

ELKE 405 Midterm Test Solutions 2009

1a)  $H_1 = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} = [I \ P^T]$

$$G_1 = [P \ I] = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

b) codewords

|       |   |                         |
|-------|---|-------------------------|
| 00000 | 0 | $A(x) = 1 + 2x^3 + x^4$ |
| 11110 | 4 |                         |
| 10101 | 3 |                         |
| 01011 | 3 |                         |

c)  $d_{\min} = 3$

d) number of detectable errors =  $d_{\min} - 1 = 2$

e) number of correctable errors =  $\left\lfloor \frac{d_{\min} - 1}{2} \right\rfloor = 1$

2.  $G_2 = [111] = [P \ I]$

$$H_2 = I \ P^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$G_2$  has  $d_{\min} = 3$  so all single errors are correctable

the correctable error patterns are

$$e_0 = 000$$

$$e_1 = 100$$

$$e_2 = 010$$

$$e_3 = 001$$

the corresponding syndromes are

$$s_0 = e_0 H^T = 00$$

$$s_1 = e_1 H^T = 10$$

$$s_2 = e_2 H^T = 01$$

$$s_3 = e_3 H^T = 11$$

$$b) \quad r H^T = 110 \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} = 11 \quad 00 \quad s_3$$

$$c = r + e_3 = 110 + 001 = 111$$

$$3) \quad 2^0 = 1 \quad 2^1 = 2 \quad 2^2 = 4 \quad 2^3 = 3 \pmod{5}$$

$\therefore 2$  is a primitive element in  $GF(5)$