Noncoherent Space-Time Block Codes with Turbo Codes in Unstructured Interference

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***Abstract*— Space-time block coding (STBC) can be used to provide transmit diversity in wireless communication systems. However, strong unstructured interference can severely degrade performance. This problem is investigated for noncoherent STBC based on a recently introduced coherent STBC decoding algorithm for interference suppression. Further, a soft-reliability metric is developed for use with turbo coding. Coding gains are achieved exceeding 8dB at a bit error rate of 10^{-2} compared to an uncoded system. Our findings also indicate that a 3dB penalty in performance occurs in the turbo-coded noncoherent STBC system over a similarly developed coherent one.**

I. Introduction

There is much interest in using multi-transmit and multi-receive antenna arrays (MIMO systems) for supporting high data rate wireless applications. This interest has increased with the arrival of an advanced modulation technique called space-time block coding (STBC). STBC is attractive because of its ability to provide full transmit diversity with a simple receiver structure. Its drawback is that no coding gain is offered. This can be remedied by using an additional outer channel code in serial concatenation.

In conjunction with the development of advanced modulation and coding techniques, wireless products are becoming smaller and faster to meet portability and usage requirements. This in turn, makes noise interference a major concern. Industry measurements have shown that the surrounding electronics contribute greatly to the ambient noise power, often leading to an unacceptable noise floor in the receiver [1].

Localized noise and interference is a significant problem in MIMO systems because it spatially correlates the received data resulting in a loss in STBC receiver performance. The development of robust space-time receivers is highly desirable to mitigate this loss.

A low complexity coherent STBC detector for unknown interference suppression was recently proposed by Larsson et al. [5] using a Gaussian noise and interference approximation. This STBC detector acquires and refines channel and noise covariance matrices based on cyclic maximization theory and a minimal amount of training data. This is denoted as the "cyclic-based maximum likelihood technique" (CML).

In this paper, the CML technique is applied to noncoherent STBC, otherwise known as differential space-time modulation (DSTM). Further, a method is developed for soft-information transfer from the DSTM receiver to a turbo code decoder.

II. DSTM and the MIMO System Model

Consider a wireless communication system employing M antennas at the transmitter and N antennas at the receiver. Let the transmitted signal be represented by groups of T data symbols $(c_1, \ldots, c_T)$ with each symbol belonging to a finite constellation of $|C|$ elements. The first step in DSTM encoding is to map each group of T symbols into an information matrix $\mathbf{G}[l]$ of dimension $M \times T$. Each information matrix is in turn differentially encoded into a codeword matrix $\mathbf{X}[l]$ of the same dimension for transmission over the wireless channel.

The (m,t) entry of $\mathbf{X}[l]$ is the signal transmitted by antenna m in time slot t. For simplicity, but noting that the system can be generalized to higher dimensions, we consider the binary constellation with $c \in (-1, 1)$, $T = 2$, and $M = 2$. Further, the full-rate DSTM group code with $\mathbf{G}[l] = \mathbf{G}_s,\ s \in \{1, \ldots, 4\}$ is considered [2]

$$\begin{aligned} \mathbf{D} &= \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, & \mathbf{G}_1 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ \mathbf{G}_2 &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, & \mathbf{G}_3 &= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \\ \mathbf{G}_4 &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \end{aligned} \tag{1}$$

where $\mathbf{D}$ is the initial training matrix.

For channel and noise estimation, consider the transmission of $(K + L)$ concatenated DSTM codeword matrices $\mathbf{X} = [\mathbf{X}[1]^{tr} \ldots \mathbf{X}[K]^{tr}, \mathbf{X}[1] \ldots \mathbf{X}[L]]$ of dimension $M \times 2(K + L)$. $\mathbf{X}$ is composed of K training matrices $\mathbf{X}[k]^{tr}$ and L information matrices $\mathbf{X}[l]$. DSTM encoding and transmission is outlined as follows. First, K training matrices $[\mathbf{X}[1]^{tr} \ldots \mathbf{X}[K]^{tr}]$ are sent with each training matrix given by $\mathbf{X}[k]^{tr} = \mathbf{D}$. This is then followed by transmission of differential codeword matrices determined by

$$\begin{aligned} \mathbf{X}[1] &= \mathbf{X}[K]^{tr}\mathbf{G}[1] \\ \mathbf{X}[l] &= \mathbf{X}[l-1]\mathbf{G}[l], l = 2, \ldots, L \end{aligned} \tag{2}$$

