A New Normalized Minimum Error Entropy Algorithm with Reduced Computational Complexity

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Outline

- Mean-Square Error Criterion

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- Entropy-of-Error Criterion
  - Minimum Entropy-of-Error (MEE) Algorithm (Erdogmus and Principe)
  - Minimum Entropy-of-Error with Self-Adjusting Step Size (MEE-SAS) Algorithm (Han, Erdogmus, and Principe)
  - Normalized Minimum Entropy-of-Error (NMEE) Algorithm (Han, Rao, Jeong, and Principe)
  - Comparison

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  - Comparison
- Proposed NMEE Algorithm
- Simulation Results
- Conclusions

Mean-Square Error Criterion *Cont'd*

- Algorithms for conventional adaptive filters that use the MSE criterion yield an *optimal solution* only if the input signal is *Gaussian* and the desired signal is *a linear function* of the input signal.

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- Often in practice, the input signal *is not Guassian* and/or the desired signal is *not a linear function* of the input signal.
- Under these circumstances, better performance can be achieved by using algorithms based on *information theoretic* criteria, which can extract information based on higher-order statistics.

Information-Potential Function Criterion

- In MEE adaptive filters, the so-called *information potential function* of the error signal is maximized in order to achieve *maximum mutual information content* between the input and output signals.

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- In MEE adaptive filters, the so-called *information potential function* of the error signal is maximized in order to achieve *maximum mutual information content* between the input and output signals.
- A discretized estimate of the information potential function over k samples can be constructed as

$$V_k(x) = \frac{1}{(k)^2} \sum_{j=1}^{k} \sum_{i=1}^{k} \kappa_\sigma(x_j - x_i)$$

where $\kappa_\sigma(x)$ is the *Gaussian kernel*.

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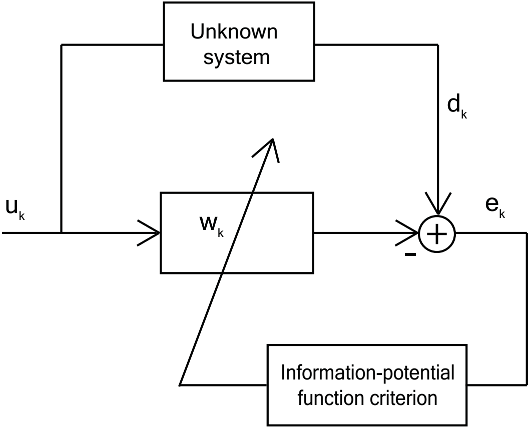
Adaptive Filter Based on Information-Potential Function Criterion



Basic MEE Algorithm

- A recursive update equation for the information potential of the error can be constructed as

$$V_k(e) = \frac{1}{k^2}\left\{(k-1)^2 V_{k-1}(e) + 2\left[\sum_{i=1}^{k-1} \kappa_\sigma(e_k - e_i) + \kappa_\sigma(0)\right]\right\}$$

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- In order to control the increasing memory requirement in the above estimator, a *forgetting factor* is introduced as

$$V_k(e) = (1-\lambda)V_{k-1}(e) + \frac{\lambda}{L}\sum_{i=k-L}^{k-1} \kappa_\sigma(e_k - e_i)$$

where L is the so-called *Parzen window length*.

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Basic MEE Algorithm *Cont'd*

- If we now use a forgetting factor $\lambda = 1$, the information potential simplifies to

$$V_k(e) = \frac{1}{L} \sum_{i=k-L}^{k-1} \kappa_\sigma(e_k - e_i)$$

Basic MEE Algorithm *Cont'd*

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$$V_k(e) = \frac{1}{L} \sum_{i=k-L}^{k-1} \kappa_\sigma(e_k - e_i)$$

- The weight update equation assumes the form

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu \boldsymbol{\nabla} V_k(e)$$

where μ is the *step size* and

$$\boldsymbol{\nabla} V_k(e) = \frac{1}{L} \sum_{i=k-L}^{k-1} \kappa_\sigma(e_k - e_i)(\mathbf{u}_k - \mathbf{u}_i)$$

is the gradient of $V_k(e)$. ($\mathbf{u}_k$ and $\mathbf{u}_i$ are input vectors.)

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MEE Algorithm with Self-Adjusting Step Size

- An MEE adaptive filter with self-adjusting step size solves the optimization problem

$$\underset{\mathbf{w}}{\text{minimize}}\ [V(0) - V(e)]^2$$

where $V(0)$ is the value of $V_k(e)$ at $e = 0$, i.e., this is the *maximum value* of $V_k(e)$.

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- The update equation for the coefficient vector is

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu[V(0) - V_k(e)]\boldsymbol{\nabla} V_k(e)$$

and the gradient of $V_k(e)$, $\boldsymbol{\nabla} V_k(e)$, is the same as for the basic MEE algorithm.

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Normalized MEE Algorithm

- The NMEE algorithm solves the optimization problem

$$\underset{\mathbf{w}}{\text{minimize}} \; ||\mathbf{w}_{k+1} - \mathbf{w}_k||^2$$

$$\text{subject to: } V(e_{p,k}) - V(0) = 0$$

where $V(e_{p,k})$ is the *a posteriori* information potential function.

