Galois Field Vector-Space Representations

GF(8)
$p(x) = x^3 + x + 1$
$\Rightarrow \alpha^3 + \alpha + 1 = 0$

$x = a + b\alpha + c\alpha^2$

x	abc
0	100
1	010
2	001
3	110
4	011
5	111
6	101

GF(16)
$p(x) = x^4 + x + 1$

0	1000
1	0100
2	0010
3	0001
4	1100
5	0110
6	0011
7	1101
8	1010
9	0101
10	1110
11	0111
12	1111
13	1011
14	1001

GF(32)
$p(x) = x^5 + x^2 + 1$

0	10000
1	01000
2	00100
3	00010
4	00001
5	10100
6	01010
7	00101
8	10110
9	01011
10	10001
11	11100
12	01110
13	00111
14	10111
15	11111
16	11011
17	11001
18	11000
19	01100
20	00110
21	00011
22	10101
23	11110
24	01111
25	10011
26	11101
27	11010
28	01101
29	10010
30	01001

GF(64)
$p(x) = x^6 + x + 1$

0	100000
1	010000
2	001000
3	000100
4	000010
5	000001
6	110000
7	011000
8	001100
9	000110
10	000011
11	110001
12	101000
13	010100
14	001010
15	000101
16	110010
17	011001
18	111100
19	011110
20	001111
21	110111
22	101011
23	100101
24	100010
25	010001
26	111000
27	011100
28	001110
29	000111
30	110011
31	101001
32	100100
33	010010
34	001001
35	110100
36	011010
37	001101
38	110110
39	011011
40	111101
41	101110
42	010111
43	111011
44	101101
45	100110
46	010011
47	111001
48	101100
49	010110
50	001011
51	110101
52	101010
53	010101
54	111010
55	011101
56	111110
57	011111
58	111111
59	101111
60	100111
61	100011
62	100001

Minimal Polynomials of Elements of GF(2^m)

GF(4)

1	(0,1,2)

GF(8)

1	(0,1,3)
3	(0,2,3)

GF(16)

1	(0,1,4)
3	(0,1,2,3,4)
5	(0,1,2)
7	(0,3,4)

GF(32)

1	(0,2,5)
3	(0,2,3,4,5)
5	(0,1,2,4,5)
7	(0,1,2,3,5)
11	(0,1,3,4,5)
15	(0,3,5)

GF(64)

1	(0,1,6)
3	(0,1,2,4,6)
5	(0,1,2,5,6)
7	(0,3,6)
9	(0,2,3)
11	(0,2,3,5,6)
13	(0,1,3,4,6)
15	(0,2,4,5,6)
21	(0,1,2)
23	(0,1,4,5,6)
27	(0,1,3)
31	(0,5,6)