The received signal is a noisy superposition of independently faded signals. Using matrix notation, and noting that the columns of $\mathbf{X}$ are transmitted during $2(K+L)$ consecutive time intervals across M antennas, the received signal in the discrete baseband domain with symbol-spaced sampling can be written as

$$\mathbf{R} = \sqrt{\frac{\rho}{M}}\mathbf{H}\mathbf{X} + \mathbf{N} + \mathbf{E}, \tag{3}$$

where ρ represents the transmitted power and $\mathbf{R} = [\mathbf{R}_1^{tr} \ldots \mathbf{R}_K^{tr}\mathbf{R}[1] \ldots \mathbf{R}[L]]$ is the received matrix. The (n,t) entry of $\mathbf{R}$ is the received signal at antenna n in time slot t. $\mathbf{N}$ is the receiver noise matrix, and $\mathbf{E}$ is the interference matrix. $\mathbf{N}$ contains spatially and temporally uncorrelated entries consisting of complex circularly-symmetric Gaussian random variables that have zero mean and unit variance, i.e. *CN(0,1)*.

The channel transfer matrix is denoted by $\mathbf{H}$ and two different channel models are considered here. The first model assumes quasi-static flat fading with constant channel coefficients for different transmit-receive (T-R) antenna pairs over the time interval required to transmit $\mathbf{X}$. $\mathbf{H}$ contains spatially and temporally white entries $h_{m,n}$ that are *CN(0,1)*. $h_{m,n}$ represents the channel coefficient linking transmit antenna m with receive antenna n. The second model relaxes the quasi-static restriction so that Doppler fading using Jakes' fading channel simulator can be included [4]. Each channel coefficient that links a T-R antenna pair now changes continuously in time with a time autocorrelation function given by

$$R_{h_{m,n}}(\tau) = J_o(\omega_m \tau), \tag{4}$$

where $R_{h_{m,n}}(\cdot)$ is the autocorrelation function, ω_m is the maximum Doppler frequency shift, and $J_o(\cdot)$ is the zero-order Bessel function of the first kind. τ denotes the relative time shift applied to the set of samples considered in the autocorrelation function.

The interference $\mathbf{E}$ is modeled as spatially correlated Gaussian noise as in [5]

$$\mathbf{E} = \sqrt{\frac{\rho_e}{M}}\mathbf{H}_e\mathbf{W}_e. \tag{5}$$

ρ_e denotes the interference power, $\mathbf{H_e}$ signifies the wireless channel matrix linking the interference to each receive antenna, and $\mathbf{W_e}$ is a Gaussian noise matrix with spatially and temporally white entries that are *CN(0,1)*. $\mathbf{H}_e$ also contains entries that are *CN(0,1)*, however, the entries are temporally uncorrelated but spatially correlated. This spatially correlated Gaussian model is a standard interference model relevant in a communication channel subject to interference and jamming [5]. An example where this model may be prevalent is when enclosed interfering electronic sources are in close proximity to each other as well as the STBC receiver. This close proximity is significant because the channels that connect the interference-receiver pair may be spatially correlated. The effect of this interference on the STBC receiver is that colored noise is introduced and the overall noise power is increased, which can significantly degrade conventional STBC receivers designed under the assumption of white noise. The covariance matrix $\mathbf{Q}$ of the combined noise and interference statistics for spatially white noise is given by $\mathbf{Q} = \sigma^2\mathbf{I}$, but in the spatially colored case $\mathbf{Q}$ is an unknown positive definite matrix. σ represents noise power and $\mathbf{I}$ denotes the identity matrix.

Note that here both the signal and interference energies are assumed normalized by the number of transmit antennas at the respective sources. Thus the average received signal and interference energy per symbol period at each receive antenna will be ρ and ρ_e, respectively.

