

3.1. (a)

$$\mathcal{Z} \left[\left(\frac{1}{2} \right)^n u[n] \right] = \sum_{n=0}^{\infty} \left(\frac{1}{2} \right)^n z^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{2z} \right)^n = \frac{1}{1 - \frac{1}{2}z^{-1}} \quad |z| > \frac{1}{2}$$

(b)

$$\begin{aligned} \mathcal{Z} \left[- \left(\frac{1}{2} \right)^n u(-n-1) \right] &= - \sum_{n=-\infty}^{-1} \left(\frac{1}{2} \right)^n z^{-n} = - \sum_{n=1}^{\infty} (2z)^n \\ &= - \frac{2z}{1-2z} = \frac{1}{1 - \frac{1}{2}z^{-1}} \quad |z| < \frac{1}{2} \end{aligned}$$

(c)

$$\mathcal{Z} \left[\left(\frac{1}{2} \right)^n u[-n] \right] = \sum_{n=-\infty}^0 (2z)^n = \frac{1}{1-2z} \quad |z| < \frac{1}{2}$$

(d)

$$\mathcal{Z}[\delta[n]] = z^0 = 1 \quad \text{all } z$$

(e)

$$\mathcal{Z}[\delta[n-1]] = z^{-1} \quad |z| > 0$$

(f)

$$\mathcal{Z}[\delta[n+1]] = z^{+1} \quad 0 \leq |z| < \infty$$

(g)

$$\mathcal{Z} \left[\left(\frac{1}{2} \right)^n (u[n] - u[n-10]) \right] = \sum_{n=0}^9 \left(\frac{1}{2z} \right)^n = \frac{1 - (2z)^{-10}}{1 - (2z)^{-1}} \quad |z| > 0$$

3.2.

$$x[n] = \begin{cases} n, & 0 \leq n \leq N-1 \\ N, & N \leq n \end{cases} = n u[n] - (n-N)u[n-N]$$

$$n x[n] \Leftrightarrow -z \frac{d}{dz} X(z) \Rightarrow n u[n] \Leftrightarrow -z \frac{d}{dz} \frac{1}{1-z^{-1}} \quad |z| > 1$$

$$n u[n] \Leftrightarrow \frac{z^{-1}}{(1-z^{-1})^2} \quad |z| > 1$$

$$x[n-n_0] \Leftrightarrow X(z) \cdot z^{-n_0} \Rightarrow (n-N)u[n-N] \Leftrightarrow \frac{z^{-N-1}}{(1-z^{-1})^2} \quad |z| > 1$$

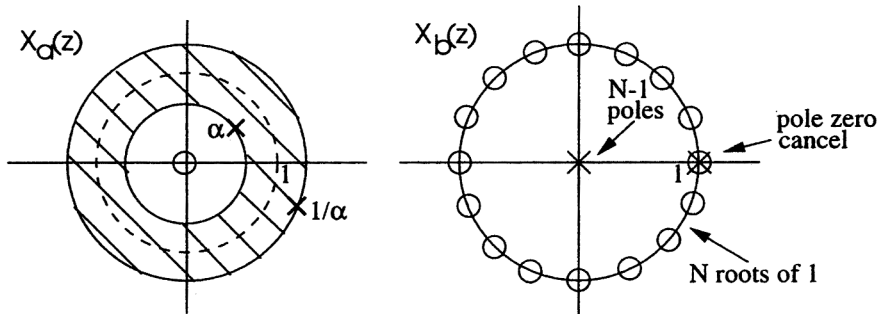
therefore

$$X(z) = \frac{z^{-1} - z^{-N-1}}{(1-z^{-1})^2} = \frac{z^{-1}(1-z^{-N})}{(1-z^{-1})^2}$$

