ELEC 310. Lecture 9.
Eigenfunctions of LTI systems

Textbook: section 2.6

! QUIZ on Wednesday Jan 27
Duration: 40 minutes
Topics covered: everything except 2.6

Allowed:
-1 single-sided sheet of handwritten formulas
-Calculator

Response of LTI systems to complex exponentials

- Exponential and sinusoidal signals are basic building blocks in the representation of DT signals
- LTI systems do not modify the shape of such signals
- Mathematically: The response of an LTI system to a complex exponential input is a scaled exponential

$$z = e^{j\omega}$$

$$z^n = H(z)z^n$$

Eigenfunctions and eigenvalues of LTI systems

- We will prove that complex exponentials are eigenfunctions of LTI systems.
- The response of the system to a complex exponential is equal to the eigenvalue associated to that exponential.

- Example. Problem 2.13 from textbook.

The frequency response of LTI filters

$$H(e^{j\omega}) = \sum_{k=-\infty}^{+\infty} h[n]e^{-j\omega n}$$

Example: Compute the frequency response of the unit delay system

The frequency response is usually a complex number, characterized by its magnitude and phase.

Why are eigenfunctions important?

- If we can decompose a general signal into a sum of eigenfunctions, then we can use:
  - the eigenfunction property
  - superposition
- for computing the system's response to the general signal
- Example:
- Informally: if the input to an LTI system is represented as a linear combination of complex exponentials, then the output can also be represented as a linear combination of the same complex exponential signals.

Figure 2.17 Ideal lowpass filter showing (a) periodicity of the frequency response and (b) one period of the periodic frequency response.

