Moving average filter.

$$y[n] = \frac{1}{M_1 + M_2 + 1} \sum_{k=-M_1}^{M_2} x[n-k].$$

This system computes the n^{th} sample of the output sequence as the average of $(M_1 + M_2 + 1)$ samples of the input sequence.

parenthesis. - particular examples.

[for the example in figure 2.7. $M_1 = 0$ $M_2 = 5$.

$$y[n] = \frac{1}{6} \sum_{k=0}^{5} x[n-k]$$

$$= \frac{1}{6} \{ x[n-5] + x[n-4] + x[n-3] + x[n-2] + x[n-1] + x[n] \}$$

The most used moving average is symmetric $(M_1 = M_2 = 1)$.

$$y[n] = \frac{1}{3} \{ x[n-1] + x[n] + x[n+1] \}.$$

It is good for reducing signal fluctuations due to noise.]

Back to the general case, How do we compute the unit impulse response from the difference eq.?

$$x_1[n] = \delta[n] \rightarrow y_1[n] = h[n].$$

$$h[n] = \frac{1}{M_1 + M_2 + 1} \sum_{k=-M_1}^{M_2} \delta[n-k]$$

or equivalent

$$h[n] = \begin{cases} \dfrac{1}{M_1+M_2+1} & -M_1 \le n \le M_2. \\ 0 & \text{otherwise.} \end{cases}$$

for computing the frequency response, we use

$$H(e^{j\omega}) = \sum_{k=-\infty}^{+\infty} h[k]\, e^{-j\omega k}.$$

Substituting $h[k]$ we obtain.

$$H(e^{j\omega}) = \frac{1}{M_1+M_2+1} \sum_{n=-M_1}^{M_2} e^{-j\omega n}.$$

From now on we will concentrate only on the causal case $(M_1 = 0)$.

For the causal case.

$$H(e^{j\omega}) = \frac{1}{M_2+1} \sum_{n=0}^{M_2} e^{-j\omega n} \qquad \sum_{n=0}^{M_2} r^n = \frac{1-r^{M_2+1}}{1-r}.$$

$$= \frac{1}{M_2+1} \, \frac{1-e^{-j\omega(M_2+1)}}{1-e^{-j\omega}} =$$

we "build" sinuses.

$$= \frac{1}{M_2+1} \, \frac{e^{-j\omega\frac{M_2+1}{2}}\left(e^{+j\omega\frac{M_2+1}{2}} - e^{-j\omega\frac{M_2+1}{2}}\right)}{e^{-j\frac{\omega}{2}}\left(e^{+j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}\right)}$$

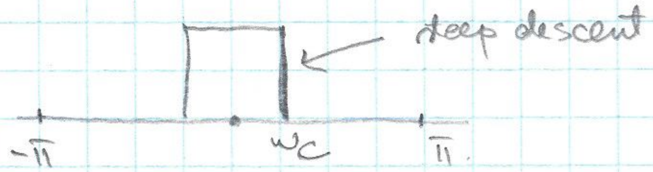
because we want to express $H(e^{j\omega})$ in polar form.

$$= \frac{1}{M_2+1}\, e^{-j\omega\frac{M_2}{2}} \, \frac{\sin\left[\omega \frac{M_2}{2}\right]}{\sin\frac{\omega}{2}}$$

The final result for the symmetric filter (2.124) is obtained through a similar mathematical development from (2.121).

Observations → $H(e^{j\omega})$ is periodic with 2π

→ falls off at high frequencies meaning that the system will smooth out rapid variations in the input sequence. (low-pass filter, but only a rough approximation, because in an ideal low-pass filter we will have a frequency response



with a steep descent at the cut-off frequency ω_c).