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- The update equation for the weight vector assumes the form

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu \frac{e_{a,k} \boldsymbol{\nabla} V(e_{p,k})}{\boldsymbol{\nabla} V(e_{p,k})^T \mathbf{x}_k}$$

where $e_{a,k} = d_k - \mathbf{w}_k^T \mathbf{u}_k$ is the *a priori* error,

$$\boldsymbol{\nabla} V(e_{p,k}) = \frac{1}{2L\sigma^2} \sum_{i=k-L}^{k-1} (e_{p,k} - e_{p,i}) \, k_\sigma(e_{p,k} - e_{p,i}) \, (\mathbf{u}_k - \mathbf{u}_i)$$

and $e_{p,i} = d_i - \mathbf{w}_{k+1}^T \mathbf{u}_i$ is the *a posteriori* error.

Comparison

- The performance of the basic MEE algorithm depends on the power of the input signal and the kernel size σ.

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- The MEE-SAS and NMEE algorithms *are faster and yield reduced residual error* relative to that of the basic MEE algorithm.

- *Increased computational complexities* are associated with the MEE-SAS and NMEE algorithms.

- The performance of the NMEE algorithm is not affected by changes in the input signal power and is not sensitive to changes in the kernel size.

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Proposed NMEE Algorithm

- The proposed NMEE algorithm is derived by minimizing the *Euclidean distance* between the weight vectors at iterations k and $k+1$ subject to the constraint that the *a posteriori* information potential function is equal to V(0).

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- The associated optimization problem can be stated as

$$\underset{\mathbf{z}}{\text{minimize}} \; ||\mathbf{w}_{k+1} - \mathbf{w}_k||^2$$

$$\text{subject to: } \; V(0) - V(e_{p,k}) = 0$$

where

$$V(e_{p,k}) = \frac{1}{L} \sum_{i=k-L}^{k-1} k_\sigma(e_{p,k} - e_{p,i})$$

is the *a posteriori* information potential function.

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Proposed NMEE Algorithm *Cont'd*

- Solving the associated optimization problem, the update equation can be obtained as

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \frac{\sum_{i=k-L}^{k-1} \nu_{k,i}\mathbf{u}_{k,i} \sum_{i=k-L}^{k-1}(e_{a,k} - e_{a,i})}{\left(\sum_{i=k-L}^{k-1} \nu_{k,i}\mathbf{u}_{k,i}\right)^T \left(\sum_{i=k-L}^{k-1} \mathbf{u}_{k,i}\right)}$$

where $\mathbf{u}_{k,i} = \mathbf{u}_k - \mathbf{u}_i$ and $\nu_{k,i} = (e_{p,k} - e_{p,i})k_\sigma(e_{p,k} - e_{p,i})$.

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- For a sufficiently small kernel size and window length, the scaling factor $\nu_{k,i}$ can be assumed to be constant over the Parzen window and, therefore, it can be factored out in the numerator and denominator to obtain the *simplified update equation*

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \frac{\bar{\mathbf{u}}_{k,i}}{\bar{\mathbf{u}}_{k,i}^T \bar{\mathbf{u}}_{k,i}} \bar{e}_{a,k,i}$$

where $\bar{\mathbf{u}}_{k,i} = \sum_{i=k-L}^{k-1} \mathbf{u}_{k,i}$, and $\bar{e}_{a,k,i} = \sum_{i=k-L}^{k-1} (e_{a,k} - e_{a,i})$.

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Proposed NMEE Algorithm *Cont'd*

- A step size μ can be introduced in the update equation as

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which will ensure that *the maximum value* of the information potential function is achieved.

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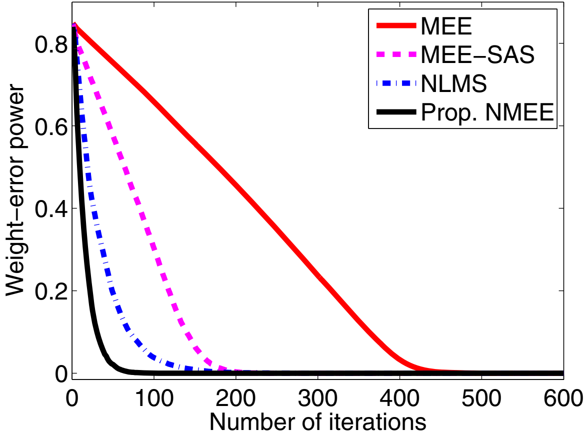
- The simplified update equation *reduces the computational complexity* significantly.