III. Receiver Design

A. Noncoherent Maximum-Likelihood Detection

Noncoherent maximum-likelihood (ML) detection for STBC coding is facilitated by considering the conditional probability density function (pdf) of the received matrix. Based on the signal and channel model with no interference, the pdf for a received matrix $\mathbf{R}[l]$ conditioned on the transmitted information matrix is [2]

$$Pr(\mathbf{R}[l]|\mathbf{G}[l]) = \frac{\exp(-Tr\{\mathbf{R}[l]\Sigma^{-1}\mathbf{R}[l]^*\})}{|\pi\Sigma|^n} \tag{6}$$

where

$$\Sigma = \mathbf{I} + \rho_t\mathbf{G}[l]^*\mathbf{G}[l].$$

$Pr(\cdot)$ stands for probability, $Tr\{\cdot\}$ is the trace operator, $\rho_t = \sqrt{\frac{\rho}{2}}$ is the transmit power per antenna, and $*$ denotes conjugate transpose.

By imposing the unitary property on noncoherent STBC code design, (6) can be simplified to produce the following differential unitary space-time quadratic receiver

$$\hat{s} = \underset{s}{\operatorname{argmax}}\ Tr\{\mathbf{R}[l]\mathbf{G}_s^*\mathbf{G}_s\mathbf{R}[l]\}, \tag{7}$$

where $\hat{s}$ is the index of the estimate of $\mathbf{G}[l]$.

Applying the DSTM group code to the differential unitary space-time quadratic receiver is complex for ML detection would require the use of the entire received sequence $\mathbf{R}$. Further simplifications are achievable, however, if the noncoherent STBC codes were designed to include group properties. The end result is a designed DSTM group code suboptimum detection scheme that relies on the last two received blocks only [2].

$$\hat{s} = \underset{s}{\operatorname{argmax}}\ \mathrm{Re}(Tr\{\mathbf{G}_s\mathbf{R}[l]^*\mathbf{R}[l-1]\}), \tag{8}$$

where $\mathrm{Re}(\cdot)$ denotes the real component.

This DSTM detector is optimal in an environment without interference. To handle interference or spatially correlated noise sources, prewhitening is required. It is well-known that a linear transformation of a Gaussian signal will still retain its Gaussian properties. Therefore, (8) can be modified to account for interference using prewhitening (see Chapter 3, [6]) giving

$$\hat{s} = \underset{s}{\operatorname{argmax}}\ \mathrm{Re}(Tr\{\mathbf{Q}^{-1}\mathbf{G}_s\mathbf{R}[l]^*\mathbf{R}[l-1]\}). \tag{9}$$

B. Cyclic-Based ML Technique

In the proposed coherent CML technique, training is used to obtain initial estimates of the channel $\mathbf{H}$ and noise $\mathbf{Q}$ matrices. These estimates are then refined through an iterative process. To obtain an initial estimate of the channel matrix $\mathbf{H}$ and noise matrix $\mathbf{Q}$, the training-based spatially white (TWML) and spatially colored (TCML) detectors compute [5]

$$\hat{\mathbf{H}} = \mathbf{Y}_{tr}\mathbf{X}_{tr}^*(\mathbf{X}_{tr}\mathbf{X}_{tr}^*)^{-1}, \tag{10}$$

provided that the training block $\mathbf{X}_{tr} = [\mathbf{X}[1]^{tr} \ldots \mathbf{X}[K]^{tr}]$ has full row rank, which is satisfied with any STBC codeword. Let $\mathbf{\Phi} = \mathbf{X}_{tr}(\mathbf{X}_{tr}^*\mathbf{X}_{tr})^{-1}\mathbf{X}^*$ and $\mathbf{\Phi}^\perp = \mathbf{I} - \mathbf{\Phi}$. The noise covariance estimates are given by

$$\begin{aligned} \hat{\mathbf{Q}}_{TWML} &= \hat{\sigma}_{ML}\mathbf{I} = \frac{1}{N \cdot K} Tr\{\mathbf{R}_{tr}\mathbf{\Phi}^\perp\mathbf{R}_{tr}^*\}\mathbf{I}, \\ \hat{\mathbf{Q}}_{TCML} &= \frac{1}{K}\mathbf{R}_{tr}\mathbf{\Phi}^\perp\mathbf{R}_{tr}^*, \end{aligned} \tag{11}$$

