Part 3: IIR Filters – Bilinear Transformation Method

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- A procedure for the design of IIR filters that would satisfy arbitrary prescribed specifications will be described.

Introduction

- A procedure for the design of IIR filters that would satisfy arbitrary prescribed specifications will be described.
- The method is based on the bilinear transformation and it can be used to design lowpass (LP), highpass (HP), bandpass (BP), and bandstop (BS), Butterworth, Chebyshev, Inverse-Chebyshev, and Elliptic filters.

  Note: The material for this module is taken from Antoniou, *Digital Signal Processing: Signals, Systems, and Filters,* Chap. 12.)

Given an analog filter with a continuous-time transfer function $H_A(s)$, a digital filter with a discrete-time transfer function $H_D(z)$ can be readily deduced by applying the bilinear transformation as follows:

$$H_D(z) = H_A(s)\Big|_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)} \qquad \text{(A)}$$

The bilinear transformation method has the following important features:

- A stable analog filter gives a stable digital filter.

Introduction *Cont'd*

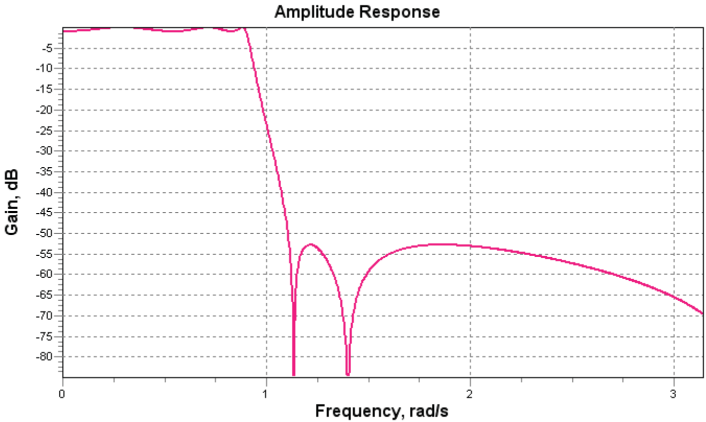
The bilinear transformation method has the following important features:

- A stable analog filter gives a stable digital filter.
- The maxima and minima of the amplitude response in the analog filter are preserved in the digital filter.

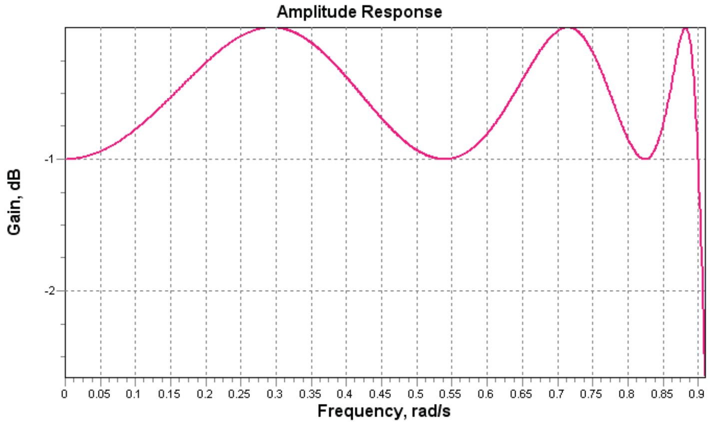
  As a consequence,

  - the passband ripple, and
  - the minimum stopband attenuation

  of the analog filter are preserved in the digital filter.


Amplitude Response
Gain, dB
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
-55
-60
-65
-70
-75
-80
0
1
2
3
Frequency, rad/s

Introduction *Cont'd*



If we let ω and Ω represent the frequency variable in the analog filter and the derived digital filter, respectively, then Eq. (A), i.e.,

$$H_D(z) = H_A(s)\Big|_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)} \tag{A}$$

gives the frequency response of the digital filter as a function of the frequency response of the analog filter as

$$H_D(e^{j\Omega T}) = H_A(j\omega)$$

provided that $s = \dfrac{2}{T}\left(\dfrac{z-1}{z+1}\right)$

or $$j\omega = \frac{2}{T}\left(\frac{e^{j\Omega T}-1}{e^{j\Omega T}+1}\right) \quad \text{or} \quad \omega = \frac{2}{T}\tan\frac{\Omega T}{2} \tag{B}$$

...

$$\omega = \frac{2}{T} \tan \frac{\Omega T}{2} \qquad \text{(B)}$$

- For $\Omega < 0.3/T$

$$\omega \approx \Omega$$

and, as a result, the digital filter has the same frequency response as the analog filter over this frequency range.