3.3. (a)

$$x_a[n] = \alpha^{|n|} \quad 0 < |\alpha| < 1$$

$$\begin{aligned} X_a(z) &= \sum_{n=-\infty}^{-1} \alpha^{-n} z^{-n} + \sum_{n=0}^{\infty} \alpha^n z^{-n} \\ &= \sum_{n=1}^{\infty} \alpha^n z^n + \sum_{n=0}^{\infty} \alpha^n z^{-n} \\ &= \frac{\alpha z}{1 - \alpha z} + \frac{1}{1 - \alpha z^{-1}} = \frac{z(1 - \alpha^2)}{(1 - \alpha z)(z - \alpha)}, \quad |\alpha| < |z| < \frac{1}{|\alpha|} \end{aligned}$$



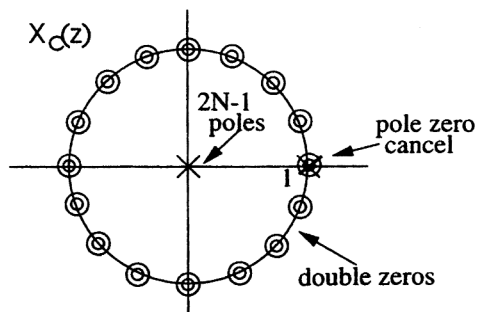
(b)

$$x_b = \begin{cases} 1, & 0 \leq n \leq N-1 \\ 0, & N \leq n \\ 0, & n < 0 \end{cases} \Rightarrow X_b(z) = \sum_{n=0}^{N-1} z^{-n} = \frac{1 - z^{-N}}{1 - z^{-1}} = \frac{z^N - 1}{z^{N-1}(z - 1)} \quad z \neq 0$$

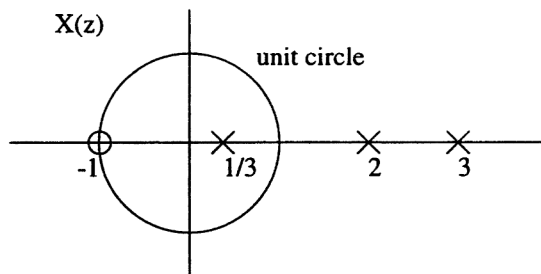
(c)

$$x_c[n] = x_b[n-1] * x_b[n] \Leftrightarrow X_c(z) = z^{-1} X_b(z) \cdot X_b(z)$$

$$X_c(z) = z^{-1} \left(\frac{z^N - 1}{z^{N-1}(z - 1)} \right)^2 = \frac{1}{z^{2N-1}} \left(\frac{z^N - 1}{z - 1} \right)^2 \quad z \neq 0, 1$$



3.4. The pole-zero plot of $X(z)$ appears below.



- (a) For the Fourier transform of $x[n]$ to exist, the z -transform of $x[n]$ must have an ROC which includes the unit circle, therefore, $|\frac{1}{3}| < |z| < |2|$.
Since this ROC lies outside $\frac{1}{3}$, this pole contributes a right-sided sequence. Since the ROC lies inside 2 and 3, these poles contribute left-sided sequences. The overall $x[n]$ is therefore two-sided.
- (b) Two-sided sequences have ROC's which look like washers. There are two possibilities. The ROC's corresponding to these are: $|\frac{1}{3}| < |z| < |2|$ and $|2| < |z| < |3|$.
- (c) The ROC must be a connected region. For stability, the ROC must contain the unit circle. For causality the ROC must be outside the outermost pole. These conditions cannot be met by any of the possible ROC's of this pole-zero plot.