under the assumptions of spatially white and spatially colored noise, respectively. Once the training estimates of $\hat{\mathbf{H}}$ and $\hat{\mathbf{Q}}$ are obtained, ML symbol detection is performed. For CML estimation, the symbol decisions provided by the training-based ML detector are then used for re-estimating $\hat{\mathbf{H}}$ and $\hat{\mathbf{Q}}$. This re-estimation can be reiterated any number of times.

In [5], the channel estimation of $\hat{\mathbf{H}}$ required a minimum of two training matrices. However, with DSTM and CML, only one training matrix is required since only an estimate of $\mathbf{Q}$ is necessary. This is done by assuming that initially there is no knowledge of the noise, i.e. $\mathbf{Q} = \mathbf{I}$. For the special case when only one training matrix is used, the performance is denoted by (TML) regardless of whether the noise is spatially white or colored. The TML noise estimates are then used for the CML iterations. Our results show that a gain in performance occurs with the colored-based detectors only, and that no gain occurs after the second iteration. Thus, we only present results for the colored-noise CML technique with one or two iterations, and these are denoted (1 Iters WML), (2 Iters WML), (1 Iters CML), and (2 Iters CML), respectively.

C. Soft-Information Transfer from the DSTM Detector

Assume that the transmission of the information matrices $\mathbf{G}_l \in (\mathbf{G}_1, \ldots, \mathbf{G}_4)$ is identically and independently distributed over the four values. The likelihood that the first transmitted symbol c_1 in a $T = 2$ symbol grouping (c_1, c_2) is equal to one is given by [3]

$$\begin{aligned} &Pr(c_1 = 1|\mathbf{R}[l]) \\ &\quad = \frac{Pr(c_1 = 1)Pr(\mathbf{R}[l]|c_1 = 1)}{\sum_{s=1}^{4} Pr(\mathbf{G}_s)Pr(\mathbf{R}[l]|\mathbf{G}_s)} \\ &\quad = \frac{Pr(c_1 = 1)Pr(\mathbf{R}[l]|c_1 = 1)}{Pr(\mathbf{R}[l])} \\ &\quad \propto Pr(\mathbf{R}[l]|c_1 = 1) \\ &\quad \propto Pr(\mathbf{R}[l]|\mathbf{G}_3) + Pr(\mathbf{R}[l]|\mathbf{G}_4), \end{aligned} \tag{12}$$

where $(c_1 = 1)$ maps directly into two of four possible information matrices in the DSTM code. For the DSTM group code considered, $c_1 = 1$ maps to $\mathbf{G}_3$ and $\mathbf{G}_4$. These calculated likelihood values can be used as soft-reliability metrics in an outer channel code. Using the same principle, we obtain

$$Pr(c_2 = 1|\mathbf{R}[l]) \propto Pr(\mathbf{R}[l]|\mathbf{G}_2) + Pr(\mathbf{R}[l]|\mathbf{G}_3). \tag{13}$$

The likelihood values calculated in (12) and (13) are in turn, treated as observations from BPSK modulation over an additive white Gaussian noise (AWGN) channel. Note that only a minor modification is required to obtain soft outputs in this DSTM detector as $Pr(\mathbf{R}[l]|\mathbf{G}_s)$ with $s \in (1, \ldots, 4)$ is necessarily calculated in a DSTM hard-decision detector.

IV. A Brief Description of Turbo Codes

In this paper, a turbo code was used used for channel coding since these are employed in most future generation wireless communication systems. We consider a rate $1/3$ parallel concatenated turbo encoder composed of two 4-state recursive systematic convolutional encoders separated by a random interleaver of size 256. The constituent encoders are identical with feedforward and feedback generator polynomials in octal form given by (5,7). An additional block interleaver was added at the encoder output to mitigate burst errors. For turbo decoding, the two most popular choices are log-MAP or SOVA decoding. The SOVA algorithm was implemented here because it provides a good trade-off between performance and complexity. SOVA requires less than half the computational complexity of log-MAP at the cost of a minor penalty of approximately 1dB in power [8]. Our results indicate that most of the gain occurs in the first 4 iterations of turbo coding. Thus results are presented only for 1 iteration and 4 iterations of turbo decoding, denoted by (1 Iter TC) and (4 Iter TC), respectively.

