

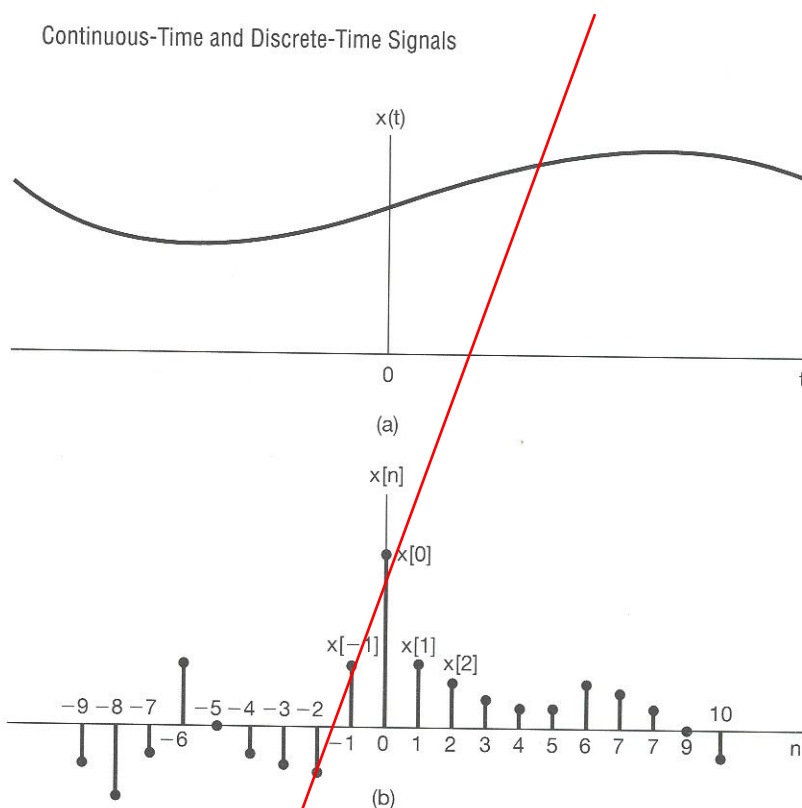


Figure 1.7 Graphical representations of (a) continuous-time and (b) discrete-time signals.

autopilot or speech and music for an audio system. Also, pictures in newspapers—or in this book, for that matter—actually consist of a very fine grid of points, and each of these points represents a sample of the brightness of the corresponding point in the original image. No matter what the source of the data, however, the signal $x[n]$ is defined only for integer values of n . It makes no more sense to refer to the $3\frac{1}{2}$ th sample of a digital speech signal than it does to refer to the average budget for a family with $2\frac{1}{2}$ family members.

Throughout most of this book we will treat discrete-time signals and continuous-time signals separately but in parallel, so that we can draw on insights developed in one setting to aid our understanding of another. In Chapter 7 we will return to the question of sampling, and in that context we will bring continuous-time and discrete-time concepts together in order to examine the relationship between a continuous-time signal and a discrete-time signal obtained from it by sampling.

1.1.2 Signal Energy and Power

From the range of examples provided so far, we see that signals may represent a broad variety of phenomena. In many, but not all, applications, the signals we consider are directly related to physical quantities capturing power and energy in a physical system. For example, if $v(t)$ and $i(t)$ are, respectively, the voltage and current across a resistor with resistance R , then the instantaneous power is

$$p(t) = v(t)i(t) = \frac{1}{R}v^2(t). \quad (1.1)$$

The total *energy* expended over the time interval $t_1 \leq t \leq t_2$ is

$$\int_{t_1}^{t_2} p(t) dt = \int_{t_1}^{t_2} \frac{1}{R} v^2(t) dt, \quad (1.2)$$

and the *average power* over this time interval is

$$\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} p(t) dt = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \frac{1}{R} v^2(t) dt. \quad (1.3)$$

Similarly, for the automobile depicted in Figure 1.2, the instantaneous power dissipated through friction is $p(t) = bv^2(t)$, and we can then define the total energy and average power over a time interval in the same way as in eqs. (1.2) and (1.3).