3.5.

$$\begin{aligned}X(z) &= (1 + 2z)(1 + 3z^{-1})(1 - z^{-1}) \\&= 2z + 5 - 4z^{-1} - 3z^{-2} \\&= \sum_{n=-\infty}^{\infty} x[n]z^{-n}\end{aligned}$$

Therefore,

$$x[n] = 2\delta[n + 1] + 5\delta[n] - 4\delta[n - 1] - 3\delta[n - 2]$$

3.6. (a)

$$X(z) = \frac{1}{1 + \frac{1}{2}z^{-1}} \quad |z| > \frac{1}{2}$$

Partial fractions: one pole $\rightarrow$ inspection, $x[n] = (-\frac{1}{2})^n u[n]$

Long division:

$$\begin{array}{r} 1 + \frac{1}{2}z^{-1} \overline{) 1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} + \dots} \\ \underline{1} \phantom{+ \frac{1}{2}z^{-1}} \\ 1 + \frac{1}{2}z^{-1} \\ \underline{-\frac{1}{2}z^{-1}} \\ -\frac{1}{2}z^{-1} - \frac{1}{4}z^{-2} \\ \underline{+\frac{1}{4}z^{-2}} \\ +\frac{1}{4}z^{-2} + \frac{1}{8}z^{-3} \end{array}$$

$$\Rightarrow x[n] = \left(-\frac{1}{2}\right)^n u[n]$$

(b)

$$X(z) = \frac{1}{1 + \frac{1}{2}z^{-1}} \quad |z| < \frac{1}{2}$$

Partial Fractions: one pole $\rightarrow$ inspection, $x[n] = -(-\frac{1}{2})^n u[-n-1]$

Long division:

$$\begin{array}{r} \frac{1}{2}z^{-1} + 1 \overline{) 2z - 4z^2 + 8z^3 + \dots} \\ \underline{1} \\ 1 + 2z \\ \underline{-2z} \\ -2z - 4z^2 \\ \underline{+4z^2} \\ +4z^2 + 8z^3 \end{array}$$

$$\Rightarrow x[n] = -\left(-\frac{1}{2}\right)^n u[-n-1]$$

(c)

$$X(z) = \frac{1 - \frac{1}{2}z^{-1}}{1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} \quad |z| > \frac{1}{2}$$

Partial Fractions:

$$X(z) = \frac{-3}{1 + \frac{1}{4}z^{-1}} + \frac{4}{1 + \frac{1}{2}z^{-1}} \quad |z| > \frac{1}{2}$$

$$x[n] = \left[-3\left(-\frac{1}{4}\right)^n + 4\left(-\frac{1}{2}\right)^n\right] u[n]$$

Long division:

$$\begin{array}{r} 1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2} \overline{) 1 + (-\frac{3}{4} - \frac{1}{2})z^{-1} + (-\frac{3}{16} + 1)z^{-2} + \dots} \\ \underline{1} \phantom{+ (-\frac{3}{4} - \frac{1}{2})z^{-1}} \\ 1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2} \\ \underline{-(-\frac{3}{4} - \frac{1}{2})z^{-1}} \\ (-\frac{3}{4} - \frac{1}{2})z^{-1} - \frac{1}{8}z^{-2} \\ \underline{+ \frac{3}{4}(-\frac{3}{4} - \frac{1}{2})z^{-2} + \frac{1}{8}(-\frac{3}{4} - \frac{1}{2})z^{-3}} \\ [-\frac{1}{8} + \frac{3}{4}(\frac{3}{4} + \frac{1}{2})]z^{-2} + \frac{1}{8}(\frac{3}{4} + \frac{1}{2})z^{-3} \end{array}$$

$$\Rightarrow x[n] = \left[-3\left(-\frac{1}{4}\right)^n + \left(-\frac{1}{2}\right)^{n-2}\right] u[n]$$

(d)

$$X(z) = \frac{1 - \frac{1}{2}z^{-1}}{1 - \frac{1}{4}z^{-2}} \quad |z| > \frac{1}{2}$$

Partial Fractions:

$$\begin{aligned} X(z) &= \frac{1 - \frac{1}{2}z^{-1}}{1 - \frac{1}{4}z^{-2}} = \frac{1}{1 + \frac{1}{2}z^{-1}} \quad |z| > \frac{1}{2} \\ x[n] &= \left(-\frac{1}{2}\right)^n u[n] \end{aligned}$$

Long division: see part (i) above.