• • •

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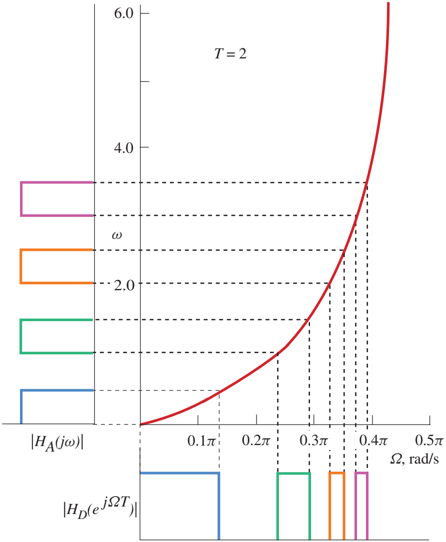
  and, as a result, the digital filter has the same frequency response as the analog filter over this frequency range.

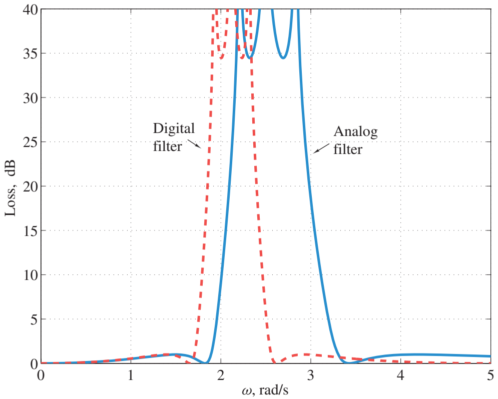
- For higher frequencies, however, the relation between ω and Ω becomes nonlinear, and *distortion is introduced in the frequency scale of the digital filter relative to that of the analog filter*.

  This is known as the *warping effect*.

The Warping Effect *Cont'd*



The warping effect changes the band edges of the digital filter relative to those of the analog filter in a nonlinear way, as illustrated for the case of a BS filter:



Prewarping

- From Eq. (B), i.e.,

$$\omega = \frac{2}{T} \tan \frac{\Omega T}{2} \qquad \text{(B)}$$

  a frequency ω in the analog filter corresponds to a frequency Ω in the digital filter and hence

$$\Omega = \frac{2}{T} \tan^{-1} \frac{\omega T}{2}$$

Prewarping

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a frequency ω in the analog filter corresponds to a frequency Ω in the digital filter and hence

$$\Omega = \frac{2}{T} \tan^{-1} \frac{\omega T}{2}$$

- If $\omega_1, \omega_2, \ldots, \omega_i, \ldots$ are the passband and stopband edges in the analog filter, then the corresponding passband and stopband edges in the derived digital filter are given by

$$\Omega_i = \frac{2}{T} \tan^{-1} \frac{\omega_i T}{2} \qquad i = 1, 2, \ldots$$

- If prescribed passband and stopband edges $\tilde{\Omega}_1$, $\tilde{\Omega}_2, \ldots, \tilde{\Omega}_i, \ldots$ are to be achieved, the analog filter must be prewarped before the application of the bilinear transformation to ensure that its band edges are given by

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$$\omega_i = \frac{2}{T} \tan \frac{\tilde{\Omega}_i T}{2}$$

- Then the band edges of the digital filter would assume their prescribed values Ω_i since

$$\begin{aligned} \Omega_i &= \frac{2}{T} \tan^{-1} \frac{\omega_i T}{2} \\ &= \frac{2}{T} \tan^{-1} \left(\frac{T}{2} \cdot \frac{2}{T} \tan \frac{\tilde{\Omega}_i T}{2} \right) \\ &= \tilde{\Omega}_i \quad \text{for} \;\; i = 1, 2, \ldots \end{aligned}$$