With simple physical examples such as these as motivation, it is a common and worthwhile convention to use similar terminology for power and energy for *any* continuous-time signal $x(t)$ or *any* discrete-time signal $x[n]$. Moreover, as we will see shortly, we will frequently find it convenient to consider signals that take on complex values. In this case, the total energy over the time interval $t_1 \leq t \leq t_2$ in a continuous-time signal $x(t)$ is defined as

$$\int_{t_1}^{t_2} |x(t)|^2 dt, \quad (1.4)$$

where $|x|$ denotes the magnitude of the (possibly complex) number x . The time-averaged power is obtained by dividing eq. (1.4) by the length, $t_2 - t_1$, of the time interval. Similarly, the total energy in a discrete-time signal $x[n]$ over the time interval $n_1 \leq n \leq n_2$ is defined as

$$\sum_{n=n_1}^{n_2} |x[n]|^2, \quad (1.5)$$

and dividing by the number of points in the interval, $n_2 - n_1 + 1$, yields the average power over the interval. It is important to remember that the terms “power” and “energy” are used here independently of whether the quantities in eqs. (1.4) and (1.5) actually are related to physical energy.¹ Nevertheless, we will find it convenient to use these terms in a general fashion.

Furthermore, in many systems we will be interested in examining power and energy in signals over an infinite time interval, i.e., for $-\infty < t < +\infty$ or for $-\infty < n < +\infty$. In these cases, we define the total energy as limits of eqs. (1.4) and (1.5) as the time interval increases without bound. That is, in continuous time,

$$E_\infty \triangleq \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt = \int_{-\infty}^{+\infty} |x(t)|^2 dt, \quad (1.6)$$

and in discrete time,

$$E_\infty \triangleq \lim_{N \rightarrow \infty} \sum_{n=-N}^{+N} |x[n]|^2 = \sum_{n=-\infty}^{+\infty} |x[n]|^2. \quad (1.7)$$

¹Even if such a relationship does exist, eqs. (1.4) and (1.5) may have the wrong dimensions and scalings. For example, comparing eqs. (1.2) and (1.4), we see that if $x(t)$ represents the voltage across a resistor, then eq. (1.4) must be divided by the resistance (measured, for example, in ohms) to obtain units of physical energy.

1.2 TRANSFORMS

Note that for some signals the integral in eq. (1.6) or sum in eq. (1.7) might not converge—e.g., if $x(t)$ or $x[n]$ equals a nonzero constant value for all time. Such signals have infinite energy, while signals with $E_\infty < \infty$ have finite energy.

In an analogous fashion, we can define the time-averaged power over an infinite interval as

$$P_\infty \triangleq \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt \quad (1.8)$$

and

$$P_\infty \triangleq \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2 \quad (1.9)$$

in continuous time and discrete time, respectively. With these definitions, we can identify three important classes of signals. The first of these is the class of signals with finite total energy, i.e., those signals for which $E_\infty < \infty$. Such a signal must have zero average power, since in the continuous time case, for example, we see from eq. (1.8) that

$$P_\infty = \lim_{T \rightarrow \infty} \frac{E_\infty}{2T} = 0. \quad (1.10)$$

An example of a finite-energy signal is a signal that takes on the value 1 for $0 \leq t \leq 1$ and 0 otherwise. In this case, $E_\infty = 1$ and $P_\infty = 0$.

A second class of signals are those with finite average power P_∞ . From what we have just seen, if $P_\infty > 0$, then, of necessity, $E_\infty = \infty$. This, of course, makes sense, since if there is a nonzero average energy per unit time (i.e., nonzero power), then integrating or summing this over an infinite time interval yields an infinite amount of energy. For example, the constant signal $x[n] = 4$ has infinite energy, but average power $P_\infty = 16$. There are also signals for which neither P_∞ nor E_∞ are finite. A simple example is the signal $x(t) = t$. We will encounter other examples of signals in each of these classes in the remainder of this and the following chapters.

1.2 TRANSFORMATIONS OF THE INDEPENDENT VARIABLE

A central concept in signal and system analysis is that of the transformation of a signal. For example, in an aircraft control system, signals corresponding to the actions of the pilot are transformed by electrical and mechanical systems into changes in aircraft thrust or the positions of aircraft control surfaces such as the rudder or ailerons, which in turn are transformed through the dynamics and kinematics of the vehicle into changes in aircraft velocity and heading. Also, in a high-fidelity audio system, an input signal representing music as recorded on a cassette or compact disc is modified in order to enhance desirable characteristics, to remove recording noise, or to balance the several components of the signal (e.g., treble and bass). In this section, we focus on a very limited but important class of elementary signal transformations that involve simple modification of the independent variable, i.e., the time axis. As we will see in this and subsequent sections of this chapter, these elementary transformations allow us to introduce several basic properties of signals and systems. In later chapters, we will find that they also play an important role in defining and characterizing far richer and important classes of systems.