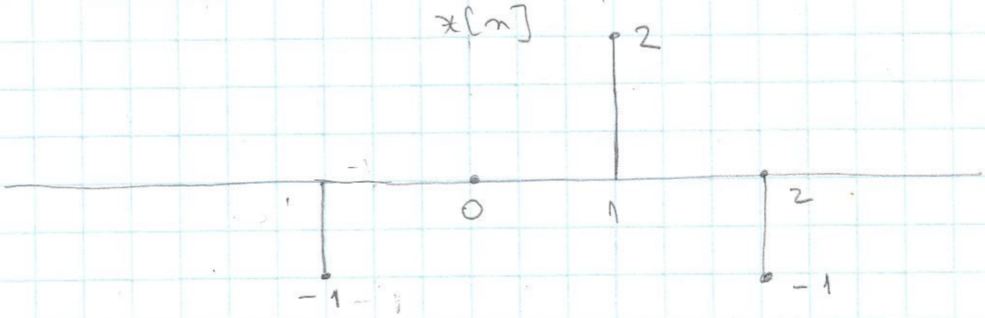
EXAMPLE 1 QUIZ REVIEW

Express the following signal $x[n]$ solely in terms of shifted and scaled superpositions of

(a) the unit impulse.

(b) the unit step.



$$x[n] = \sum_{n=-\infty}^{+\infty} x[k]\,\delta[n-k] = -\delta[n+1] + \delta[n-1] - \delta[n-2]$$

$$\boxed{x[n] = -\delta[n+1] + 2\delta[n-1] - \delta[n-2]}. \quad (a)$$

From expression (a) we can derive the representation in terms of $u[n-n_0]$, knowing that $\delta[n] = u[n] - u[n-1]$.

$$x[n] = -\{u[n+1] - u[n]\} + 2\{u[n-1] - u[n-2]\} -$$

$$- \{u[n-2] - u[n-3]\} =$$

$$= -u[n+1] + u[n] + 2u[n-1] - 3u[n-2] + u[n-3].$$

{Convolution example} EXAMPLE 3 QUIZ REVIEW.

$$x[n] = u[n-4] \qquad h[n] = 2^n u[-n-1].$$

$$x[n] * h[n] = \sum_{k=-\infty}^{+\infty} u[n-k-4] \cdot 2^k u[-k-1]$$

$$u[n-k-4] \cdot u[-k-1] \neq 0 \text{ if } \begin{cases} n-k-4 \geq 0, \\ -k-1 \geq 0 \end{cases}$$

$$\left.\begin{aligned} k &< n-4 \\ k &\leq -1. \end{aligned}\right\} \Rightarrow \boxed{k < \min(n-4, -1)}$$

We need to compare the numbers $n-4$ and -1

$n-4 \cancel{\geq} -1 \Rightarrow \boxed{n < 3}$ The sum limit is -1

$$\sum_{k=-\infty}^{-1} 2^k = \sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k = \frac{\left(\frac{1}{2}\right)^{+1}}{1 - \frac{1}{2}.} = 1$$

$n-4 \leq -1 \Rightarrow \boxed{n \leq 3}$ The sum limit is $n-4$

$$\sum_{k=-\infty}^{n-4} 2^k = \sum_{k=4-n}^{\infty} \left(\frac{1}{2}\right)^k = \frac{\left(\frac{1}{2}\right)^{4-n.}}{1 - \frac{1}{2}.} = \left(\frac{1}{2}\right)^{3-n} = 2^{n-3}$$

$$y_2[n] = \begin{cases} 2^{n-3} & \text{for } n \leq 3. \\ 1 & \text{for } n > 3. \end{cases}$$

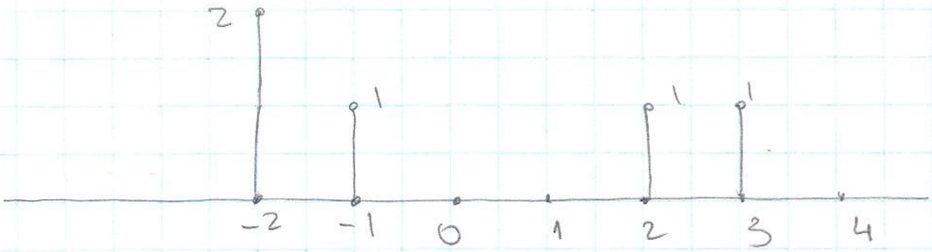
EXAMPLE 4 QUIZ REVIEW.