Design Procedure

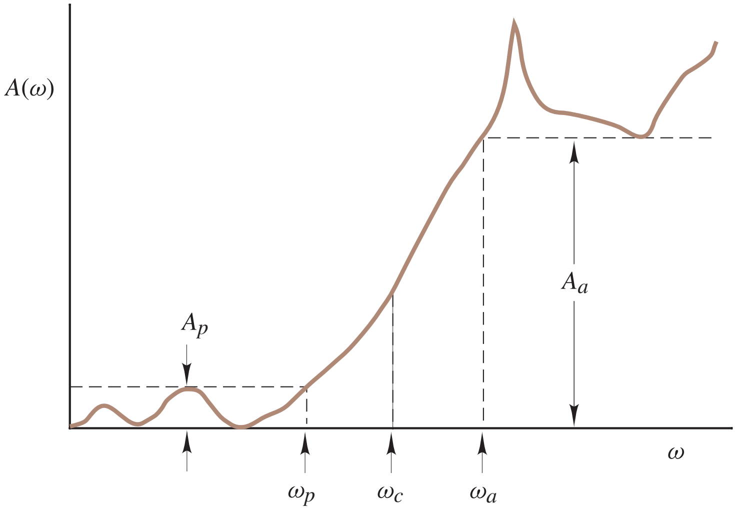
Consider a normalized analog LP filter characterized by $H_N(s)$ with an attenuation

$$A_N(\omega) = 20 \log \frac{1}{|H_N(j\omega)|}$$

(also known as loss) and assume that

$$0 \leq A_N(\omega) \leq A_p \quad \text{for} \;\; 0 \leq |\omega| \leq \omega_p$$
$$A_N(\omega) \geq A_a \quad \text{for} \;\; \omega_a \leq |\omega| \leq \infty$$

Note: The transfer functions of analog LP filters are reported in the literature in normalized form whereby the passband edge is typically of the order of unity.



$A(\omega)$
A_p
A_a
ω
ω_p
ω_c
ω_a

Design Procedure

A denormalized LP, HP, BP, or BS filter that has the same passband ripple and minimum stopband attenuation as a given normalized LP filter can be derived from the normalized LP filter through the following steps:

1. Apply the transformation $s = f_X(\bar{s})$

$$H_X(\bar{s}) = H_N(s)\Big|_{s=f_X(\bar{s})}$$

   where $f_X(\bar{s})$ is one of the four standard analog-filters transformations, given by the next slide.

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$$H_X(\bar{s}) = H_N(s)\Big|_{s=f_X(\bar{s})}$$

   where $f_X(\bar{s})$ is one of the four standard analog-filters transformations, given by the next slide.

2. Apply the bilinear transformation to $H_X(\bar{s})$, i.e.,

$$H_D(z) = H_X(\bar{s})\Big|_{\bar{s}=\frac{2}{T}\left(\frac{z-1}{z+1}\right)}$$

Standard forms of $f_X(\bar{s})$

X	$f_X(\bar{s})$
LP	$\lambda\bar{s}$
HP	$\lambda/\bar{s}$
BP	$\frac{1}{B}\left(\bar{s}+\dfrac{\omega_0^2}{\bar{s}}\right)$
BS	$\dfrac{B\bar{s}}{\bar{s}^2+\omega_0^2}$

- The digital filter designed by this method will have the required passband and stopband edges only if the parameters λ, ω_0, and B of the analog-filter transformations and the order of the continuous-time normalized LP transfer function, $H_N(s)$, are chosen appropriately.