(e)

$$X(z) = \frac{1 - az^{-1}}{z^{-1} - a} \quad |z| > |a^{-1}|$$

Partial Fractions:

$$\begin{aligned} X(z) &= -a - \frac{a^{-1}(1 - a^2)}{1 - a^{-1}z^{-1}} \quad |z| > |a^{-1}| \\ x[n] &= -a\delta[n] - (1 - a^2)a^{-(n+1)}u[n] \end{aligned}$$

Long division:

$$\begin{array}{r} -a + z^{-1} \overline{) \begin{array}{l} -\frac{1}{a} - (\frac{a^{-1}-a}{a})z^{-1} - (\frac{a^{-1}-a}{a^2})z^{-2} + \dots \\ 1 \quad -az^{-1} \\ 1 \quad -az^{-1} \\ \hline (a^{-1}-a)z^{-1} \quad \dots \end{array}} \\ \Rightarrow x[n] = -a\delta[n] - (1 - a^2)a^{-(n+1)}u[n] \end{array}$$

3.7. (a)

$$x[n] = u[-n-1] + \left(\frac{1}{2}\right)^n u[n]$$

$$\Rightarrow X(z) = \frac{-1}{1-z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}} \quad \frac{1}{2} < |z| < 1$$

Now to find $H(z)$ we simply use $H(z) = Y(z)/X(z)$; i.e.,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{-\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1+z^{-1})} \cdot \frac{(1-z^{-1})(1-\frac{1}{2}z^{-1})}{-\frac{1}{2}z^{-1}} = \frac{1-z^{-1}}{1+z^{-1}}$$

$H(z)$ causal $\Rightarrow$ ROC $|z| > 1$.

- (b) Since one of the poles of $X(z)$, which limited the ROC of $X(z)$ to be less than 1, is cancelled by the zero of $H(z)$, the ROC of $Y(z)$ is the region in the z -plane that satisfies the remaining two constraints $|z| > \frac{1}{2}$ and $|z| > 1$. Hence $Y(z)$ converges on $|z| > 1$.

(c)

$$Y(z) = \frac{-\frac{1}{3}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{1}{3}}{1+z^{-1}} \quad |z| > 1$$

Therefore,

$$y[n] = -\frac{1}{3} \left(\frac{1}{2}\right)^n u[n] + \frac{1}{3}(-1)^n u[n]$$

3.8. The causal system has system function

$$H(z) = \frac{1 - z^{-1}}{1 + \frac{3}{4}z^{-1}}$$

and the input is $x[n] = \left(\frac{1}{3}\right)^n u[n] + u[-n - 1]$. Therefore the z -transform of the input is

$$X(z) = \frac{1}{1 - \frac{1}{3}z^{-1}} - \frac{1}{1 - z^{-1}} = \frac{-\frac{2}{3}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - z^{-1})} \quad \frac{1}{3} < |z| < 1$$

(a) $h[n]$ causal $\Rightarrow$

$$h[n] = \left(-\frac{3}{4}\right)^n u[n] - \left(-\frac{3}{4}\right)^{n-1} u[n-1]$$

(b)

$$\begin{aligned} Y(z) &= X(z)H(z) = \frac{-\frac{2}{3}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 + \frac{3}{4}z^{-1})} \quad \frac{3}{4} < |z| \\ &= \frac{-\frac{8}{13}}{1 - \frac{1}{3}z^{-1}} + \frac{\frac{8}{13}}{1 + \frac{3}{4}z^{-1}} \end{aligned}$$

Therefore the output is

$$y[n] = -\frac{8}{13} \left(\frac{1}{3}\right)^n u[n] + \frac{8}{13} \left(-\frac{3}{4}\right)^n u[n]$$

(c) For $h[n]$ to be causal the ROC of $H(z)$ must be $\frac{3}{4} < |z|$ which includes the unit circle. Therefore, $h[n]$ absolutely summable.

