TRANSFORMATIONS OF THE INDEPENDENT VARIABLE FOR DISCRETE-TIME SIGNALS.

a) time shift $x_1[n] = x[n-n_0]$.

b) time reversal $x_2[n] = x[-n]$.

c) time scale $x_3[n] = x[an]$.

Transformations in DT are analogous to those in continuous time. However, there are a few differences to consider:

(a) Only integer time shifts are permitted

(b) When time scaling with $a > 1$ (compression) we lose information.

$x_1[n] = x[2n]$. we lose all values of $x[n]$ located at odd times.

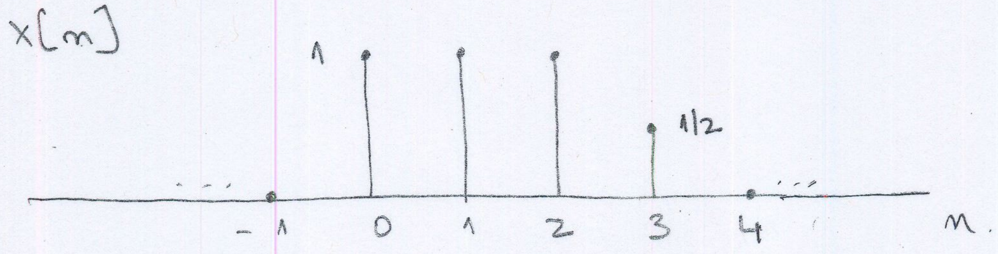
Some issues to be further explored:

- How can we time shift $x[n]$ by a non-integer delay? $x_1[n] = x\left[n - \frac{1}{2}\right]$.

  Hint: we transform DT into CT via interpolation, then time-shift the CT signal, then resample to get the final shifted DT signal.

– How can we expand $x[n]$ to $x\left[\frac{1}{2}n\right]$, how do we "fill in the blanks"?

MULTIPLE TRANSFORMATIONS



Given $x[n]$, we want to compute $x[2n+6]$.

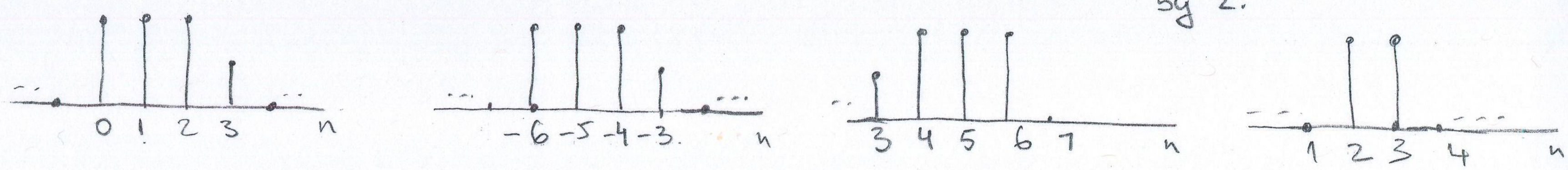
We need to use all 3 types of transformations.

We can do these transformations in ANY order.

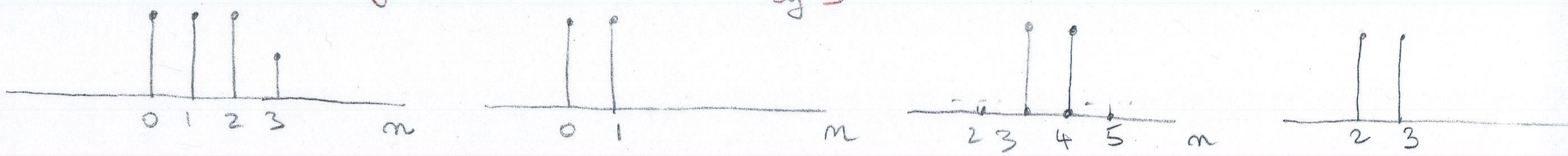
However, the amount and direction of the shift depends on whether it is performed before or after the reversal and scale.

To demonstrate this, let's do the transformation in 3 different orders.

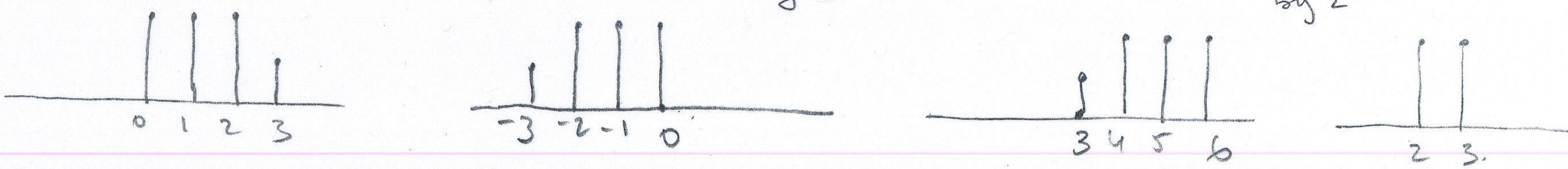
$$x[n] \xrightarrow{\text{advance by } 6} x[n+6] \xrightarrow{\text{reverse}} x[-n+6] \xrightarrow[\text{by } 2]{\text{compress}} x[-2n+6]$$



$$x[n] \xrightarrow[\text{by } 2]{\text{compress}} x[2n] \xrightarrow[\text{by } 3]{\text{advance}} x[2(n+3)] \xrightarrow{\text{reverse}} x[2(-n+3)]$$



$$x[n] \xrightarrow{\text{reverse}} x[-n] \xrightarrow[\text{by } 6]{\text{Delay}} x[-(n-6)] \xrightarrow[\text{by } 2]{\text{Compress}} x[-2(n-6)]$$



A common mistake.

INCORRECT $x[n] \xrightarrow[\text{"advance by 6"}]{} x[n+6] \xrightarrow[\text{"Reverse"}]{} x[-(n+6)] \xrightarrow[\text{"Compress"}]{} x[-2(n+6)]$

CORRECT.

$$x[n] \xrightarrow[\text{"advance by 6}]{} x[n+6] \xrightarrow[\text{Reverse}]{} x[-n+6] \xrightarrow[\text{"Compress"}]{} x[-2n+6]$$

The correct version replaces the independent variable n (not the entire argument of the function) by some function of n.

$g[n] = n + 6$ for time shift

$h[n] = -n$ for reversal

$k[n] = 2n$ for scaling.