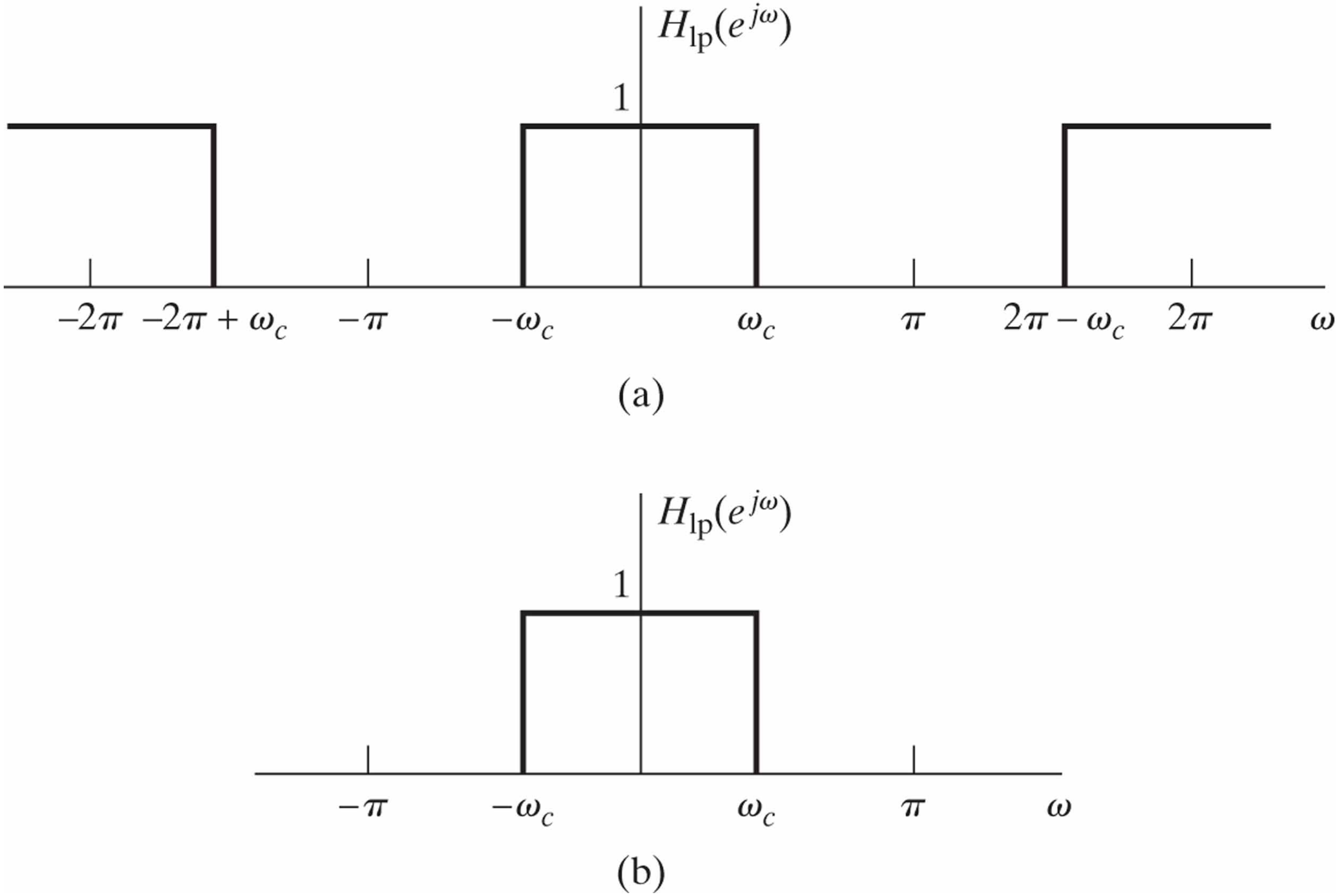
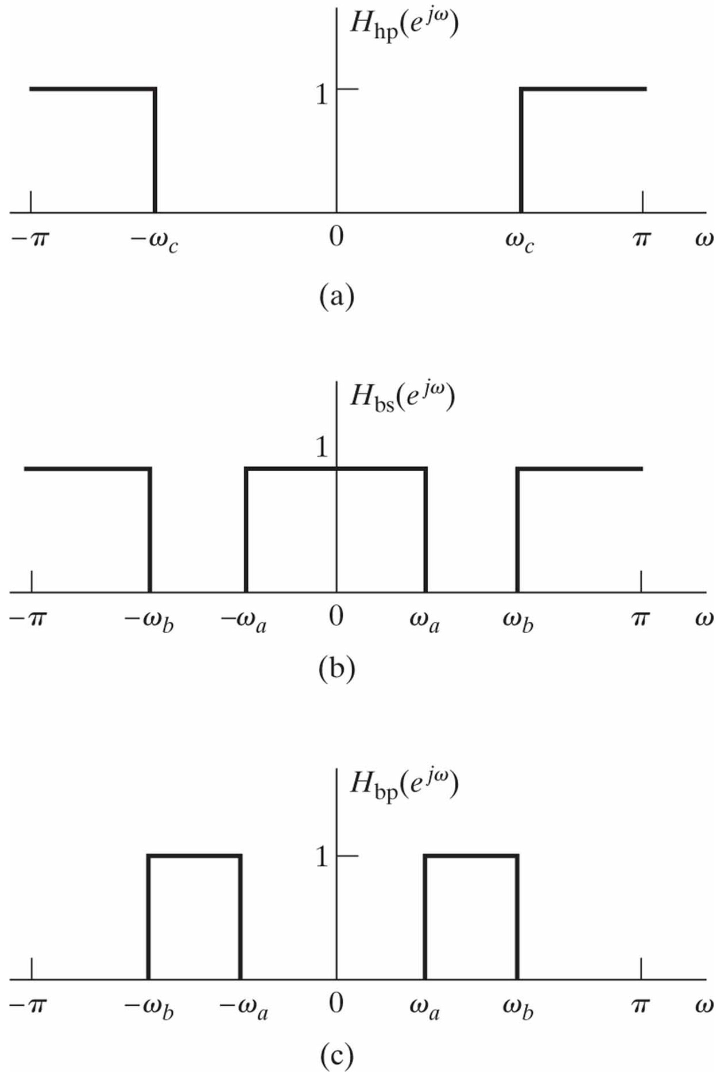


Figure 2.18 Ideal frequency-selective filters. (a) Highpass filter. (b) Bandstop filter. (c) Bandpass filter. In each case, the frequency response is periodic with period 2π. Only one period is shown.



Example

- Problem 2.11 from textbook