Consider the causal LTI system initially at rest

and described by

$$y[n] - \frac{1}{2}\, y[n-1] = 2\, x[n] - x[n-2].$$

Find the response of the system to the input



$$y[n] = \frac{1}{2}\, y[n-1] + 2x[n] - x[n-2].$$

$$y[-2] = 0 + 2\cdot 2 - 0 = 4$$

$$y[-1] = \left(\frac{1}{2}\right)\cdot 4 + 2\cdot 1 - 0 = 4$$

$$y[0] = \left(\frac{1}{2}\right)\cdot 4 + 0 - 2 = 0$$

$$y[1] = 0 + 0 - 1 = -1$$

$$y[2] = -\left(\frac{1}{2}\right)\cdot 1 + 2\cdot 1 - 0 = \frac{3}{2}$$

$$y[3] = \left(\frac{1}{2}\right)\cdot \frac{3}{2} + 2\cdot 1 - 0 = \frac{11}{4}$$

$$y[4] = \left(\frac{1}{2}\right)\cdot \frac{11}{4} + 0 - 1 = \frac{3}{8}$$

$$y[5] = \left(\frac{1}{2}\right)\cdot \frac{3}{8} - 1 = \frac{13}{16}$$

$$y[6] = \frac{1}{2}\, y[5] \quad \text{etc.}$$

$$\boxed{y[n] = -\left(\frac{1}{2}\right)^{n-5}\cdot \frac{13}{16}}$$

EXAMPLE 2 QUIZ REVIEW.

Check the Causality, Linearity, and Time-Invariance of the system given by:

$$y[n] = (x[n])^2.$$

(A) not linear. We prove this using a counterexample.

Let $x_1[n] = n$ and $x_2[n] = 2n$.

$$x_1[n] + x_2[n] \longrightarrow (3n)^2 = 9n^2$$

$$\neq x_1^2[n] + x_2^2[n] = 5n^2$$

(B) Time-Invariance: Yes

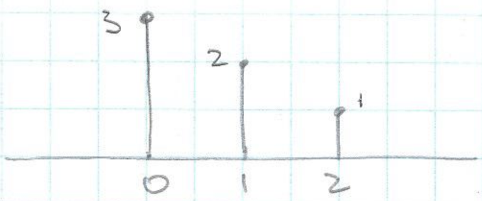
If $(x[n])^2 \longrightarrow y[n]$.

then. $(x[n-1])^2 \longrightarrow y[n-1]$

(C) Causality Yes, the output does not depend on future values of n.

Example 5 Quiz Review

Given $x[n]$ as sketched



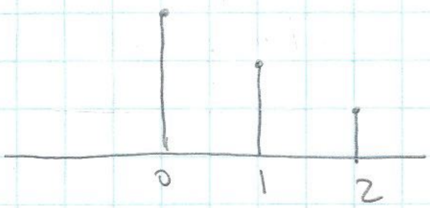
Compute $x[2-n]$ via the following sequence of transformations:

(a) time-shifting followed by time reversal

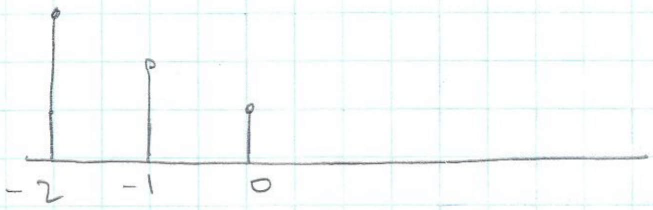
(b) time reversal followed by time shifting.

(a)

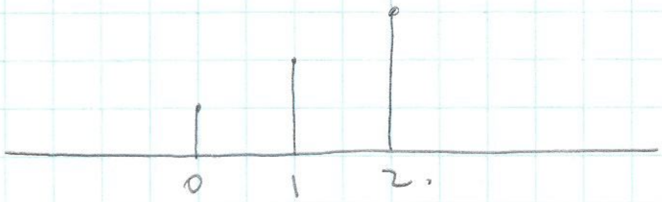
$x[n]$



$x[n+2]$
advence with 2 units

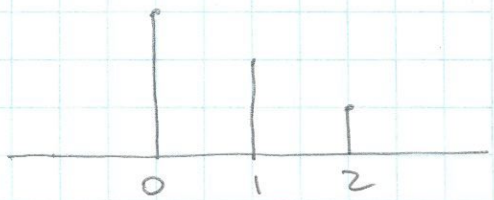


$x[-n+2] = x[2-n]$
time reverse

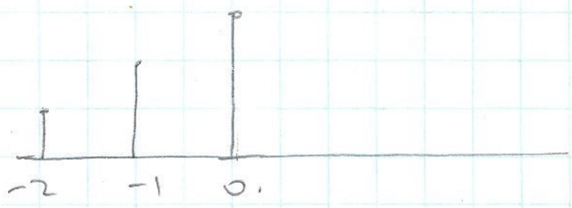


(b)

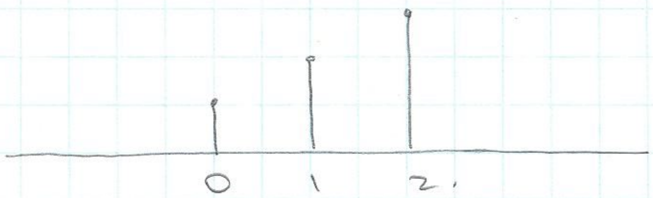
$x[n]$.



$x[-n]$
time reverse



$x[-(n-2)] = x[2-n]$
delay with 2



Two ways of computing $x[2-n]$.

Left, Time shifting followed by time reversal

Right, Time reversal followed by time shifting

Both approaches yield the same result.

let's verify ourselves. $x_1[n] = x[-n+2]$

$x_1[0] = x[2] = 1$

$x_1[1] = x[1] = 2$

$x_1[2] = x[0] = 3.$