Design Procedure *Cont'd*

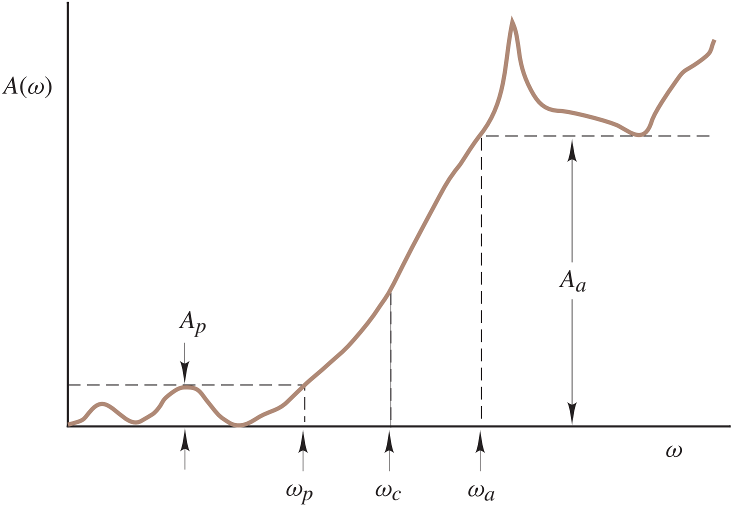
- The digital filter designed by this method will have the required passband and stopband edges only if the parameters λ, ω_0, and B of the analog-filter transformations and the order of the continuous-time normalized LP transfer function, $H_N(s)$, are chosen appropriately.
- This is obviously a difficult problem but general solutions are available for LP, HP, BP, and BS, Butterworth, Chebyshev, inverse-Chebyshev, and Elliptic filters.

General Design Procedure for LP Filters

An outline of the methodology for the derivation of general solutions for LP filters is as follows:

1. Assume that a continuous-time normalized LP transfer function, $H_N(s)$, is available that would give the required passband ripple, A_p, and minimum stopband attenuation (loss), A_a.

   Let the passband and stopband edges of the analog filter be ω_p and ω_a, respectively.



Attenuation characteristic of continuous-time normalized LP filter

2. Apply the LP-to-LP analog-filter transformation to $H_N(s)$ to obtain a denormalized discrete-time transfer function $H_{LP}(\bar{s})$.

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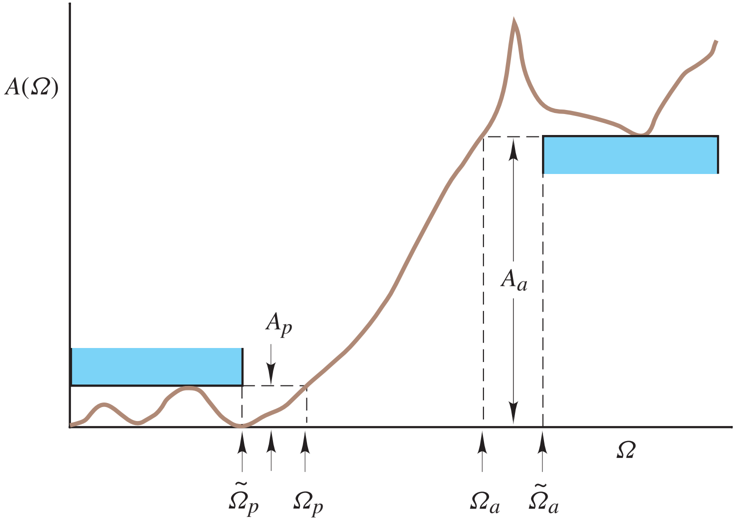
3. Apply the bilinear transformation to $H_{LP}(\bar{s})$ to obtain a discrete-time transfer function $H_D(z)$.
4. At this point, assume that the derived discrete-time transfer function has passband and stopband edges that satisfy the relations

$$\tilde{\Omega}_p \leq \Omega_p \quad \text{and} \quad \Omega_a \leq \tilde{\Omega}_a$$

where $\tilde{\Omega}_p$ and $\tilde{\Omega}_a$ are the *prescribed* passband and stopband edges, respectively.

In effect, we assume that the digital filter has passband and stopband edges that *satisfy* or *oversatisfy* the required specifications.



Attenuation characteristic of required LP digital filter

5. Solve for λ, the parameter of the LP-to-LP analog-filter transformation.

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6. Find the minimum value of the ratio ω_p/ω_a for the continuous-time normalized LP transfer function.

