ELEC 310. Lecture 8.
Linear constant-coefficient difference equations

Textbook: section 2.5

Ways to describe discrete LTI systems

- 1) via the impulse response
- **2) via difference equations**
  - $y[n]$=some formula in $x[n]$, and possibly $x[n-n_0]$, $y[n-n_0]$
- 3) via the frequency response
- 4) via the transfer function (Z transform)

Examples

1) Find the difference equation that characterizes the LTI system given by the following impulse response:

$$h[n]=\delta[n+1]-\delta[n]$$

2) Difference equation representation for the accumulator system

Description of LTi discrete systems via difference equations

- This is an implicit specification of the system
  - Difference equations describe a relationship between the input and the output

    rather than

    an explicit expression for the system output as a function of its input

  - For finding this explicit expression, we must solve the differential equation
  - To find a solution, we need more information than provided by the differential equation alone
  - This information is specified by **auxiliary conditions**.

General formula

- A linear constant-coefficient difference equation of order N looks like:

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

- All solutions y[n] can be expressed as a sum

$$y_h[n]+y_p[n]$$

Difference equation rewritten

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

can be rewritten as:

$$y[n] = \frac{1}{a_0}\left\{\sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k]\right\}$$

We need to know the input for all n as well as a set of N auxiliary conditions such as y[-N], y[-N+1], …, y[-1] in order to be able to solve the equation

Condition of initial rest

- We need auxiliary conditions to accompany the difference equation
- Initial rest is the simplest and the most widely used auxiliary condition in LTI systems:

An input $x[n]=0$ for $n<n_0$ leads to an output $y[n]=0$ for $n<n_o$

A causal input $x[n]=0$ for $n<0$ leads to a causal output $y[n]=0$ for $n<0$

Special case: non-recursive equation

$$y[n] = \frac{1}{a_0}\left\{\sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k]\right\}$$

If N=0, then

$$y[n] = \sum_{k=0}^{M}\left(\frac{b_k}{a_0}\right)x[n-k]$$

Finite impulse response (FIR)

To summarize

- A linear constant coefficient difference equation does not uniquely specify the system. The output for a given input is not uniquely specified. Auxiliary conditions are required

- If auxiliary information is given as N sequential values of the output, we rearrange the difference equation as a recurrence equation and solve it for future and past values of the output.

- If we know that the system is LTI and causal, then the system is fully specified by the LCCDE plus the condition of initial rest.

Example

- 2.20 textbook

Consider the difference equation representing a causal LTI system

$$y[n] - (1/a)y[n-1]=x[n-1]$$

a) Find the impulse response of the system
b) For what ranges of a will the system be stable?