Simulation Results

- Convergence plots for a plant identification problem:



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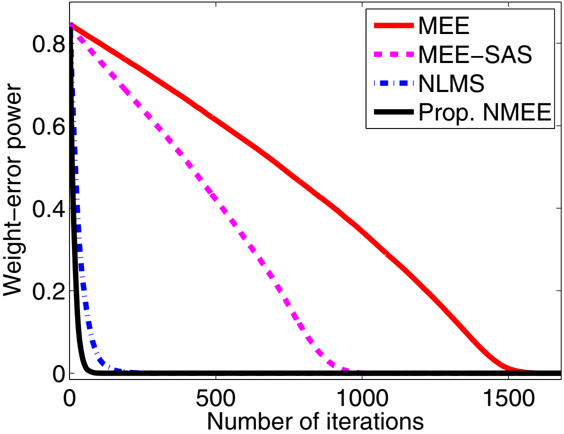
- Convergence plots with the input power increased by a factor of 10:



Simulation Results *Cont'd*

- Residual error and required number of iterations for a white input signal (1p=unit power, 10p=10 times unit power)

	MEE	MEE-SAS	NLMS	Prop. NMEE
Res. Err.	1.115×10^{-25}	1.888×10^{-29}	2.221×10^{-14}	3.741×10^{-32}
Iters., 1p	450	200	200	90
Res. Err.	1.944×10^{-7}	2.675×10^{-31}	2.211×10^{-28}	3.833×10^{-32}
Iters., 10p	1550	950	280	100

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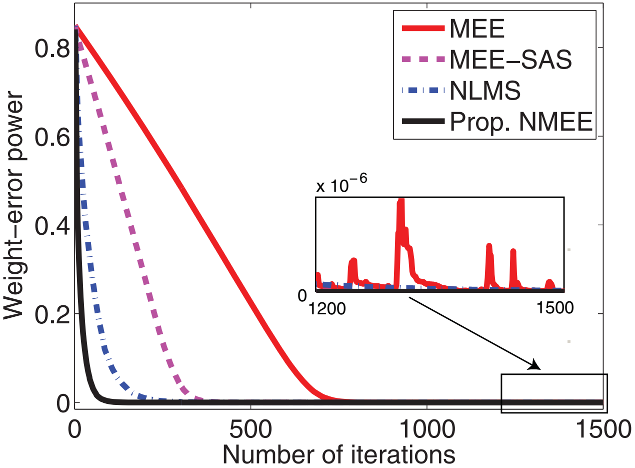
Simulation Results *Cont'd*

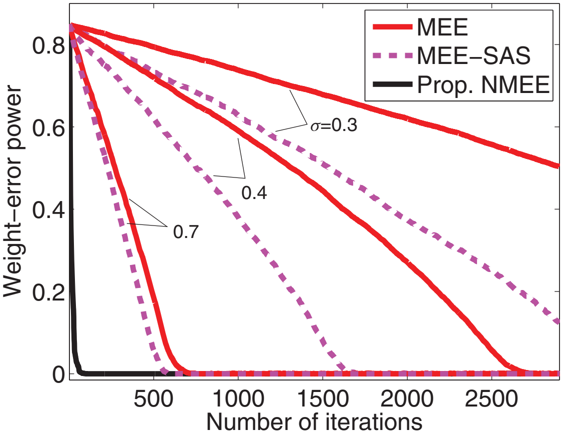
- Convergence plots for a color input signal of unit power:



- Residual error for colored input signal of unit power:

	MEE	MEE-SAS	NLMS	Prop. NMEE
Res. Err.	2.343×10^{-8}	1.389×10^{-11}	1.033×10^{-7}	3.474×10^{-32}

- Convergence plots for different kernel sizes for a color input signal with the power level increased by a factor of 10:



- Residual error for a colored input signal with the power increased by a factor of 10:

kernel size	MEE	MEE-SAS	Prop. NMEE
0.4	1.061×10^{-3}	1.129×10^{-3}	2.960×10^{-32}
0.7	4.937×10^{-4}	1.720×10^{-5}	3.192×10^{-32}

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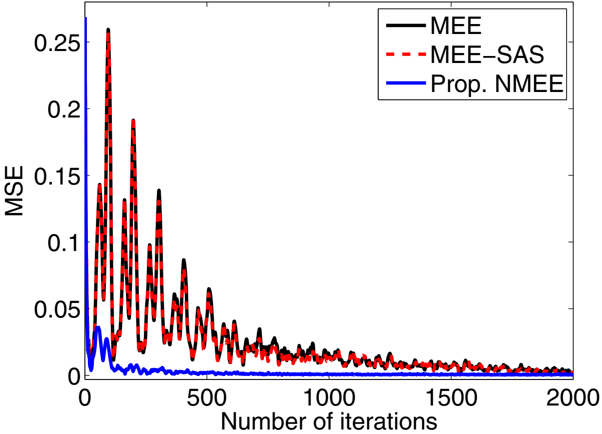
- Convergence plots for a signal prediction application: We used an FIR filter of length 6 to predict a Mackey-Glass time series with delay parameter $\tau = 30$.



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- It offers *reduced computational complexity* as well as *better tracking* performance.
- It yields *reduced residual error* relative to the known MEE algorithms.
- It offers improved performance for the case of a color input signal as it reuses the past L input signal vectors and the corresponding error signals in every iteration.

Thank you for your attention.