   The ratio ω_p/ω_a is a fraction less than unity and it is a measure of the steepness of the transition characteristic. It is often referred to as the *selectivity* of the filter.

   The selectivity of a filter *dictates the minimum order* to achieve the required specifications.

   Note: As the selectivity approaches unity, the filter-order tends to infinity!

7. The same methodology is applicable for HP filters, except that the LP-HP analog-filter transformation is used in Step 2.

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8. The application of this methodology yields the formulas summarized by the table shown in the next slide.

Formulas for LP and HP Filters

LP	$\dfrac{\omega_p}{\omega_a} \ge K_0$ $\lambda = \dfrac{\omega_p T}{2\tan(\tilde{\Omega}_p T/2)}$
HP	$\dfrac{\omega_p}{\omega_a} \ge \dfrac{1}{K_0}$ $\lambda = \dfrac{2\omega_p \tan(\tilde{\Omega}_p T/2)}{T}$
where	$K_0 = \dfrac{\tan(\tilde{\Omega}_p T/2)}{\tan(\tilde{\Omega}_a T/2)}$

Design of LP Filters *Cont'd*

- The table of formulas presented is applicable to all the classical types of analog filters, namely,
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  - Chebyshev
  - Inverse-Chebyshev
  - Elliptic

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  - Elliptic
- Formulas that can be used to design digital versions of these filters will be presented later.

An outline of the methodology for the derivation of general solutions for BP filters is as follows:

1. Assume that a continuous-time normalized LP transfer function, $H_N(s)$, is available that would give the required passband ripple, A_p, and minimum stopband attenuation, A_a.

   Let the passband and stopband edges of the analog filter be ω_p and ω_a, respectively.

2. Apply the LP-to-BP analog-filter transformation to $H_N(s)$ to obtain a denormalized discrete-time transfer function $H_{BP}(\bar{s})$.

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3. Apply the bilinear transformation to $H_{BP}(\bar{s})$ to obtain a discrete-time transfer function $H_D(z)$.

4. At this point, assume that the derived discrete-time transfer function has passband and stopband edges that satisfy the relations

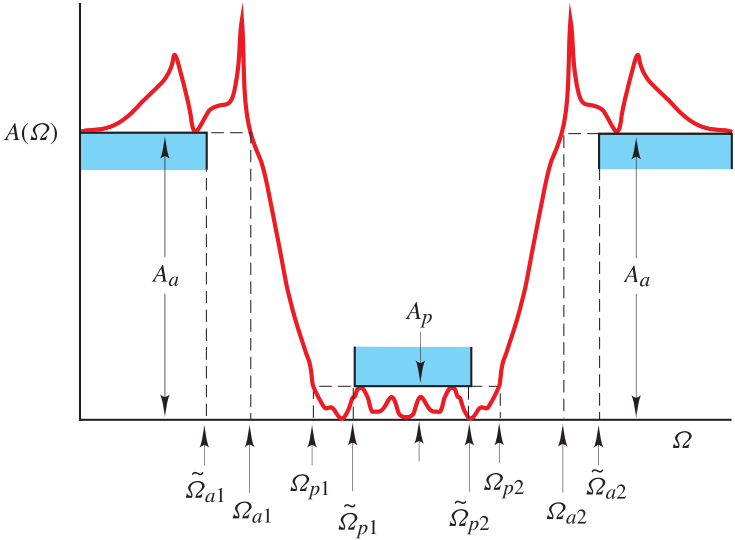
$$\Omega_{p1} \leq \tilde{\Omega}_{p1} \quad \Omega_{p2} \geq \tilde{\Omega}_{p2}$$

and

$$\Omega_{a1} \geq \tilde{\Omega}_{a1} \quad \Omega_{a2} \leq \tilde{\Omega}_{a2}$$

where

- Ω_{p1} and Ω_{p2} are the actual lower and upper passband edges,
- $\tilde{\Omega}_{p1}$ and $\tilde{\Omega}_{p2}$ are the *prescribed* lower and upper passband edges,
- Ω_{a1} and Ω_{a2} are the actual lower and upper stopband edges,
- $\tilde{\Omega}_{p1}$ and $\tilde{\Omega}_{p2}$ are the *prescribed* lower and upper stopband edges, respectively.