V. Simulation Results

CML performance in a DSTM coded system is presented under several different conditions. For reference purposes, the performance of a coherent STBC system assuming either known or estimated $\mathbf{H}$ and $\mathbf{Q}$ is provided when appropriate. Estimates of $\mathbf{H}$ and $\mathbf{Q}$ are obtained using the CML technique. For coherent decoding, orthogonal STBC (OSTBC) codes were used as described in [5]. Unless specified otherwise, each frame of data is composed of $L = 15$ information matrices and $K = 1$ training matrix with equal transmit signal and interference powers, i.e. a signal-to-interference ratio (SIR) of 0dB. $L = 15$ and $K = 1$ gives a training overhead of 6.75%. Note that with $K = 1$, an initial assumption of no knowledge of the noise is made, i.e. $\hat{\mathbf{Q}}_{TML} = \mathbf{I}$. Otherwise, the training methods given in [5] are used.

Figure 1 presents CML performance with interference. Although a relatively high error floor exists, recall that only one training matrix is employed and SIR=0dB with no initial knowledge of the noise. In Figure 2, CML interference suppression is demonstrated by varying the SIR. Results indicate that a SIR of -1dB SIR is sufficient for a BER of 10^{-2}. Further, the CML technique with two iterations requires approximately

6dB less power for the same BER as with conventional TML training.

CML performance can be further improved if the number of training matrices is increased. Figure 3 shows that $K = 3$ provides adequate BER performance. However, this improvement comes at a great cost in efficiency since the training overhead is 18.75% in this case.

Figure 4 shows the advantage of noncoherent DSTM over coherent STBC as the channel changes from a slow to fast fading environment. BER performance results in a constantly changing channel indicate that the initial channel estimate obtained in a coherent system is an inadequate channel estimate. The crossover point in performance occurs at a low normalized Doppler frequency of approximately 0.008.

Figure 5 provides two different comparisons. The first shows that coding gains of more than 8dB over an uncoded DSTM system can be obtained with 4 iterations of turbo coding (4 Iter TC), at a bit error rate of 10^{-2}. The second comparison shows the suitability of the proposed soft-reliability metric for DSTM. A 3dB loss in performance is noted for the coded DSTM system relative to a coded OSTBC system with CML channel and noise estimation. In the coded coherent STBC system, a minimal number of training matrices as well as the soft metric described in [5] were used.

VI. Conclusions

We have extended a recently proposed technique for coherent STBC interference suppression for noncoherent DSTM. The effect of training symbols and Doppler fading on DSTM detector performance was presented. A means of soft-information transfer was developed for use with turbo coding. Favourable results were achieved that included coding gains exceeding 8dB at a bit error rate of 10^{-2} over an uncoded system.

Acknowledgments

The authors wish to acknowledge the financial support of Sierra-Wireless Inc., and in particular Mr. Peter McConnell, the company representative who motivated this work.

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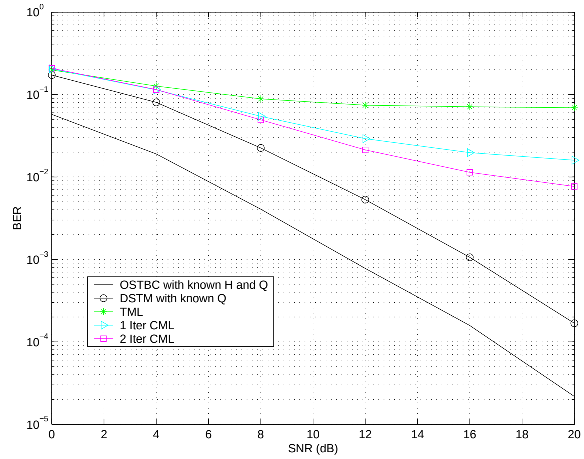

Fig. 1. CML performance in a DSTM system with one strong interferer.