3.9.

$$H(z) = \frac{1 + z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{4}z^{-1})} = \frac{2}{(1 - \frac{1}{2}z^{-1})} - \frac{1}{(1 + \frac{1}{4}z^{-1})}$$

(a) $h[n]$ causal $\Rightarrow$ ROC outside $|z| = \frac{1}{2} \Rightarrow |z| > \frac{1}{2}$.

(b) ROC includes $|z| = 1 \Rightarrow$ stable.

(c)

$$\begin{aligned} y[n] &= -\frac{1}{3} \left(-\frac{1}{4}\right)^n u[n] - \frac{4}{3} (2)^n u[-n-1] \\ Y(z) &= \frac{-\frac{1}{3}}{1 + \frac{1}{4}z^{-1}} + \frac{\frac{4}{3}}{1 - 2z^{-1}} \\ &= \frac{1 + z^{-1}}{(1 + \frac{1}{4}z^{-1})(1 - 2z^{-1})} \quad \frac{1}{4} < |z| < 2 \\ X(z) &= \frac{Y(z)}{H(z)} = \frac{(1 - \frac{1}{2}z^{-1})}{(1 - 2z^{-1})} \quad |z| < 2 \\ x[n] &= -(2)^n u[-n-1] + \frac{1}{2} (2)^{n-1} u[-n] \end{aligned}$$

(d)

$$h[n] = 2 \left(\frac{1}{2}\right)^n u[n] - \left(-\frac{1}{4}\right)^n u[n]$$

3.10. (a)

$$\begin{aligned} x[n] &= \left(\frac{1}{2}\right)^n u[n-10] + \left(\frac{3}{4}\right)^n u[n-10] \\ &= \left(\frac{1}{2}\right)^n u[n] + \left(\frac{3}{4}\right)^n u[n] \\ &\quad - \left[\left(\left(\frac{1}{2}\right)^n + \left(\frac{3}{4}\right)^n \right) (u[n] - u[n-11]) \right] \end{aligned}$$

The last term is finite length and converges everywhere except at $z = 0$.

Therefore, ROC outside largest pole $\frac{3}{4} < |z|$.

(b)

$$x[n] = \begin{cases} 1, & -10 \leq n \leq 10 \\ 0, & \text{otherwise} \end{cases}$$

Finite length but has positive and negative powers at z in its $X(z)$. Therefore the ROC is $0 < |z| < \infty$.

(c)

$$\begin{aligned} x[n] &= 2^n u[-n] = \left(\frac{1}{2}\right)^{-n} u[-n] \\ x[-n] &\leftrightarrow X(1/z) \\ \left(\frac{1}{2}\right)^n u[n] &\Rightarrow \text{ROC is } |z| > \frac{1}{2} \\ \left(\frac{1}{2}\right)^{-n} u[-n] &\Rightarrow \text{ROC is } |z| < 2 \end{aligned}$$

(d)

$$x[n] = \left[\left(\frac{1}{4}\right)^{n+4} - (e^{j\pi/3})^n \right] u[n-1]$$

$x[n]$ is right-sided, so its ROC extends outward from the outermost pole $e^{j\pi/3}$. But since it is non-zero at $n = -1$, the ROC does not include ∞ . So the ROC is $1 < |z| < \infty$.

(e)

$$\begin{aligned} x[n] &= u[n+10] - u[n+5] \\ &= \begin{cases} 1, & -10 \leq n \leq -6 \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

$x[n]$ is finite-length and has only positive powers of z in its $X(z)$. So the ROC is $|z| < \infty$.

(f)

$$x[n] = \left(\frac{1}{2}\right)^{n-1} u[n] + (2+3j)^{n-2} u[-n-1]$$

$x[n]$ is two-sided, with two poles. Its ROC is the ring between the two poles: $\frac{1}{2} < |z| < \left| \frac{1}{2+3j} \right|$, or $\frac{1}{2} < |z| < \frac{1}{\sqrt{13}}$.

