolution is distributive

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$$

- What property of LTI systems is accountable for the distributivity of the convolution?
- We interpret this property in terms of system interconnections
- Parallel connection: summation of the individual impulse responses of the systems connected in parallel

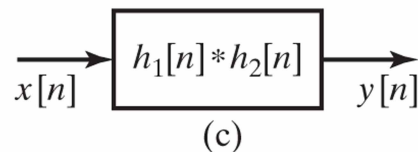
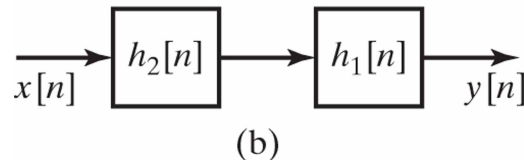
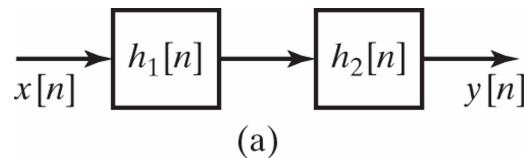


Convolution is associative

$$x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$$

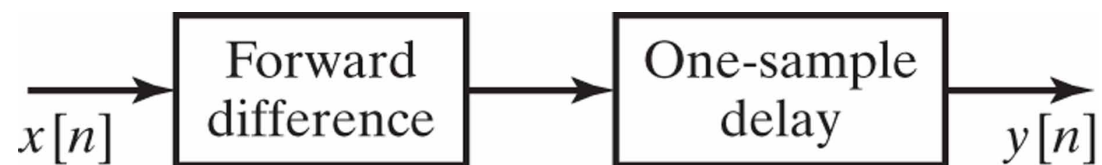
$$x[n] * (h_1[n] * h_2[n]) = x[n] * (h_2[n] * h_1[n]) = (x[n] * h_2[n]) * h_1[n]$$

- Again, we interpret this property in terms of system interconnections
- Serial (cascade) connection

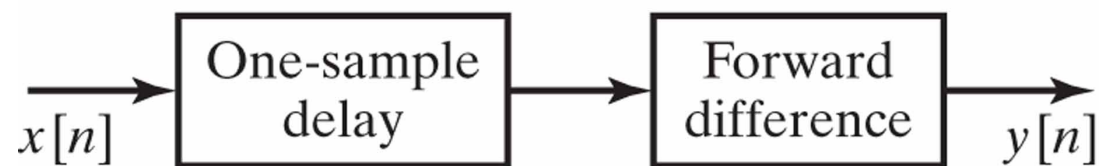


Example

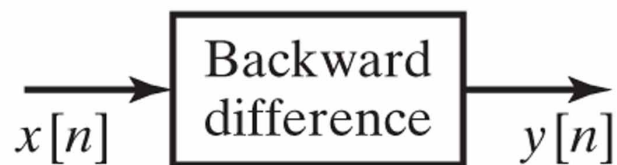
Figure 2.13 Equivalent systems found by using the commutative property of convolution.



(a)

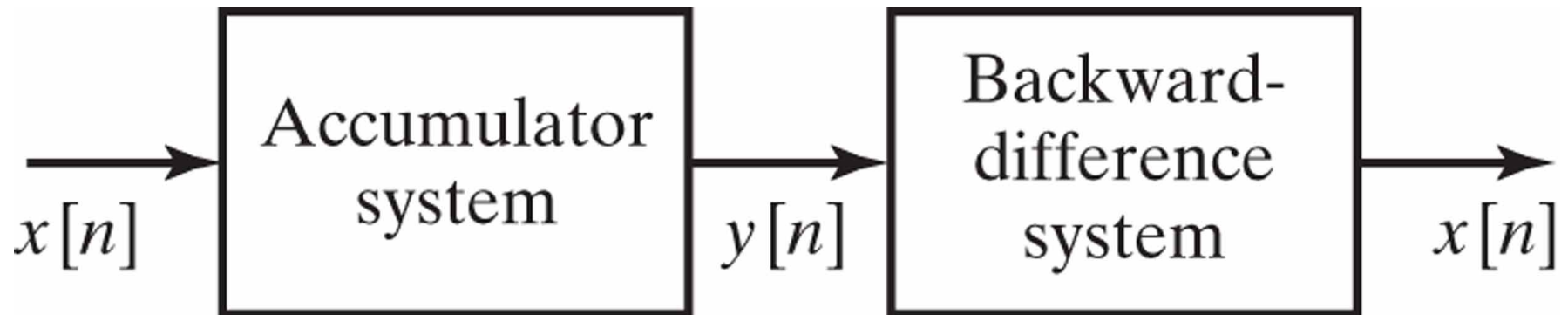


(b)



(c)

Figure 2.14 An accumulator in cascade with a backward difference. Since the backward difference is the inverse system for the accumulator, the cascade combination is equivalent to the identity system.



Properties of LTI systems based on their unit impulse response

Causality

Stability

Invertibility