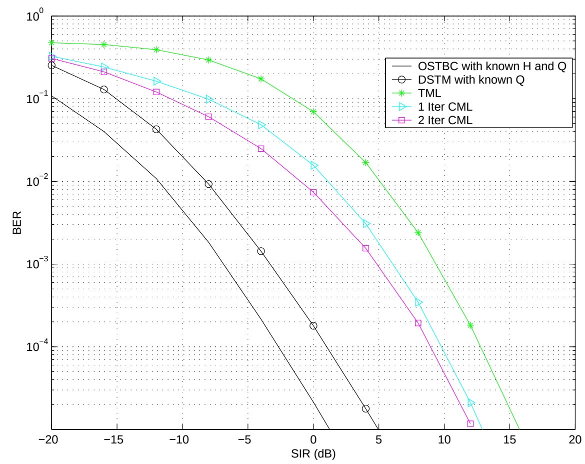

Fig. 2. CML vs. SIR performance in a DSTM system with SNR = 15dB.

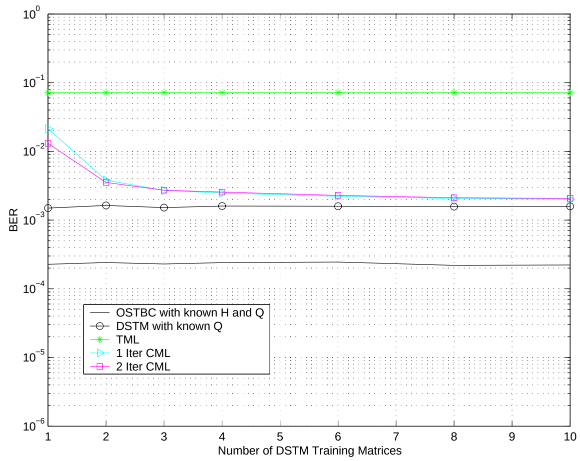

Fig. 3. Performance versus the number of training symbols with SNR = 15dB and SIR = 0dB.

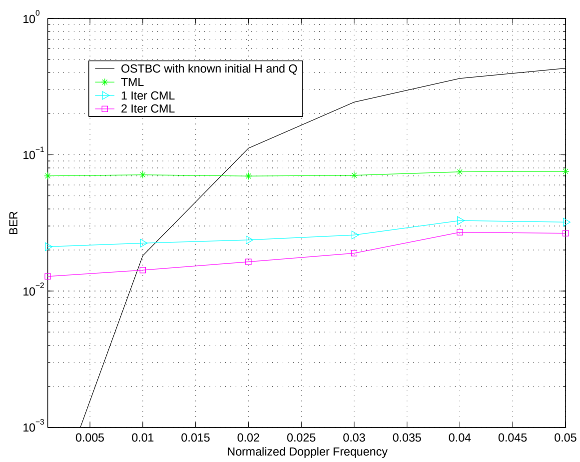

Fig. 4. The impact of Doppler fading with SNR = 15dB and SIR = 0dB.

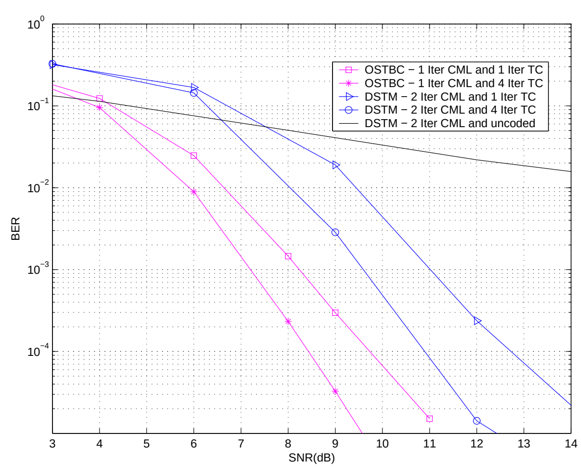

Fig. 5. Comparison between coherent and noncoherent turbo-coded systems, and an uncoded system.