3.11.

$$x[n] \text{ causal} \Rightarrow X(z) = \sum_{n=0}^{\infty} x[n]z^{-n}$$

which means this summation will include *no* positive powers of z . This means that the closed form of $X(z)$ must converge at $z = \infty$, i.e., $z = \infty$ must be in the ROC of $X(z)$, or $\lim_{z \rightarrow \infty} X(z) \neq \infty$.

(a)

$$\lim_{z \rightarrow \infty} \frac{(1 - z^{-1})^2}{(1 - \frac{1}{2}z^{-1})} = 1 \quad \text{could be causal}$$

(b)

$$\lim_{z \rightarrow \infty} \frac{(z - 1)^2}{(z - \frac{1}{2})} = \infty \quad \text{could not be causal}$$

(c)

$$\lim_{z \rightarrow \infty} \frac{(z - \frac{1}{4})^5}{(z - \frac{1}{2})^6} = 0 \quad \text{could be causal}$$

(d)

$$\lim_{z \rightarrow \infty} \frac{(z - \frac{1}{4})^6}{(z - \frac{1}{2})^5} = \infty \quad \text{could not be causal}$$

3.12. (a)

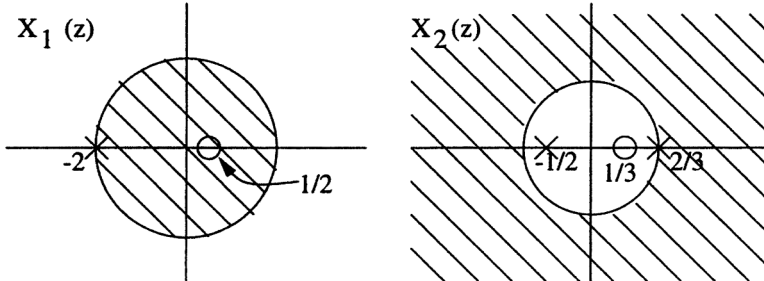
$$X_1(z) = \frac{1 - \frac{1}{2}z^{-1}}{1 + 2z^{-1}}$$

The pole is at -2, and the zero is at 1/2.

(b)

$$X_2(z) = \frac{1 - \frac{1}{3}z^{-1}}{(1 + \frac{1}{2}z^{-1})(1 - \frac{2}{3}z^{-1})}$$

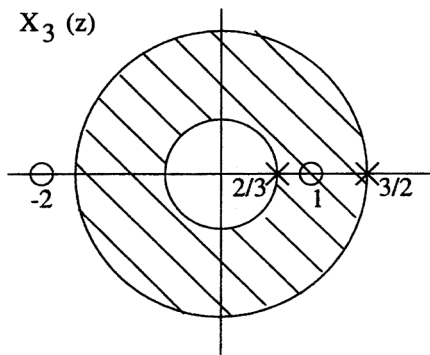
The poles are at -1/2 and 2/3, and the zero is at 1/3. Since $x_2[n]$ is causal, the ROC extends from the outermost pole: $|z| > 2/3$.



(c)

$$X_3(z) = \frac{1 + z^{-1} - 2z^{-2}}{1 - \frac{13}{6}z^{-1} + z^{-2}}$$

The poles are at 3/2 and 2/3, and the zeros are at 1 and -2. Since $x_3[n]$ is absolutely summable, the ROC must include the unit circle: $2/3 < |z| < 3/2$.



3.13.

$$\begin{aligned} G(z) &= \sin(z^{-1})(1 + 3z^{-2} + 2z^{-4}) \\ &= \left(z^{-1} - \frac{z^{-3}}{3!} + \frac{z^{-5}}{5!} - \frac{z^{-7}}{7!}\right)(1 + 3z^{-2} + 2z^{-4}) \\ &= \sum_n g[n]z^{-n} \end{aligned}$$

$g[11]$ is simply the coefficient in front of z^{-11} in this power series expansion of $G(z)$:

$$g[11] = -\frac{1}{11!} + \frac{3}{9!} - \frac{2}{11!}$$

3.14.