Attenuation characteristic of required BP digital filter

5. Solve for B and ω_0, the parameters of the LP-to-BP analog-filter transformation.

Design of LP Filters *Cont'd*

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7. The same methodology can be used for the design of BS filters except that the LP-to-BS transformation is used in Step 2.
8. The application of this methodology yields the formulas summarized in the next two slides.

Formulas for the design of BP Filters

BP

$$\omega_0 = \frac{2\sqrt{K_B}}{T}$$

$$\frac{\omega_p}{\omega_a} \geq \begin{cases} K_1 & \text{if } K_C \geq K_B \\ K_2 & \text{if } K_C < K_B \end{cases}$$

$$B = \frac{2K_A}{T\omega_p}$$

where

$$K_A = \tan\frac{\tilde{\Omega}_{p2}T}{2} - \tan\frac{\tilde{\Omega}_{p1}T}{2} \qquad K_B = \tan\frac{\tilde{\Omega}_{p1}T}{2}\tan\frac{\tilde{\Omega}_{p2}T}{2}$$

$$K_C = \tan\frac{\tilde{\Omega}_{a1}T}{2}\tan\frac{\tilde{\Omega}_{a2}T}{2} \qquad K_1 = \frac{K_A\tan(\tilde{\Omega}_{a1}T/2)}{K_B - \tan^2(\tilde{\Omega}_{a1}T/2)}$$

$$K_2 = \frac{K_A\tan(\tilde{\Omega}_{a2}T/2)}{\tan^2(\tilde{\Omega}_{a2}T/2) - K_B}$$

Formulas for the Design of BS Filters

BS	$\omega_0 = \dfrac{2\sqrt{K_B}}{T}$
	$\dfrac{\omega_p}{\omega_a} \geq \begin{cases} \dfrac{1}{K_2} & \text{if } K_C \geq K_B \\ \dfrac{1}{K_1} & \text{if } K_C < K_B \end{cases}$
	$B = \dfrac{2K_A\omega_p}{T}$

where $K_A = \tan\dfrac{\tilde{\Omega}_{p2}T}{2} - \tan\dfrac{\tilde{\Omega}_{p1}T}{2}$ $\quad K_B = \tan\dfrac{\tilde{\Omega}_{p1}T}{2}\tan\dfrac{\tilde{\Omega}_{p2}T}{2}$

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$K_2 = \dfrac{K_A\tan(\tilde{\Omega}_{a2}T/2)}{\tan^2(\tilde{\Omega}_{a2}T/2) - K_B}$

Formulas for ω_p and n

- The formulas presented apply to any type of normalized analog LP filter with an attenuation that would satisfy the following conditions:

$$0 \le A_N(\omega) \le A_p \quad \text{for } \ 0 \le |\omega| \le \omega_p$$
$$A_N(\omega) \ge A_a \quad \text{for } \ \omega_a \le |\omega| \le \infty$$

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- However, the values of the normalized passband edge, ω_p, and the required filter order, n, depend on the type of filter.

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- However, the values of the normalized passband edge, ω_p, and the required filter order, n, depend on the type of filter.
- Formulas for these parameters for Butterworth, Chebyshev, and Elliptic filters are presented in the next three slides.

Formulas for Butterworth Filters

LP	$K = K_0$
HP	$K = \dfrac{1}{K_0}$
BP	$K = \begin{cases} K_1 & \text{if } K_C \geq K_B \\ K_2 & \text{if } K_C < K_B \end{cases}$
BS	$K = \begin{cases} \dfrac{1}{K_2} & \text{if } K_C \geq K_B \\ \dfrac{1}{K_1} & \text{if } K_C < K_B \end{cases}$

$$n \geq \frac{\log D}{2\log(1/K)}, \quad D = \frac{10^{0.1A_a} - 1}{10^{0.1A_p} - 1}$$

