Design of Digital Filters
Satisfying Prescribed Specifications

Introduction

Tutorial ISCAS 2007

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State-of-the-art methods for the design of digital filters will be presented placing special emphasis on the design of filters that would satisfy prescribed specifications.
Example

In an application, an equiripple bandstop digital filter is required which should satisfy the following specifications:

- Maximum passband ripple $A_p : 0.5 \text{ dB}$
- Minimum stopband attenuation $A_a : 50.0 \text{ dB}$
- Lower passband edge $\omega_{p1} : 0.8 \text{ rad/s}$
- Upper passband edge $\omega_{p2} : 2.2 \text{ rad/s}$
- Lower stopband edge $\omega_{a1} : 1.2 \text{ rad/s}$
- Upper stopband edge $\omega_{a2} : 1.8 \text{ rad/s}$
- Sampling frequency $\omega_s : 2\pi \text{ rad/s}$

Design the lowest-order filter that will satisfy the specifications.
The tutorial is organized in four parts as follows:

- **Part 1**: FIR (nonrecursive) filters using the window method.
- **Part 2**: FIR filters using the weighted-Chebyshev method.
- **Part 3**: IIR (recursive) filters based on the bilinear transformation method.
- **Part 4**: IIR filters based on the optimization approach.
The window method uses the Fourier series in conjunction with a class of functions known as *window functions*.
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Prescribed specifications can be achieved by using a *design technique proposed by Kaiser*.
Window Method \textit{Cont'd}

Advantages:

- Closed-form method.
Window Method *Cont’d*

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- Easy to apply.
Window Method \textit{Cont’d}

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- Closed-form method.
- Easy to apply.
- The design entails a relatively insignificant amount of computation.
Disadvantages:

- Designs are suboptimal, i.e., the filter order needed to satisfy a given set of prescribed specifications is not the lowest, i.e., other methods are available that yield a lower-order filter (e.g., the weighted-Chebyshev method).
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- Designs are suboptimal, i.e., the filter order needed to satisfy a given set of prescribed specifications is not the lowest, i.e., other methods are available that yield a lower-order filter (e.g., the weighted-Chebyshev method).
- A higher-order filter means more computations per sample, which implies that these filters are slower and less efficient in real-time applications.
The weighted-Chebyshev method is an iterative multi-variable optimization method based on a very efficient optimization algorithm, the so-called *Remez Exchange Algorithm*.
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Prescribed specifications can be achieved by using a prediction technique due to Herrmann, Rabiner, and Chan.
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- Designs are optimal, i.e., the required filter order for a set of prescribed specifications is the lowest that can be achieved.
- Minimum filter order implies a more efficient and faster filter for real-time applications.
- The method is very flexible – it can be used to design filters, differentiators, Hilbert transformers, etc.
- The solutions achieved are equiripple.
Disadvantages:

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- Not suitable for applications where the design has to be carried out in real- or quasi-real time, for example, in programmable or adaptable filters.
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- The optimization approach.
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Prescribed specifications can be achieved by using a design method proposed by Antoniou some years ago.

This method essentially involves deducing the parameters of the transformations involved as well as the minimum filter order from the required specifications.
Advantages

- The bilinear transformation method leads to a complete description of the transfer function \textit{in closed form} either in terms of its zeros and poles or its coefficients.
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- The filters designed are *always stable*. 
Disadvantages

- Its main disadvantage is that it is applicable only for the design of *filters with piecewise-constant amplitude responses*, i.e., filters whose passband and stopband gains are constant and zero, respectively, to within prescribed tolerances.
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As the value of the norm approaches zero, the resulting amplitude and/or phase response approaches the desired amplitude and/or phase response.
Advantages

- The optimization approach is very flexible in that it can be used to design filters with arbitrary amplitude and/or phase responses.
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- The optimization approach is very flexible in that it can be used to design filters with arbitrary amplitude and/or phase responses.
- A great variety of optimization algorithms can be used such as least-squares, minimax, Newton, quasi-Newton, even genetic algorithms.
Disadvantages

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- The use of unconstrained optimization often leads to unstable designs.

  However, techniques are available that can be used to convert an unstable into a stable design.

- For filters with piecewise constant amplitude responses, the bilinear transformation method is preferred.
The problem of instability can also be overcome by using constrained optimization.

However, the amount of computation required to complete a design is increased considerably, sometimes by a factor of 10 or more.
Disadvantages *Cont’d*

- The problem of instability can also be overcome by using constrained optimization.

  However, the amount of computation required to complete a design is increased considerably, sometimes by a factor of 10 or more.

- For FIR filters, there is a technique for predicting the minimum filter order to achieve any prescribed set of specifications.

  Unfortunately, no such technique is available for IIR filters designed by optimization.

  Typically, the problem is solved through a cut-and-try approach.
This slide concludes the Introduction. Thank you for your attention.