$$\begin{aligned}H(z) &= \frac{1}{1 - \frac{1}{4}z^{-2}} \\&= \frac{1}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{2}z^{-1})} \\&= \frac{0.5}{1 - \frac{1}{2}z^{-1}} + \frac{0.5}{1 + \frac{1}{2}z^{-1}}\end{aligned}$$

Taking the inverse z-Transform:

$$h[n] = \frac{1}{2}\left(\frac{1}{2}\right)^n u[n] + \frac{1}{2}\left(-\frac{1}{2}\right)^n u[n]$$

So,

$$A_1 = \frac{1}{2}; \quad \alpha_1 = \frac{1}{2}; \quad A_2 = \frac{1}{2}; \quad \alpha_2 = -\frac{1}{2};$$

3.15. Using long division, we get

$$\begin{aligned} H(z) &= \frac{1 - \frac{1}{1024}z^{-10}}{1 - \frac{1}{2}z^{-1}} \\ &= \sum_{n=0}^{n=9} \left(\frac{1}{2}\right)^n z^{-n} \end{aligned}$$

Taking the inverse z-transform,

$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & n = 0, 1, 2, \dots, 9 \\ 0, & \text{otherwise} \end{cases}$$

Since $h[n]$ is 0 for $n < 0$, the system is causal.

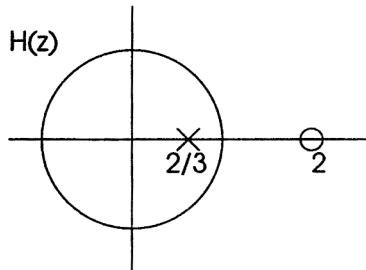
3.16. (a) To determine $H(z)$, we first find $X(z)$ and $Y(z)$:

$$\begin{aligned} X(z) &= \frac{1}{1 - \frac{1}{3}z^{-1}} - \frac{1}{1 - 2z^{-1}} \\ &= \frac{-\frac{5}{3}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - 2z^{-1})}, \quad \frac{1}{3} < |z| < 2 \\ Y(z) &= \frac{5}{1 - \frac{1}{3}z^{-1}} - \frac{5}{1 - \frac{2}{3}z^{-1}} \\ &= \frac{-\frac{5}{3}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - \frac{2}{3}z^{-1})}, \quad |z| > \frac{2}{3} \end{aligned}$$

Now

$$\begin{aligned} H(z) &= \frac{Y(z)}{X(z)} \\ &= \frac{1 - 2z^{-1}}{1 - \frac{2}{3}z^{-1}} \quad |z| > \frac{2}{3} \end{aligned}$$

The pole-zero plot of $H(z)$ is plotted below.



(b) Taking the inverse z -transform of $H(z)$, we get

$$\begin{aligned} h[n] &= \left(\frac{2}{3}\right)^n u[n] - 2\left(\frac{2}{3}\right)^{n-1} u[n-1] \\ &= \left(\frac{2}{3}\right)^n (u[n] - 3u[n-1]) \end{aligned}$$

(c) Since

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 2z^{-1}}{1 - \frac{2}{3}z^{-1}},$$

we can write

$$Y(z)\left(1 - \frac{2}{3}z^{-1}\right) = X(z)(1 - 2z^{-1}),$$

whose inverse z -transform leads to

$$y[n] - \frac{2}{3}y[n-1] = x[n] - 2x[n-1]$$

(d) The system is stable because the ROC includes the unit circle. It is also causal since the impulse response $h[n] = 0$ for $n < 0$.

3.17. We solve this problem by finding the system function $H(z)$ of the system, and then looking at the different impulse responses which can result from our choice of the ROC.

