QUIZ 1 SOLUTION.

1. (a) Compute

$$\left\{\left(\frac{1}{2}\right)^{n-2} u[n-2]\right\} \otimes u[n+2]$$

Let $x_1[n] = \left(\frac{1}{2}\right)^{n-2} u[n-2]$

and

$x_2[n] = u[n+2]$.

$$x_1[n] * x_2[n] = \sum_{k=-\infty}^{+\infty} x_1[k]\, x_2[n-k] =$$

$$= \sum_{k=-\infty}^{+\infty} \left(\frac{1}{2}\right)^{k-2} u[k-2]\; u[n-k+2].$$

$$= \sum_{k=2}^{+\infty} \left(\frac{1}{2}\right)^{k-2} u[n-k+2].$$

$k' = k-2$
$$= \sum_{k'=0}^{\infty} \left(\frac{1}{2}\right)^{k'} u[n-k'].$$

$$u[n-k'] = \begin{cases} 1 & \text{for } n-k' > 0 \\ 0 & \text{otherwise.} \end{cases}$$

For $n < 0$ $\quad \sum_{k'=0}^{\infty} \left(\frac{1}{2}\right)^{k'} \underbrace{u[n-k']}_{<0} = 0.$

for $n > 0.$ $\quad \sum_{k'=0}^{\infty} \left(\frac{1}{2}\right)^{k'} u[n-k'] = \sum_{k'=0}^{n} \left(\frac{1}{2}\right)^{k'} = \dfrac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}.$

$$y[n] = \begin{cases} 2\left(1-\left(\frac{1}{2}\right)^{n+1}\right) & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

$$\boxed{y[n] = 2\left[1-\left(\frac{1}{2}\right)^{n+1}\right] u[n]}$$

QUIZ 1

1 (b)

$$x[n] = 2\delta[n-1] - \delta[n-3]$$

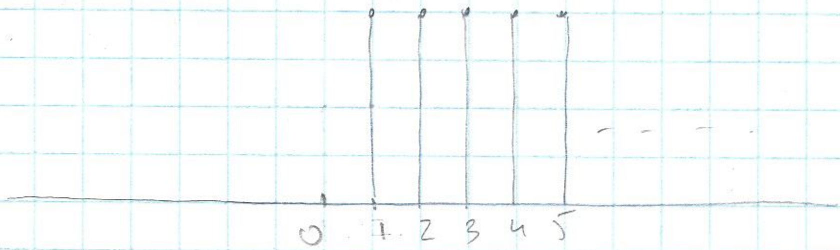
$$h[n] = 2u[n].$$

The easiest way to compute this convolution is to use the properties of δ.

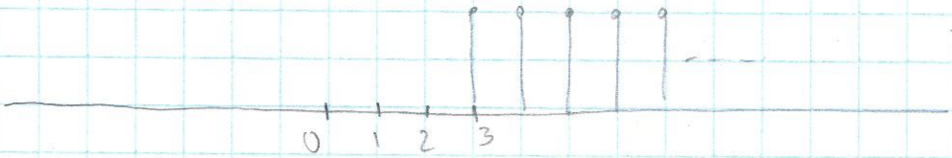
$$x[n] * h[n] = \{2\delta[n-1] - \delta[n-3]\} * 2u[n].$$

$$= 4\delta[n-1] * u[n] - 2\delta[n-3] * u[n]$$

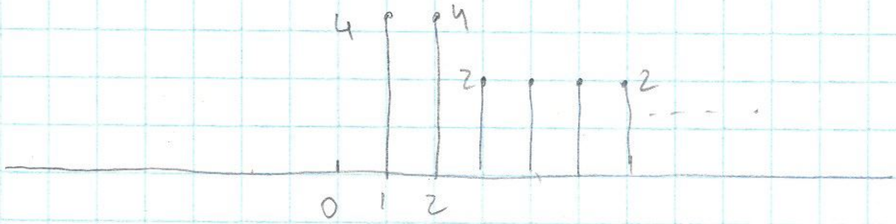
$$= 4u[n-1] - 2u[n-3].$$



$4u[n-1].$



$2u[n-3]$



$y[n].$

Quiz 1

Question 2

(a) $x_1[n] = x_1[n+N]$. for all N.

$$\cos\left(\frac{3\pi n}{4}\right) = \cos\left(\frac{3\pi}{4}(n+N)\right) = \cos\left(\frac{3\pi}{4}n + 2k\pi\right)$$

$$\frac{3\pi}{4}N = 2k\pi$$

$N = \frac{8k}{3}$ the smallest integer N satisfying this relation is $N = 8$ (for $k = 3$)

(b). Using the same reasoning as in a we reach

$N = \frac{8k}{5} \longrightarrow N = 8$ fundamental

(c) $x_1[n] = x_2[n]$. YES, TRUE.

$$\cos\left(\frac{3\pi}{4}n\right) = \cos\left(2\pi - \frac{3\pi}{4}n\right) = \cos\left(\frac{5\pi}{4}\right)n.$$