$$\omega_p = (10^{0.1A_p} - 1)^{1/2n}$$

Formulas for Chebyshev Filters

LP	$K = K_0$
HP	$K = \dfrac{1}{K_0}$
BP	$K = \begin{cases} K_1 & \text{if } K_C \geq K_B \\ K_2 & \text{if } K_C < K_B \end{cases}$
BS	$K = \begin{cases} \dfrac{1}{K_2} & \text{if } K_C \geq K_B \\ \dfrac{1}{K_1} & \text{if } K_C < K_B \end{cases}$

$$n \geq \frac{\cosh^{-1}\sqrt{D}}{\cosh^{-1}(1/K)}, \quad D = \frac{10^{0.1A_a} - 1}{10^{0.1A_p} - 1}$$

$$\omega_p = 1$$

Formulas for Elliptic Filters

	k	ω_p
LP	$K = K_0$	$\sqrt{K_0}$
HP	$K = \dfrac{1}{K_0}$	$\dfrac{1}{\sqrt{K_0}}$
BP	$K = \begin{cases} K_1 & \text{if } K_C \geq K_B \\ K_2 & \text{if } K_C < K_B \end{cases}$	$\sqrt{K_1}$ $\sqrt{K_2}$
BS	$K = \begin{cases} \dfrac{1}{K_2} & \text{if } K_C \geq K_B \\ \dfrac{1}{K_1} & \text{if } K_C < K_B \end{cases}$	$\dfrac{1}{\sqrt{K_2}}$ $\dfrac{1}{\sqrt{K_1}}$

$$n \geq \frac{\cosh^{-1}\sqrt{D}}{\cosh^{-1}(1/K)}, \qquad D = \frac{10^{0.1A_a} - 1}{10^{0.1A_p} - 1}$$

Example – HP Filter

An HP filter that would satisfy the following specifications is required:

$$A_p = 1 \text{ dB}, \quad A_a = 45 \text{ dB}, \quad \tilde{\Omega}_p = 3.5 \text{ rad/s},$$

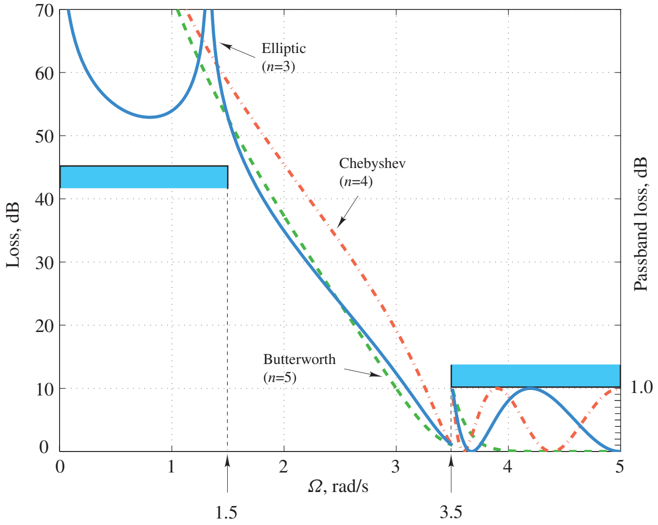
$$\tilde{\Omega}_a = 1.5 \text{ rad/s}, \quad \omega_s = 10 \text{ rad/s}.$$

Design a Butterworth, a Chebyshev, and then an Elliptic digital filter.

Solution

Filter type	n	ω_p	λ
Butterworth	5	0.873610	5.457600
Chebyshev	4	1.0	6.247183
Elliptic	3	0.509526	3.183099