Taking the z -transform of the difference equation, we get

$$Y(z)(1 - \frac{5}{2}z^{-1} + z^{-2}) = X(z)(1 - z^{-1}),$$

and thus

$$\begin{aligned} H(z) &= \frac{Y(z)}{X(z)} = \frac{1 - z^{-1}}{1 - \frac{5}{2}z^{-1} + z^{-2}} \\ &= \frac{1 - z^{-1}}{(1 - 2z^{-1})(1 - \frac{1}{2}z^{-1})} \\ &= \frac{2/3}{1 - 2z^{-1}} + \frac{1/3}{1 - \frac{1}{2}z^{-1}} \end{aligned}$$

If the ROC is

(a) $|z| < \frac{1}{2}$:

$$\begin{aligned} h[n] &= -\frac{2}{3}2^n u[-n-1] - \frac{1}{3}\left(\frac{1}{2}\right)^n u[-n-1] \\ &\Rightarrow h[0] = 0. \end{aligned}$$

(b) $\frac{1}{2} < |z| < 2$:

$$\begin{aligned} h[n] &= -\frac{2}{3}2^n u[-n-1] + \frac{1}{3}\left(\frac{1}{2}\right)^n u[n] \\ &\Rightarrow h[0] = \frac{1}{3}. \end{aligned}$$

(c) $|z| > 2$:

$$\begin{aligned} h[n] &= \frac{2}{3}2^n u[n] + \frac{1}{3}\left(\frac{1}{2}\right)^n u[n] \\ &\Rightarrow h[0] = 1. \end{aligned}$$

(d) $|z| > 2$ or $|z| < \frac{1}{2}$:

$$\begin{aligned} h[n] &= \frac{2}{3}2^n u[n] - \frac{1}{3}\left(\frac{1}{2}\right)^n u[n-1] \\ &\Rightarrow h[0] = \frac{2}{3}. \end{aligned}$$

3.18. (a)

$$\begin{aligned} H(z) &= \frac{1 + 2z^{-1} + z^{-2}}{(1 + \frac{1}{2}z^{-1})(1 - z^{-1})} \\ &= -2 + \frac{\frac{1}{3}}{1 + \frac{1}{2}z^{-1}} + \frac{\frac{8}{3}}{1 - z^{-1}} \end{aligned}$$

Taking the inverse z-transform:

$$h[n] = -2\delta[n] + \frac{1}{3}\left(-\frac{1}{2}\right)^n u[n] + \frac{8}{3}u[n].$$

(b)

Given

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{(1 + \frac{1}{2}z^{-1})(1 - z^{-1})},$$

$z = 2$ is inside the ROC. Therefore,

$$\begin{aligned} y[n] &= H(z)\big|_{z=2} 2^n \\ &= \frac{18}{5} 2^n. \end{aligned}$$

3.19. The ROC($Y(z)$) includes the intersection of ROC($H(z)$) and ROC($X(z)$).

(a)

$$Y(z) = \frac{1}{(1 + \frac{1}{2}z^{-1})(1 - \frac{1}{4}z^{-1})}$$

The intersection of ROCs of $H(z)$ and $X(z)$ is $|z| > \frac{1}{2}$. So the ROC of $Y(z)$ is $|z| > \frac{1}{2}$.

(b) The ROC of $Y(z)$ is exactly the intersection of ROCs of $H(z)$ and $X(z)$: $\frac{1}{3} < |z| < 2$.

(c)

$$Y(z) = \frac{1}{(1 - \frac{1}{5}z^{-1})(1 + \frac{1}{3}z^{-1})}$$

The ROC is $|z| > \frac{1}{3}$.

3.20. In both cases, the ROC of $H(z)$ has to be chosen such that $\text{ROC}(Y(z))$ includes the intersection of $\text{ROC}(H(z))$ and $\text{ROC}(X(z))$.

(a)

$$H(z) = \frac{1 - \frac{3}{4}z^{-1}}{1 + \frac{2}{3}z^{-1}}$$

The ROC is $|z| > \frac{2}{3}$.

(b)

$$H(z) = \frac{1}{1 - \frac{1}{6}z^{-1}}$$

The ROC is $|z| > \frac{1}{6}$.