Example *Cont'd*



Example – BP Filter

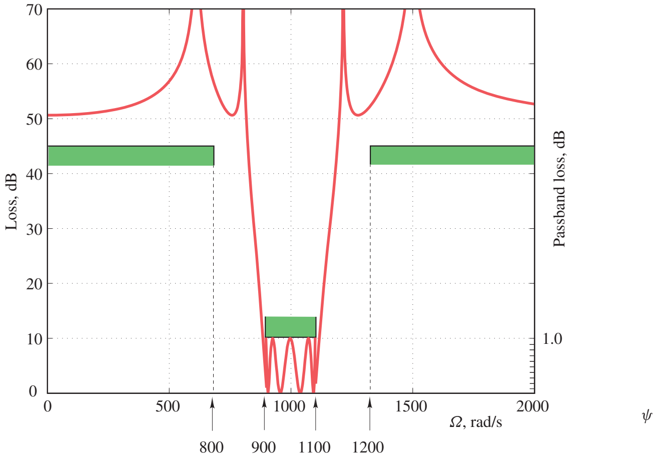
Design an Elliptic BP filter that would satisfy the following specifications:

$$A_p = 1 \text{ dB}, \quad A_a = 45 \text{ dB}, \quad \tilde{\Omega}_{p1} = 900 \text{ rad/s}, \quad \tilde{\Omega}_{p2} = 1100 \text{ rad/s},$$
$$\tilde{\Omega}_{a1} = 800 \text{ rad/s}, \quad \tilde{\Omega}_{a2} = 1200 \text{ rad/s}, \quad \omega_s = 6000 \text{ rad/s}.$$

Solution

$$\begin{aligned} k &= 0.515957 \\ \omega_p &= 0.718302 \\ n &= 4 \\ \omega_0 &= 1,098.609 \\ B &= 371.9263 \end{aligned}$$

Example – BP Filter *Cont'd*



Example – BS Filter

Design a Chebyshev BS filter that would satisfy the following specifications:

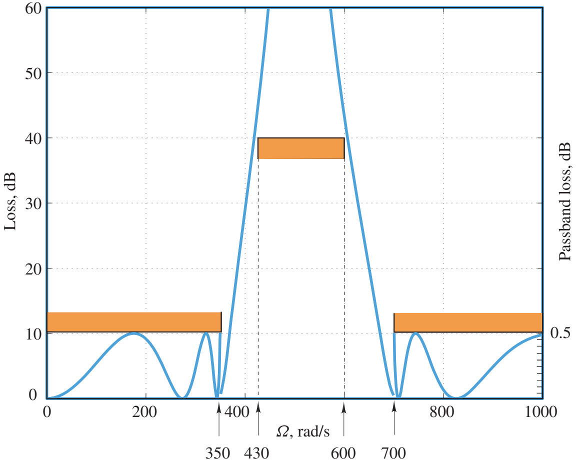
$$A_p = 0.5\,\text{dB}, \quad A_a = 40\,\text{dB}, \quad \tilde{\Omega}_{p1} = 350\,\text{rad/s}, \quad \tilde{\Omega}_{p2} = 700\,\text{rad/s},$$

$$\tilde{\Omega}_{a1} = 430\,\text{rad/s}, \quad \tilde{\Omega}_{a2} = 600\,\text{rad/s}, \quad \omega_s = 3000\,\text{rad/s}.$$

Solution

$$\begin{aligned} \omega_p &= 1.0 \\ n &= 5 \\ \omega_0 &= 561,4083 \\ B &= 493,2594 \end{aligned}$$

Example – BS Filter *Cont'd*



D-Filter

A DSP software package that incorporates the design techniques described in this presentation is *D-Filter*. Please see

http://www.d-filter.ece.uvic.ca

for more information.

Summary

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- The method requires *very little computation* and leads to very precise *optimal* designs.
- It can be used to design LP, HP, BP, and BS filters of the Butterworth, Chebyshev, Inverse-Chebyshev, Elliptic types.

Summary

- A design method for IIR filters that leads to a complete description of the transfer function *in closed form* either in terms of its zeros and poles or its coefficients has been described.
- The method requires *very little computation* and leads to very precise *optimal* designs.
- It can be used to design LP, HP, BP, and BS filters of the Butterworth, Chebyshev, Inverse-Chebyshev, Elliptic types.
- All these designs can be carried out by using DSP software package D-Filter.

References

- A. Antoniou, *Digital Signal Processing: Signals, Systems, and Filters*, Chap. 15, McGraw-Hill, 2005.

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This slide concludes the presentation.
Thank you for your attention.